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Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

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PROBLEMS FOR CHAPTER 6 Dynamic Games with Asymmetric Information

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This appendix contains answers to the odd-numbered problems in the gourth edition of *Games and Information* by Eric Rasmusen, which I am working on now and perhaps will come out in 2005. The answers to the even- numbered problems are available to instructors or self-studiers on request to me at Erasmuse@indiana.edu.

Other books which contain exercises with answers include Bierman & Fernandez (1993), Binmore (1992), Fudenberg & Tirole (1991a), J. Hirshleifer & Riley (1992), Moulin (1986), and Gintis (2000). I must ask pardon of any authors from whom I have borrowed without attribution in the problems below; these are the descendants of problems that I wrote for teaching without careful attention to my sources.

PROBLEMS FOR CHAPTER 6 Dynamic Games with Asymmetric Information

6.1. Cournot Duopoly Under Incomplete Information About Costs
This problem introduces incomplete information into the Cournot model of Chapter 3 and allows for a continuum of player types.

(a) Modify the Cournot Game of Chapter 3 by specifying that Apex's average cost of production is c per unit, while Brydox' remains zero. What are the outputs of each firm if the costs are common knowledge? What are the numerical values if $c = 10$?

<u>Answer.</u> The payoff functions are

$$\begin{array}{l} \pi_{Apex} = (120 - q_a - q_b - c)q_a \\ \pi_{Brydox} = (120 - q_a - q_b - c)q_b \end{array} \quad (1)$$

The first order conditions are then

$$\begin{array}{l} \frac{\partial \pi_{Apex}}{\partial q_a} = 120 - 2q_a - q_b - c = 0 \\ \frac{\partial \pi_{Brydox}}{\partial q_b} = 120 - q_a - 2q_b = 0 \end{array} \quad (2)$$

Solving the first order conditions together gives

$$\begin{array}{l} q_a = 40 - \frac{2c}{3} \\ q_b = 40 + \frac{c}{3} \end{array} \quad (3)$$

If $c = 10$, Apex produces 33 1/3 and Brydox produces 43 1/3. Apex's higher costs make it cut back its output, which encourages Brydox to produce more.

(b) Let Apex's cost c be c_{max} with probability θ and 0 with probability $(1 - \theta)$, so Apex is one of two types. Brydox does not know Apex's type. What are the outputs of each firm?

<u>Answer.</u> Apex's payoff function is the same as in part (a), because

$$\pi_{Apex} = (120 - q_a - q_b - c)q_a, \quad (4)$$

which yields the reaction function

$$q_a = 60 - \frac{q_b + c}{2}. \quad (5)$$

Brydox's expected payoff is

$$\pi_{Brydox} = (1 - \theta)(120 - q_a(c = 0) - q_b)q_b + \theta(120 - q_a(c = c_{max}) - q_b)q_b. \quad (6)$$

The first order condition is

$$\frac{\partial \pi_{Brydox}}{\partial q_b} = (1 - \theta)(120 - q_a(c = 0) - 2q_b) + \theta(120 - q_a(c = c_{max}) - 2q_b) = 0. \quad (7)$$

Now substitute the reaction function of Apex, equation (5), into (7) and condense a few terms to obtain

$$120 - 2q_b - [1-\theta][60 - \frac{q_b + 0}{2}] - \theta[60 - \frac{q_b + c_{max}}{2}] = 0. \tag{8}$$

Solving for q_b yields

$$q_b = 40 + \frac{\theta c_{max}}{3} \tag{9}$$

One can then use equations (5) and (9) to find

$$q_a = 40 - \frac{\theta c_{max}}{6} - \frac{c}{2}. \tag{10}$$

Note that the outputs do not depend on θ or c_{max} separately, only on the expected value of Apex's cost, θc_{max}.

(c) Let Apex's cost c be drawn from the interval $[0, c_{max}]$ using the uniform distribution, so there is a continuum of types. Brydox does not know Apex's type. What are the outputs of each firm?

<u>*Answer.*</u> Apex's payoff function is the same as in parts (a) and (b),

$$\pi_{Apex} = (120 - q_a - q_b - c)q_a, \tag{11}$$

which yields the reaction function

$$q_a = 60 - \frac{q_b + c}{2}. \tag{12}$$

Brydox's expected payoff is (letting the density of possible values of c be $f(c)$)

$$\pi_{Brydox} = \int_0^{c_{max}} (120 - q_a(c) - q_b)q_b f(c)dc. \tag{13}$$

The probability density is uniform, so $f(c) = \frac{1}{c_{max}}$. Substituting this into (13), the first order condition is

$$\frac{\partial \pi_{Brydox}}{\partial q_b} = \int_0^{c_{max}} (120 - q_a(c) - 2q_b)\left(\frac{1}{c_{max}}\right) dc = 0. \tag{14}$$

Now substitute in the reaction function of Apex, equation (12), which gives

$$\int_0^{c_{max}} (120 - [60 - \frac{q_b + c}{2}] - 2q_b)\left(\frac{1}{c_{max}}\right) dc = 0. \tag{15}$$

Simplifying by integrating out the terms in (15) which depend on c only through the probability density yields

$$60 - \frac{3q_b}{2} + \int_0^{c_{max}} \left(\frac{c}{2c_{max}}\right) dc = 0. \tag{16}$$

Integrating and rearranging yields

$$q_b = 40 + \frac{c_{max}}{6} \tag{17}$$

One can then use equations (12) and (17) to find

$$q_a = 40 - \frac{c_{max}}{12} - \frac{c}{2}. \tag{18}$$

(d) Outputs were 40 for each firm in the zero- cost game in Chapter 3. Check your answers in parts (b) and (c) by seeing what happens if $c_{max} = 0$.

Answer. If $c_{max} = 0$, then in part (b), $q_a = 40 - \frac{0}{6} - \frac{0}{2} = 40$ and $q_b = 40 + \frac{0}{3} = 40$, which is as it should be.

If $c_{max} = 0$, then in part (c), $q_a = 40 - \frac{0}{12} - \frac{0}{2} = 40$ and $q_b = 40 + \frac{0}{6} = 40$, which is as it should be.

(e) Let $c_{max} = 20$ and $\theta = 0.5$, so the expectation of Apex's average cost is 10 in parts (a), (b), and (c) . What are the average outputs for Apex in each case?

Answer. In part (a), under full information, the outputs were $q_a = 33\ 1/3$ and $q_b = 43\ 1/3$. In part (b), with two types, $q_b = 43\ 1/3$ from equation (9), and the average value of q_a is

$$Eq_a = (1-\theta)(40 - \frac{0.5(20)}{6} - \frac{0}{2}) + \theta(40 - \frac{0.5(20)}{6} - \frac{20}{2}) = 33\ \ 1/3. \tag{19}$$

In part (c), with a continuum of types, $q_b = 43\ \ 1/3$ and q_a is found from

$$\begin{aligned} Eq_a = & \int_0^{c_{max}} (40 - \tfrac{c_{max}}{8} - \tfrac{c}{2}) \left(\tfrac{1}{c_{max}}\right) dc \\ & = 40 - \tfrac{20}{8} - \tfrac{c_{max}^2}{4c_{max}} = 33\ \ 1/3. \end{aligned} \tag{20}$$

(f) Modify the model of part (b) so that $c_{max} = 20$ and $\theta = 0.5$, but somehow $c = 30$. What outputs do your formulas from part (b) generate? Is there anything this could sensibly model?

Answer. The purpose of Nature's move is to represent Brydox's beliefs about Apex, not necessarily to represent reality. Here, Brydox believes that Apex's costs are either 0 or 20 but he is wrong and they are actually 30. In this game that does not cause problems for the analysis. Using equations (9) and (10), the outputs are and $q_a = 23\ 1/3$ $(= 40 - \frac{0.5(20)}{6} - \frac{30}{2})$ and $q_b = 43\ 1/3$ $(= 40 + \frac{0.5(20)}{3})$.

If the game were dynamic, however, a problem would arise. When Brydox observes the first-period output of $q_a = 24\ 1/6$, what is he to believe about Apex's costs? Should he deduce that $c = 30$, or increase his belief that $c = 20$, or believe something else entirely? This departs from standard modelling.

6.3. Symmetric Information and Prior Beliefs

In the *Expensive-Talk Game* of Table 1, the *Battle of the Sexes* is preceded by by a communication move in which the man chooses *Silence* or *Talk*. *Talk* costs 1 payoff unit, and consists of a declaration by the man that he is going to the prize fight. This declaration is just talk; it is not binding on him.

Table 1: Subgame Payoffs in the Expensive-Talk Game

		Woman	
		Fight	*Ballet*
Man:	*Fight*	3,1	0, 0
	Ballet	0, 0	1,3

Payoffs to: (Man, Woman).

(a) Draw the extensive form for this game, putting the man's move first in the simultaneous-move subgame.

<u>*Answer.*</u> See Figure A6.1.

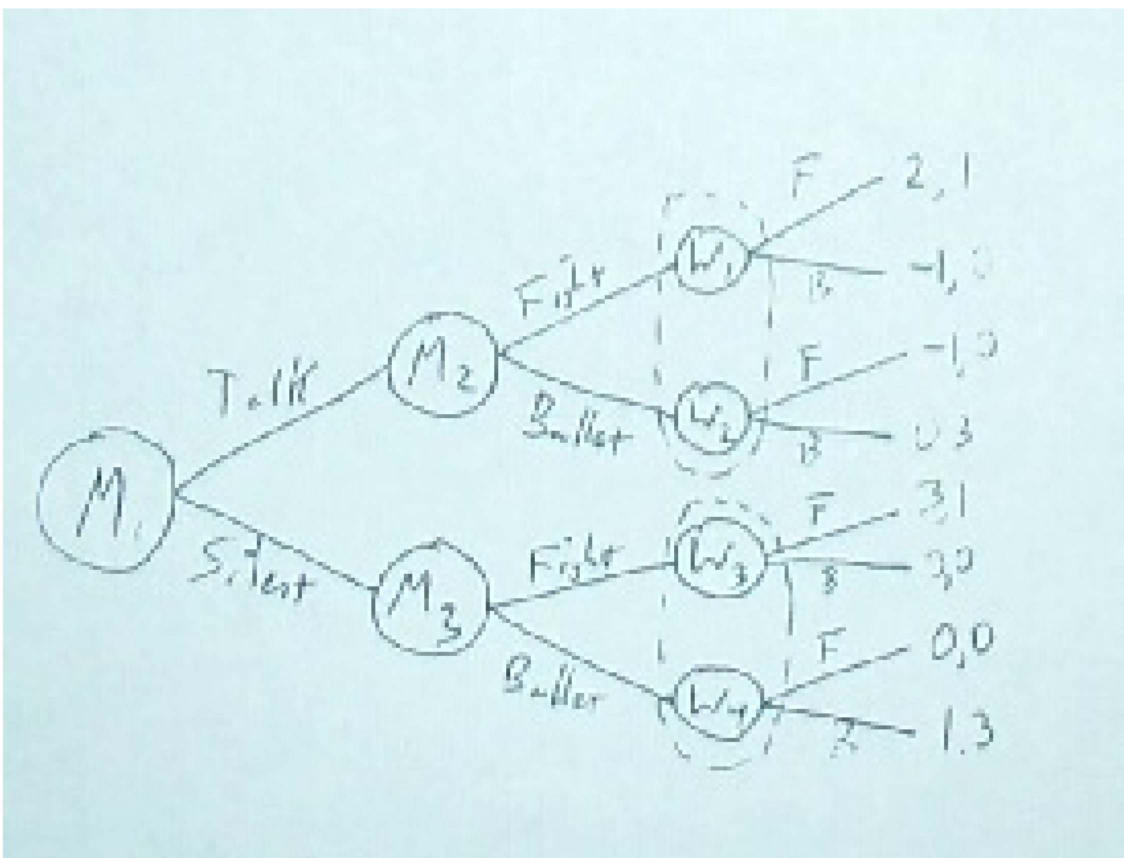

Figure A6.1: The Extensive Form for the Expensive Talk Game

(b) What are the strategy sets for the game? (start with the woman's)

Answer. The woman has two information sets at which to choose moves, and the man has three. Table A6.1 shows the woman's four strategies.

Table A6.1: The Woman's Strategies in "The Expensive Talk Game"

Strategy	W_1, W_2	W_3, W_4
1	F	F
2	F	B
3	B	F
4	B	B

Table A6.2 shows the man's eight strategies, of which only the boldfaced four are important, since the others differ only in portions of the game tree that the man knows he will never reach unless he trembles at M_1.

Table A6.2: The Man's Strategies in the Expensive Talk Game

Strategy	M_1	M_2	M_3
1	**T**	**F**	F
2	T	F	B
3	**T**	**B**	B
4	T	B	F
5	**S**	F	**F**
6	S	B	F
7	**S**	B	**B**
8	S	F	B

(c) What are the three perfect pure-strategy equilibrium outcomes in terms of observed actions? (Remember: strategies are not the same thing as outcomes.)

Answer. SFF, SBB, TFF.

The equilibrium that supports SBB is $[(S, B|S, B|T), (B|S, B|T)]$.

TBB is not an equilibrium outcome. That is because the Man would deviate to Silence, saving 1 payoff unit without changing the actions each player took.

(d) Describe the equilibrium strategies for a perfect equilibrium in which the man chooses to talk.

Answer. Woman: $(F|T, B|S)$ and Man: $(T, F|T, B|S)$.

(e) The idea of "forward induction" says that an equilibrium should remain an equilibrium even if strategies dominated in that equilibrium are removed from the game and the procedure is iterated. Show that this procedure rules out SBB as an equilibrium outcome.

See Van Damme (1989). In fact, this procedure rules out TFF *(Talk, Fight, Fight)* also.

Answer. First delete the man's strategy of (T, B), which is dominated by (S, B) whatever the woman's strategy may be. Without this strategy in the game, if the woman sees the man deviate and choose $Talk$, she knows that the man must choose

$Fight$. Her strategies of $(B|T, F|S)$ and $(B|T, B|S)$ are now dominated, so let us drop those. But then the man's strategy of (S, B) is dominated by $(T, F|T, B|S)$. The man will therefore choose to $Talk$, and the SBB equilibrium is broken.

This is a strange result. More intuitively: if the equilibrium is SBB, but the man chooses $Talk$, the argument is that the woman should think that the man would not do anything purposeless, so it must be that he intends to choose $Fight$. She therefore will choose $Fight$ herself, and the man is quite happy to choose $Talk$ in anticipation of her response. Taking forward induction one step further: TFF is not an equilibrium, because now that SBB has been ruled out, if the man chooses *Silence*, the woman should conclude it is because he thinks he can thereby get the SFF payoff. She decides that he will choose $Fight$, and so she will choose it herself. This makes it profitable for the man to deviate to SFF from TFF.