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ARE EQUILIBRIUM STRATEGIES UNAFFECTED BY INCENTIVES?

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ABSTRACT

In a mixed-strategy Nash equilibrium, changing one player's payoffs affects only the other player's equilibrium strategy mix. This 'Payoff Irrelevance Proposition' (PIP) appears to undercut the main foundations of economic policy analysis since, allegedly, equilibrium behavior will not respond to changes in incentives. We show, in contrast, that: (1) When the policy-maker has the first move in a sequential-move game, the PIP does not hold. (2) Even in a simultaneous-move game, the PIP holds only when the policy space is discrete, and for sufficiently small payoff revisions. Thus, incentives do generally affect behavior in equilibrium.

KEY WORDS • game theory • incentives and payoffs • mixed strategies • the Police Game

At a mixed-strategy Nash equilibrium of a two-person non-cooperative game, changing one player's payoffs affects only the other player's equilibrium strategy mix.[1] A series of papers by George Tsebelis (1989, 1990a,b) uses this well-known theorem to draw startling inferences about policies aimed at deterring undesired actions. Among the assertions are that: (1) in crime control, increasing the size of penalties will not reduce the number of offenses; (2) in international affairs, imposing economic sanctions will not lead the targeted nation to modify its actions; and (3) in hierarchical systems, supervision will not improve the behavior of subordinates. The policy-maker, despite being able to influence the payoffs, supposedly cannot affect the actual equilibrium choice (mixed strategy) of the targeted parties. We call this assertion the Payoff Irrelevance Proposition (PIP).

The PIP, to the extent that it is applicable in some social context, evidently undercuts standard economic reasoning about how behavior might be influenced by policy-makers. In analyzing the trade-off between probability of detection and size of penalties, for example, Becker (1968) and Ehrlich (1973) presumed that incentives do affect the choices of rational criminals; their analyses require drastic revision if sanctions do not affect how

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1. Provided that the changes in the first player's payoffs do not affect the *elements* entering, with non-zero probability, into his strategy mix.

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criminals behave. More generally, if the PIP applies, the usual arguments for and against policy measures like tariffs, taxes, subsidies and regulations are all gravely weakened.[2] (The PIP does not imply that policy is totally useless, however. If criminal penalties are increased, although the PIP predicts that the amount of crime will not change, the police may be able to economize by investing less effort in enforcing the laws.)

Rising to the challenge, we show that the PIP is applicable only in what are, from a policy point of view, very special and indeed limiting cases. So the foundation of the economic approach to policy - the premise that incentives do affect the equilibrium behavior of impacted parties - is solid after all. While other critics[3] have argued that policy situations ought to be modelled as repeated games or that two-person games are unrealistic, our analysis is based upon the order of moves and the number and range of allowed strategies.

I. The Police Game and the Payoff Irrelevance Proposition (PIP)

In the characteristic situation ('the Police Game') described by Tsebelis, the police are the policy-making authority, choosing between *patrolling* to enforce the law and *not patrolling* (strategies P and NP). The potential criminals choose between *committing* and *not committing* crimes (strategies C and NC). Table 1 shows the respective payoffs abstractly. Following Tsebelis's assumptions, for the criminals $c_4 > c_3$ and $c_2 > c_1$, while for the police $p_4 > p_3$ and $p_2 > p_1$. (The criminals will choose C if they know the police are not patrolling, but NC if the police are patrolling; the police will choose P if they anticipate that crimes will be committed, but NP if they do not expect any offenses.) Table 2 is a numerical illustration consistent with these specifications, bigger numbers representing more desired outcomes.

This is a discoordination game, with no Nash equilibrium in pure strategies. As the arrows in the tables indicate, one player or the other will want to deviate from any combination of pure strategies. Following standard procedures, the mixed-strategy equilibrium (where θ_c is the probability that criminals use their C strategy and θ_p is similarly the probability that police use their P strategy) is given by:

2. There is a seeming resemblance between the Payoff Irrelevance Proposition and 'rational expectations' theorems (for example, Barro, 1974, on the ineffectiveness of fiscal policy). But the similarity is superficial. The rational expectations argument is that policy is ineffective *because it has been fully anticipated*; strategic uncertainty is not involved. The Payoff Irrelevance Proposition, in contrast, is based upon the nature of the mixed-strategy equilibrium under strategic uncertainty.

3. See Bianco et al. (1990).

Table 1. Payoffs in the 2 × 2 Police Game

		Police	
		NP	*P*
Criminals	*NC*	c_3, p_4 ←	c_2, p_3
		↓	↑
	C	c_4, p_1 →	c_1, p_2

$c_4 > c_3,\ c_2 > c_1$

$p_4 > p_3,\ p_2 > p_1$

Table 2. The 2 × 2 Police Game-Numerical Example

		Police	
		NP	*P*
Criminals	*NC*	3, 4 ←	2, 3
		↓	↑
	C	4, 1 →	1, 2

$$\begin{aligned} \theta_c &= (p_4 - p_3)/(p_2 - p_1 + p_4 - p_3) \\ \theta_p &= (c_4 - c_3)/(c_2 - c_1 + c_4 - c_3) \end{aligned} \tag{1}$$

It is easy to verify that, for the specific numerical matrix of Table 2, $\theta_c = \theta_p = 1/2$.

Equations (1) show the Payoff Irrelevance Proposition at work. The probability mixture chosen by the criminals is a function only of the police payoffs, and that of the police is a function only of the criminal payoffs. If penalties for crime were increased, therefore (that is, if c_1 were reduced), θ_c would remain the same but θ_p would change. The probability of committing crime would be left unchanged in the new equilibrium; only the probability of patrolling would change.

Intuitively, the reasoning underlying the PIP is the following. At the mixed-strategy equilibrium, the criminals are willing to choose randomly between *C* and *NC*, so it must be that their expected payoffs from *C* and *NC* are identical. After a change in the criminals' payoff parameters - say an increase in the penalties for crime - since *C* and *NC* would no longer yield the same expected payoffs, the initial response of the criminals would be to choose one or the other. So, if the overall conditions still dictate a mixed-strategy equilibrium, it must be that the *police* behavior changes suitably. Specifically, the police will change their strategy mix to make the criminal yields from choosing *C* and *NC* once again equal. And in fact the same exact proportions of *C* and *NC* must be restored in order to keep the police indifferent between their strategies *P* and *NP*.

II. Simultaneous versus Sequential Play

The first substantive issue to be addressed is whether the Police Game ought to be modelled as a simultaneous-play or sequential-play game. These terms are to be understood in an informational rather than a calendar-time sense. If the police move first in time but their chosen move remains unknown to the criminals, the two sides are still effectively playing a simultaneous-move game. It is the asymmetric ability of one side to respond to the other's known choice that characterizes the sequential-play game. Put another way, in a simultaneous-move game both sides have to behave strategically, whereas in a sequential-play game only the first-mover is involved in a strategic choice - the last-mover has all the information needed for making a simple optimizing choice.

In the nature of the case, policy-makers like the police in the Police Game have to reason strategically. In deciding whether to allocate more resources to the south end of town than to the north end, the police must rationally ask themselves how potential offenders would respond. If the decision is to patrol the north end more heavily, the criminals, it can usually be anticipated, will become aware of this and shift their depredations to the south end.[4] The criminals, on the other hand, as individually small actors, will normally optimize in a non-strategic way, like 'price-takers' in microeconomic theory. An individual offender could not reasonably say to himself, 'If I shift to the south end of town, the police will change their planned allocation of effort to come after me there, so I won't derive any advantage.' Most criminal activities - from murder and theft to fraud and insider trading - are engaged in by decentralized actors behaving non-strategically. (As an evident exception, however, if crime were cartelized through a Mafia-type organization, both the police and the criminal planners would have to play strategically.)

Setting this exception aside, the Police Game should normally be analyzed in terms of a sequential-play protocol in which the authorities make the initial move.[5] Applying the usual 'subgame-perfect equilibrium' concept, in which the first-mover makes a rational choice on the assumption that the last-mover will also behave rationally, we see in Table 1 that the police would choose *P* if $p_3 > p_1$ and *NP* if the opposite holds. That is, the police would enforce the law if they prefer the outcome 'Patrol, Not commit' over 'Not patrol, Commit' - while if their preferences were reversed, they would not patrol. In the specific illustration of Table 2 the police have the first of these rankings ($p_3 > p_1$), with the desirable consequence that

4. The authorities may well attempt to publicize their actions. One of us recently heard a radio ad of the Chicago Transit Authority advising criminals (truthfully?) that enforcement levels had been increased.

5. The corresponding assumption, that one firm is the 'leader' in setting output quantity, leads to the (asymmetrical) Stackelberg solution in duopoly theory. The simultaneous-play protocol, that is, the assumption that each firm chooses its output in ignorance of the other's decision, leads to the (symmetrical) Cournot equilibrium.

all crimes are deterred. But given the cost of patrolling this may not always be the case. As an economic choice, the community in general and the police as their representatives routinely tolerate relatively minor crimes such as possession of small amounts of marijuana. For such offenses they may prefer 'Not patrol, Commit' - $p_1 > p_3$.[6]

Whichever way the ordering goes for different types of offenses, the crucial point is that in this sequential-move game[7] the PIP generally fails. Specifically, we saw that under the conditions of Table 1 the police would patrol if they prefer the outcome 'Patrol, Not commit' over 'Not patrol, Commit' - while if their preferences were reversed, they would not enforce the law. It is true, however, that a change in the criminal payoffs will still leave the criminals' optimal responses to the prior police moves unaffected - provided that the payoff changes do not alter the assumed rankings $c_4 > c_3$ and $c_2 > c_1$ which define the Police Game. But if a payoff change were to modify these rankings, as it easily might, then the criminal last-move responses and the police first-move actions will both typically be affected.

This conclusion will be stated as our first listed result:

RESULT 1. *Most often, policy interactions are better modelled as sequential games, not as simultaneous-move games. In such sequential games, the Payoff Irrelevance Proposition does not generally hold. Changes in own-payoffs may affect the choices of the policy-making authorities (who have the first move), of the targeted individuals and groups (who respond non-strategically), or both.*

6. Instead of the numbers in Table 2, for example, the police payoffs could be:

	NP	*P*
NC	4	1
C	2	3

Here $p_1 = 2$ exceeds $p_3 = 1$.

7. Tsebelis (1989: 83) concedes that, in the Police Game, it is 'more realistic' to assume that the police have the first move, so that the conditions for the sequential equilibrium apply. But he incorrectly asserts that the sequential solution will be the *same* as the solution to the simultaneous-play game. His reasoning appears to be based on the premise that the police, although having the first move, would find it advisable to keep their strategy choice secret. This would in effect throw the parties back into the simultaneous-move game, with the same solution as before. But secrecy (giving up the first move) is not *generally* more profitable than making the first move and openly announcing it. In the numerical illustration of Table 2, for example, the best sequential play leads to the outcome 'Patrol, Not commit' with a payoff to the police of $p_3 = 3$. The simultaneous-play equilibrium $\theta_c = \theta_p = 1/2$ achievable under secrecy has a payoff to the police of only 2 1/2. Thus, in this case, the police would be unwise to sacrifice the advantage of the first move.

III. Simultaneous-Play Equilibrium with a Strategy Continuum

In the preceding section we indicated that policy-makers normally will be playing a sequential-move rather than a simultaneous-move game with the affected parties. There are, however, a number of important exceptions, among them: (1) if the targeted party or parties are as much of a centrally organized entity as the policy-makers themselves, and (2) if the policy-makers, although having the option of the first move, find it more advantageous to act secretly. In this section we are postulating that one or the other of these exceptions applies, so that the simultaneous-move protocol is indeed applicable. Even so, we shall see, the PIP has only a limited range of applicability.

A natural way to approach the simultaneous-move Police Game is to postulate that both police and criminals can choose over a strategy continuum. Instead of the restriction to the discrete options C versus NC on the one side and P versus NP on the other, let criminals choose crime level $0 \leqslant C \leqslant 1$ while the police simultaneously choose patrolling level $0 \leqslant P \leqslant 1$.

A standard method of solution for games with continuous strategy spaces is to determine the Reaction Curves showing each side's optimal action as a function of the other's choice. The intersection of the Reaction Curves represents the simultaneous-play Nash–Cournot equilibrium. Some simple examples are illuminating.

Example 1. Suppose the payoff functions, for the criminals and police respectively, are:

$$\begin{aligned} V_c &= \alpha_c C - \beta_c C^2/2 - \gamma_c P \\ V_p &= \alpha_p P - \beta_p P^2/2 - \gamma_p C \end{aligned} \tag{2}$$

where the Greek letters signify positive parameters. Here each side's payoff is a quadratic function of its own level of activity and a negative-linear function of the opponent's level of activity.

The implied Reaction Curves are:[8]

$$\begin{aligned} RC_c &: C = \alpha_c/\beta_c \\ RC_p &: P = \alpha_p/\beta_p \end{aligned} \tag{3}$$

In this first example, even though the payoff V_c varies negatively with P, the chosen level of crime C is independent of P. Instead, it responds only to the criminals' own payoffs: C increases as the criminals' 'gain parameter' α_c rises and decreases as their 'diminishing returns parameter' β_c rises.

8. Found by setting the first derivatives $\partial V_c/\partial C$ and $\partial V_p/\partial P$ equal to zero and solving for C and P.

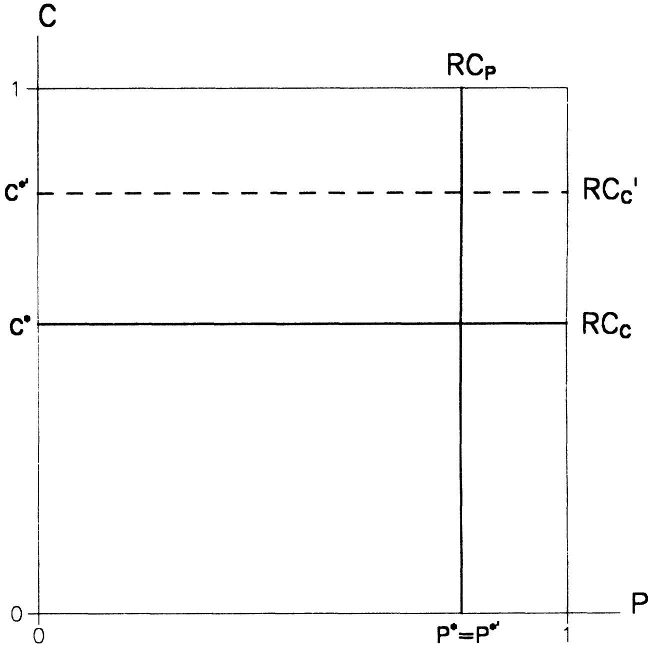

Figure 1. Simultaneous-Move Police Game with Strategy Continuum: Effect of Increased Criminal Payoffs when Equilibrium Strategy Choices Depend Upon Own Payoff Parameters Only

Similarly, the patrolling effort P will depend only upon the police 'gain parameter' α_p and 'diminishing returns parameter' β_p. As shown in Figure 1, the criminals' Reaction Curve RC_c is the solid horizontal line at $C = \alpha_c/\beta_c$, while the police Reaction Curve RC_p is a vertical line at $P = \alpha_p\beta_p$. The equilibrium is of course at the intersection of the two Reaction Curves, the solution being:

$$C^* = \alpha_c/\beta_c \quad \text{and} \quad P^* = \alpha_p/\beta_p \tag{4}$$

Now we ask, what if the criminals' payoff parameters were to change? The dashed line in Figure 1 shows how the criminals' Reaction Curve RC_c shifts in response to an increase in the ratio α_c/β_c – which of course could be the consequence either of a rise in the 'gain parameter' α_c or a fall in the 'diminishing returns parameter' β_c. Evidently, the criminals' Reaction Curve shifts upward so that the new equilibrium involves a larger amount of criminal activity with unchanged police activity. While this is a special case, it demonstrates that, to the exact contrary of the PIP, in the simultaneous-play game it is perfectly possible to have each side's optimal choice depend solely upon its own payoff parameters and not at all upon the opponents' payoffs.[9]

Nevertheless, that the PIP *might* possibly hold is illustrated by a second example.

Example 2. Suppose the payoff functions, for the criminals and police respectively, are instead:

$$V_c = (\alpha_c - \beta_c P) C - \gamma_c P$$
$$V_p = (-\alpha_p + \beta_p C) P - \gamma_p C \tag{5}$$

with $\gamma_p > \beta_p$ and all the parameters positive as before. Maximizing each payoff with respect to its control variable leads to the solution:

$$C^* = \alpha_p/\beta_p \quad \text{and} \quad P^* = \alpha_c/\beta_c \tag{6}$$

Here the equilibrium strategy for each side depends only upon the *opponent's* payoff parameters. Specifically, as shown in Figure 2, a rise in α_c or a fall in β_c leads only to an increase in police activity P^*.

It is of interest to remark on several aspects of this second example: (i) The PIP applies here as a pure-strategy equilibrium in a continuous strategy space, rather than as a mixed-strategy equilibrium in a discrete strategy space. (ii) The payoff functions have the undesirable feature of failing to display diminishing returns. The marginal return to police activity, for example, varies with C but remains constant as P increases. (iii) The implications for the respective Reaction Curves, as pictured in Figure 2, are rather strange. The police, for example, will respond with $P = 0$ to any $C < C^*$ and with $P = 1$ to any $C > C^*$. So a change in crime level C will, over most of its range, lead to no response at all, but at the critical point $C = \alpha_p/\beta_p$ the tiniest change in C would trigger a total swing in police activity toward one extreme or the other. So the two Reaction Curves would each be a discontinuous step-function, the intersection being at the respective points of discontinuity.

We have seen that it is possible to have payoff functions in which the equilibrium levels of activity depend only on the player's own payoff parameters (Example 1); or, less plausibly, only upon the opponent's payoff parameters (Example 2). The general and most reasonable case, of course, is where the equilibrium depends upon the payoff parameters *of both sides*. Typically, the Reaction Curves will be sloping as illustrated in Figure 3. Consistent with the spirit of the Police Game, the criminals'

9. The criminals are maximizing V_c with respect to C while the police are maximizing V_p with respect to P. It is easy to see that the first derivative $\partial V_c/\partial C$ does not depend upon P, and similarly $\partial V_p/\partial P$ does not depend upon C.

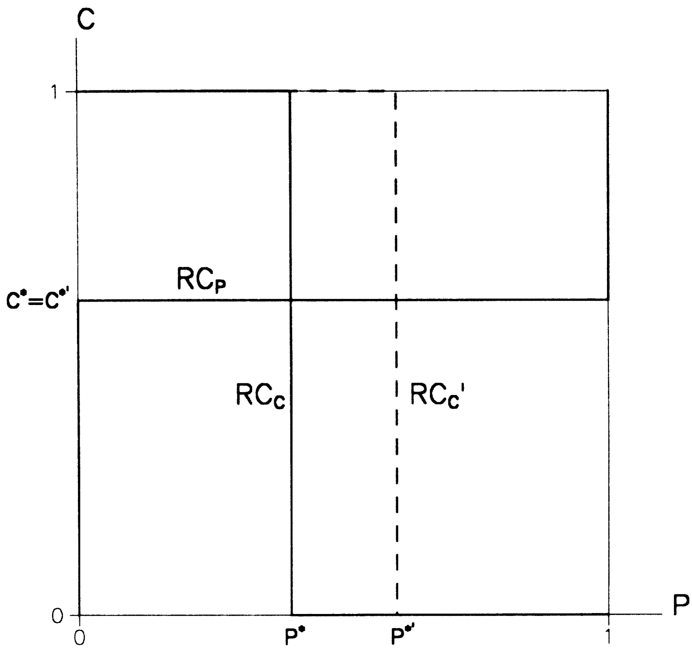

Figure 2. Simultaneous-Move Police Game with Strategy Continuum: Effect of Increased Criminal Payoffs when Equilibrium Strategy Choices Depend Upon Opponent's Payoff Parameters Only

Reaction Curve RC_c would be negatively sloped: as the police increase patrolling, the criminals prefer a lower level of criminal activity. And, correspondingly, the police Reaction Curve RC_p would be positively sloped: as offenders increase C, the police would prefer a higher level of patrolling.

Example 3. The illustration in Figure 3 is based upon the payoff functions:

$$\begin{aligned} V_c &= \alpha_c C - \beta_c P C^2/2 \\ V_p &= -\alpha_p P - \beta_p C/P \end{aligned} \tag{7}$$

where all the constants are positive as before.

These payoff functions retain the desirable property that the marginal returns, to both the criminals' efforts C and the police efforts P, are always diminishing. The implied Reaction Curves are:

$$\begin{aligned} RC_c&: C = \alpha_c/(\beta_c P) \\ RC_p&: P = (\beta_p C/\alpha_p)^{1/2} \end{aligned} \tag{8}$$

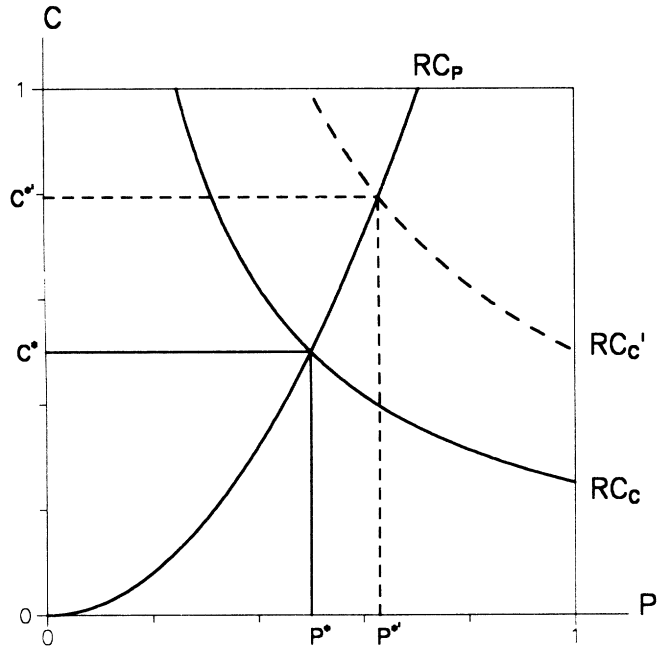

Figure 3. Simultaneous-Move Police Game with Strategy Continuum: Effect of Increased Criminal Payoffs when Equilibrium Strategy Choices Depend Upon Both Sides' Payoff Parameters

The equilibrium levels of C and P, obtained by solving equations (8) simultaneously, are:

$$\begin{aligned} C^* &= (\alpha_c^2 \alpha_p / \beta_c^2 \beta_p)^{1/3} \\ P^* &= (\alpha_c \beta_p / \alpha_p \beta_c)^{1/3} \end{aligned} \tag{9}$$

The solid curves in Figure 3 correspond to the parameter values $(\alpha_c, \beta_c, \alpha_p, \beta_p) = (1, 4, 2, 1)$. The equilibrium strategy-pair, in pure strategies of course, is then $C^* = P^* = 1/2$. That each party's equilibrium choice does depend upon its own as well as upon the other's payoff parameters is evident from the form of equations (9). If, for example, the criminal 'gain parameter' α_c rises from 1 to 2, the criminals' Reaction Curve RC_c shifts upward as illustrated by the dashed curve in the diagram. At the new solution, $(C^*, P^*) = (.7937, .6300)$. Thus, an increase in criminal payoffs has led to a rise in both the amount of crime and the amount of patrolling. Summarizing:

RESULT 2. *In simultaneous-move games, if the players have continuous strategy spaces there will typically be a pure-strategy equilibrium for which the Payoff Irrelevance Proposition does not generally hold.*[10]

The paradoxical PIP result, we can now see, is essentially an artifact due to lumpiness. Rational choice involves trade-offs, and lumpiness of the options available reduces what can be done in the way of trade-offs. Suppose a consumer initially finds apples too expensive to buy, but then the price falls. If the choice is between buying 50 apples or none, such a consumer may still take none - whereas, offered the opportunity of buying single apples, he might buy two or three instead. Similarly, a choice between a discrete C or NC is less susceptible to the influence of payoff changes than a choice over the entire range of options in between.

IV. Simultaneous-Play Equilibrium with Discrete Strategies

Still under the simultaneous-play protocol, suppose now that while the underlying situation remains the continuous-strategy space as in the figures, for some reason only selected discrete options and not the entire continuum are available to the parties. As can be seen, the case for the PIP is strongest here. It is convenient to state the result first, with the development to follow.

RESULT 3. *The Payoff Irrelevance Proposition may or may not hold for simultaneous-move games if the strategy space is discrete. For it to hold, the payoff changes must be 'sufficiently small' (in a sense to be made precise below).*

Let equations (7) continue to represent the payoff functions, as in our previous illustration. But now suppose that the parties can no longer choose C and P over the continuum. Instead, C for the criminals and P for the police must be chosen from the set $\{.2, .4, .6, .8, 1\}$. As can be seen in Figure 3, the strategy options .4 and .6 for each side are the 'immediate neighbors' bracketing the (no longer available) continuous-strategy equilibrium choices $C^* = .5 = P^*$. It is a plausible procedure, valid here though unfortunately not universally correct, to consider only these immediate

10. A technical qualification: the Reaction Curves must actually intersect in the interior of the strategy space. If they do not, there may be a corner solution in which players choose extreme behavior that may not alter when parameters change. If electrocuting burglars reduces burglary to zero, then increasing the penalty to boiling in oil will have no effect. Sufficient conditions for an interior intersection are that the strategy sets be compact and convex, and that a player's payoff be quasi-concave in his own strategy (see Rasmusen 1989: 124–5).

Table 3. The Police Game with Five Discrete Strategies

		Police				
		.2	.4	.6	.8	1
Criminals	.2	.184, −1.4	.168, −1.3	.152, −1.533	.136, −1.85	.12, −2.2
	.4	.336, −2.4	.272, −1.8	.208, −1.867	.144, −2.1	.08, −2.4
	.6	.456, −3.4	.312, −2.3	.168, −2.2	.024, −2.35	−.12, −2.6
	.8	.544, −4.4	.288, −2.8	.032, −2.533	−.244, −2.6	−.48, −2.8
	1	.6, −5.4	.2, −3.3	−.2, −2.867	−.6, −2.85	−1, −3

Table 4. Strategies of Table 3 That Are Used in Equilibrium

		Police	
		.4	.6
Criminals	.4	.272, −1.8	.208, −1.867
	.6	.312, −2.3	.168, −2.2

Table 5. A Truncated Strategy Set from Table 3

		Police	
		.4	1
Criminals	.4	.272, −1.8	.08, −2.4
	1	.2, −3.3	−1, −3

neighbors as candidates for a possible mixed-strategy equilibrium.[11]

Table 3 illustrates the entire range of payoffs, while Table 4 is a condensation showing only the immediate-neighbor options: $C \in \{.4, .6\}$ and $P \in \{.4, .6\}$.[12] There is no pure-strategy equilibrium for this game. Using equations (1) to find the equilibrium mix of the two neighboring strategies, over the strategy sets $\{.2, .4, .6, .8, 1\}$ the equilibrium mixtures are: for the criminals, $\theta_c = (0, .6, .4, 0, 0)$ and for the police $\theta_p = (0, .5, .5, 0, 0)$.

Thus we have shown by construction that a mixed-strategy equilibrium is *possible* given a discrete strategy space for the Police Game. That a mixed-strategy outcome is not *inevitable* is illustrated in Table 5. Here the available strategy options are, by assumption, .4 and 1.0 on each side.[13] In Table 5, $C^* = P^* = .4$ is a *pure-strategy* equilibrium.

11. More extreme pure strategies might enter into a mixed-strategy equilibrium, even conceivably in place of a 'neighboring' strategy, since the payoff functions might take on any of a wide variety of forms. The payoff functions need not necessarily be monotonic or even bitonic in the C and P variables, and the interaction can be formulated in many different ways.

12. Table 4 is an allowable condensation, since in Table 3 all but the .4 and .6 row and column strategies can be ruled out by iterated strict dominance. Specifically: (1) the $P = 1$ and $P = .2$ columns can be deleted as they are dominated by $P = .8$ and $P = .4$ respectively; (2) then the $C = .2$ row is dominated by $C = .4$, and $C = .8$ and $C = 1$ are both dominated by $C = .6$; (3) and finally, the $P = .8$ column is then dominated by $P = .6$. This process leaves only the .4 and .6 strategies on each side.

13. Whereas Table 4 was a condensation of the underlying Table 3 showing the strategies

Table 6. Table 3 with Increased Criminal Gain Parameter

		Police				
		.2	.4	.6	.8	1
Criminals	.2	−.384, −1.4	.368, −1.3	.352, −1.533	.336, −1.85	.32, −2.2
	.4	.736, −2.4	.672, −1.8	.608, −1.867	.544, −2.1	.48, −2.4
	.6	1.056, −3.4	.912, −2.3	.768, −2.2	.624, −2.35	.48, −2.6
	.8	1.344, −4.4	1.088, −2.8	.832, −2.533	.576, −2.6	.32, −2.8
	1	1.6, −5.4	1.2, −3.3	.8, −2.867	.4, −2.85	0, −3

These results suggest that there is likely to be a pure-strategy equilibrium when the available choices are asymmetrically placed - one of the strategy-pairs being located close to and the others far away from the pure-strategy equilibrium choices of the underlying continuum. Conversely, when the available choices are more or less evenly distant from the equilibrium of the continuous game, a mixed-strategy equilibrium is likely.[14]

Finally, let us return to the claim that the PIP holds at least within the window of conditions leading to mixed-strategy equilibria. Intuition suggests that if the equilibrium strategy mixture is not to be affected, only payoff changes within a limited range are allowable. More specifically, our previous analysis suggests that for such insensitivity to hold, the payoff parameter variations must be small enough so as not to cause a shift either to a pure-strategy equilibrium or to a mixed equilibrium involving different strategy elements.

Looking at Figure 3, the continuous-strategy intersection involving the dashed RC_c curve was generated by a change in the criminals' gain parameter from $\alpha_c = 1$ to $\alpha_c = 2$. Is this change 'sufficiently small' for insensitivity to hold in the discrete-strategy game described above, where the options on each side ranged from .2 to 1 in steps of .2 each? That is, will the criminals' optimal strategy remain the mixture of $C = .4$ and $C = .6$ with probabilities .6 and .4 respectively? Notice that the intersection involving the new RC_c curve no longer lies between .4 and .6, which may lead us to suspect that this parameter change is not 'sufficiently small' to leave the criminals' optimum mix unaffected. And in fact, Table 6 - like Table 3 but calculated in terms of the criminal 'gain parameter' $\alpha_c = 2$

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entering into the equilibrium mixture on each side, Table 5 is quite different. It represents a quite different game where, by assumption, all but the two specified strategies on each side have been disallowed.

14. This is only a tendency rather than a strict rule, since (as previously noted) distance as measured in the strategy space need not correlate well with distance in terms of the payoffs.

instead of $\alpha_c = 1$ - reveals that $(C^*, P^*) = (.8, .6)$ is now a pure-strategy equilibrium. That this strategy-pair is a pure-strategy solution is not surprising since it is very close to (.7937, .6300), which was the equilibrium of the corresponding *continuous-strategy* game using the changed α_c parameter.

Thus, we have verified Result 3 by example. In particular, we have shown that, with a discrete strategy space, there may or may not be a mixed-strategy equilibrium. If there is a mixed-strategy equilibrium, then the PIP will be valid only for payoff changes that are 'sufficiently small' in the sense of not affecting the strategies entering into the equilibrium mixture.

V. Conclusion

In discrete two-strategy simultaneous-move games with a mixed-strategy equilibrium, a change in one player's payoffs affects only the other player's equilibrium mix. We call this the PIP. It has been alleged that the PIP vitiates the economic arguments for or against policy initiatives; although policy-makers can alter the payoffs received from alternative strategies, changes in payoffs allegedly do not modify the equilibrium strategy mix of the affected parties. In particular, in the Police Game an increase in penalties will assertedly not affect the criminals' mix between committing and not committing crimes.

In this paper we showed that:

1. Policy-making is ordinarily better modelled not in terms of a simultaneous-move protocol but as a sequential-move game in which the authorities have the first move. Using the standard subgame-perfect equilibrium concept for sequential-move games, the PIP will not generally apply. Changes in incentives on either side will ordinarily affect the equilibrium behavior of the policy-makers themselves and of the individuals or groups they are trying to influence.
2. Even in the simultaneous-move game, if the strategy space is a continuum there will typically be a pure-strategy Nash equilibrium in which changes in payoffs affect the behavior of both sides. Only in very special cases will the PIP hold.
3. The case for the PIP is strongest when, in the simultaneous-move game, the strategy space consists of discrete options. If choices are sufficiently lumpy, a game may have a mixed-strategy equilibrium so that the PIP does apply over a certain range. Specifically, it holds only for payoff changes that are 'sufficiently small' in the sense of not shifting the strategy elements entering into the equilibrium.

We conclude that the PIP is only rarely applicable in actual policy-making situations. Incentives do, almost always, affect behavior in equilibrium.

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