Old Test Questions

March 1, 1998

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Abstract

These are old test questions for G601, a PhD course in game theory, information economics, and modelling.

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1. The Rules of the Game

1.1 Find the Nash equilibria of the following game, illustrated in Table 1.1. Can any of them be reached by iterated dominance?

Table 1.1 A Midterm Game

		Column		
		Left	*Middle*	*Right*
	Up	10,10	0, 0	−1, 15
Row:	*Sideways*	−12, 1	**8,8**	−1, −1
	Down	**15,1**	8,−1	0, 0

Payoffs to: (Row, Column).

ANSWER. The Nash equilibria are boldfaced. DL can be reached by iterated dominance. Iterate in the order: UP, SIDEWAYS, MIDDLE, RIGHT.

1.2. What is a Nash equilibrium in Table 1.2, if it is a simultaneous-move game?
(a) *Flavor, Flavor*
@ (b) *Flavor, Texture*
@ (c) *Texture, Flavor*
(d) *Texture, Texture*

Table 1.2: Flavor and Texture I

		Brydox	
		Flavor	*Texture*
Apex:	*Flavor*	-2,0	0,1
	Texture	-1,-1	0,-2

Payoffs to: (Apex, Brydox).

1.3. Using iterated dominance, what is the equilibrium in Table 1, if it is a simultaneous-move game?
(a) *Flavor, Flavor*
(b) *Flavor, Texture*
@ (c) *Texture, Flavor*
(d) *Texture, Texture*
(e) There is no such equilibrium

1.4. What is the Nash equilibrium in Table 1.3, if it is a simultaneous-move game?
(a) *Flavor, Flavor*
@ (b) *Flavor, Texture*
@(c) *Texture, Flavor*
(d) *Texture, Texture*
(e) There is no such equilibrium

Table 1.3: Flavor and Texture II

		Brydox	
		Flavor	*Texture*
	Flavor	20,7	28,8
Apex:			
	Texture	25,5	28,3

Payoffs to: (Apex, Brydox).

1.5. What is the Nash equilibrium in Table 1.4, if it is a simultaneous-move game?
(a) *Flavor, Flavor*
@ (b) *Flavor, Texture*
@ (c) *Texture, Flavor*
(d) *Texture, Texture*

Table 1.4: Flavor and Texture III

		Brydox	
		Flavor	*Texture*
	Flavor	120, 107	128,108
Apex:			
	Texture	125, 105	128,103

Payoffs to: (Apex, Brydox).

1.6. Using iterated dominance, what is the equilibrium in Table 1.4, if it is a simultaneous-move game?
(a) *Flavor, Flavor*
(b) *Flavor, Texture*
@ (c) *Texture, Flavor*
(d) *Texture, Texture*
(e) There is no such equilibrium

1.7. The following is the payoff matrix for
(a.) a version of the Battle of the Sexes.
(b.) a version of the Prisoner's Dilemma.
(c.) a version of Pure Coordination.
(d.) a version of the Legal Settlement Game.
@ (e.) none of the above.

		COL	
		A	B
ROW	A	3,3	0,1
	B	5,0	-1,-1

1.8. The following game has how many pure strategy Nash equilibria?
(a.) zero
(b.) one
@(c.) two
(d.) three

			COL
		Up	Down
ROW	Right	1,250	-1, 22
	Left	1,1	0,0

1.9. The problem of deciding whether to adopt IBM or HP computers by two offices in a company is like
(a.) the prisoner's dilemma
(b.) the welfare game
@(c.) The battle of the sexes.

1.10. Find the pure-strategy Nash equilibria of the following game.

Table 1.7 A Final Game

		Column		
		Left	*Middle*	*Right*
	Up	9,2	0, 0	0, 9
Row:	*Sideways*	7, 1	8,8	5,5
	Down	8,4	8,2	0, 0

Payoffs to: (Row, Column).

ANSWER: (*Sideways, Middle*) is the only Nash equilibrium. It happens to be weak, but it is still unique.

1.11. The large Wall Street investment banks have recently agreed not to make campaign contributions to state treasurers, which up till now has been a common practice. What was the game in the past, and why can the banks expect this agreement to hold fast?

ANSWER. This game was like a Prisoner's Dilemma. Suppose there are two investment banks. The one who made the largest contribution would get the state's bond issuing business, but all profits would be eaten up in contributions if both made contributions. Both would be better off if they refrained from making any contributions. These are the payoffs of a Prisoner's Dilemma.

This is a repeated game, however, and if one bank deviates by making contributions, other banks will resume making contributions, so the gain will be temporary. Moreover, if the contributions are public, the other banks can in fact respond immediately, before the underwriter for the bond issue is decided upon, so the deviating bank does not even get a temporary advantage.

Note that less efficient banks, which probably includes the Small regional ones like the Stephens bank in Arkansas, would prefer the old system, since they have a comparative advantage in corruption
.

1.12. Identify any dominated strategies and any Nash equilibria in pure strategies in the following game.

Table 1.8: A Game for the 1997 Final

		Column		
		Left	**Middle**	*Right*
	Up	**1,4**	$5,-1$	$0,1$
Row:	**Sideways**	$-1,0$	-2,-2	$-3,4$
	Down	$0,3$	9,-1	$5,0$

Payoffs to: (Row, Column).

ANSWER: Middle and Sideways are dominated. *Up, Left* is Nash. Note that I did not ask about iterated dominance, which is a separate issue entirely. Using iterated dominance will not tell you what strategies are dominated or give you a complete set of Nash equilibria.

2 Information

2.1. The boss is trying to decide whether Smith's energy level is high or low. He can only look in on Smith once during the day. He knows if Smith's energy is low, he will be yawning with a 50 percent probability, but if it is high, he will be yawning with a 10 percent probability. Before he looks in on him, the boss thinks that there is an 80 percent probability that Smith's energy is high, but then he sees him yawning. What probability of high energy should the boss now assess?

ANSWER: What we want to find is $Prob(High|Yawn)$. The information is that $Prob(High) = .80$, $Prob(Yawn|High) = .10$, and $Prob(Yawn|Low) = .50$. Using Bayes Rule,

$$Prob(High|Yawn) = \frac{Prob(High)Prob(Yawn|High)}{Prob(High)Prob(Yawn|High) + Prob(Low)Prob(Yawn|Low)} = \frac{(.8)(.1)}{(.8)(.1) + (.2)(.5)} = .44.$$

3 Continuous and Mixed Strategies

3.1. Industry output is
(a.) lowest with monopoly, highest with a Cournot equilibrium
@(b.) lowest with monopoly, highest with a Stackelberg equilibrium.
(c.) lowest with a Cournot, highest with a Stackelberg equilibrium.
(d.) lowest with a Stackelberg, highest with a Cournot equilibrium.

3.2. Three firms producing an identical product face the demand curve $P = 240 - \alpha Q$, and produce at marginal cost β. Each firm picks its quantity simultaneously. If $\alpha = 1$ and $\beta = 40$, the equilibrium output of the industry is in the interval
(a) $[0, 20]$
(b) $[20, 100]$
(c) $[100, 130]$
@ (d) $[130, 200]$
(e) $[200, \infty]$

3.3. Is this triopoly game supermodular?
(a) Yes
@(b) No
(c) Only under some values of α
(d) Not enough information is provided to answer

3.4. In this triopoly game, if β increases then the industry output
(a) Rises
@(b) Falls
(c) Might either rise or fall
(d) Stays the same

3.5. If a player uses mixed strategies in equilibrium,
(a.) All players are indifferent among all their strategies.
(b.) That player is indifferent among all his strategies.
@ (c.) That player is indifferent among the strategies he has a positive probability of choosing in equilibrium.
(d.) That player is indifferent among all his strategies except the ones that are weakly dominated.
(e) None of the above.

3.6. Find the unique Nash equilibrium of the game in Table 3.1.

Table 3.1: A Game for the 1996 Midterm

		Column		
		Left	*Middle*	*Right*
	Up	1,0	$10, -1$	$0, 1$
Row:	*Sideways*	$-1, 0$	-2,-2	$-12, 4$
	Down	$0, 2$	823,-1	$2, 0$

Payoffs to: (Row, Column).

ANSWER. The equilibrium is in mixed strategies. Denote Row's probability of *Up* by γ and Column's probability of *Left* by θ. Strategies *Sideways* and *Middle* are strongly dominated strategies, so we can forget about them. Row has no reason ever to choose *Sideways*, and Column has no reason ever to choose *Middle*.

In equilibrium, Row must be indifferent between *Up* and *Down*, so

$$\pi_R(Up) = \theta(1) + (1-\theta)(0) = \pi_R(Down) = \theta(0) + (1-\theta)(2)$$

This yields $\theta^* = 2/3$. Column must be indifferent between *Left* and *Right*, so

$$\pi_R(Left) = \gamma(0) + (1-\gamma)(2) = \pi_R(Right) = \gamma(1) + (1-\gamma)(0)$$

This yields $\gamma^* = 2/3$.

3.7. Three companies provide tires to the Australian market. The total cost curve for a firm making Q tires is TC = 5 + 20Q, and the demand equation is P = 100-N, where N is the total number of tires on the market.

According to the Cournot model, in which the firms's simultaneously choose quantities, what will the total industry output be?

ANSWER. Marginal cost is 20 for each firm. For firm 1, revenue is

$$R_1 = PQ_1 = (100 - Q_1 - Q_2 - Q_3)Q_1,$$

so marginal revenue is $100 - 2Q_1 - Q_2 - Q_3$. Setting this equal to marginal cost yields $20 = 100 - 2Q_1 - Q_2 - Q_3$. Since each firm produces the same quantity in equilibrium, $4Q_1 = 80$, and $Q_1 = 20$. Total industry output is therefore 60.

3.8 (hard). On his job visit, Professor Schaffer of Michigan told me that in a Cournot model with a linear demand curve $P = \alpha - \beta Q$ and constant marginal cost C_i for firm i, the equilibrium industry output Q depends on $\Sigma_i C_i$, but not on the individual levels of C_i. I may have misremembered. Prove or disprove this assertion. Would your conclusion be altered if we made some other assumption on demand? Discuss.

ANSWER. Everybody had trouble with this. A good approach when stymied is to start with a simple case. Here, the two-firm problem is the obvious simpler case. Prove the proposition for the simple case, and then use that as a pattern to extend it. (Also, you can disprove a general proposition using a simple counterexample, though you cannot prove one using a simple example.)

Note that you cannot assume symmetry of strategies in this game. It is plausible, though not always correct (remember Chicken), when players are identical, but they are not here— firms have different costs. So we would expect their equilibrium outputs to differ.

Also, remember to answer all parts of test questions. The second part of this question asked about nonlinear demand functions, and it is actually the easier part.

The proposition is true.

$$\pi_j = (\alpha - \beta \Sigma_i Q_i - C_j)Q_j,$$

so

$$\frac{d\pi_j}{dQ_j} = \alpha - \beta \Sigma_{i \neq j} Q_i - 2\beta Q_j - C_j = 0,$$

and

$$Q_j = \frac{C_j - \alpha - \beta \Sigma_{i \neq j} Q_i}{2\beta}.$$

Industry output is

$$\Sigma_j Q_j = \Sigma_j \frac{C_j - \alpha - \beta \Sigma_{i \neq j} Q_i}{2\beta} = \Sigma_j \frac{C_j - \alpha}{2\beta} - \Sigma_j \frac{\Sigma_{i \neq j} Q_i}{2}.$$

The first term of this last expression depends on the sum of the firms' cost parameters, but not on their individual levels. The second term adds up the outputs of all but one firm N times, and so equals $(N-1)$ times the sum of the output, $(N-1)\Sigma_j Q_j$. Thus,

$$\Sigma_j Q_j = \Sigma_j \frac{C_j - \alpha}{2\beta N}.$$

This does not depend on the cost parameters except through their sum. Q.E.D.

Chris Pope pointed out one caveat. This proof implicitly assumed that every firm had low enough costs that it would produce positive output. If it produces zero output, it is at a corner solution, and the first order condition does not hold, so the proof fails. Thus, the validity of the proposition depends on the following being true for every j:

$$Q_j = \frac{C_j - \alpha - \beta \Sigma_{i \neq j} Q_i}{2\beta} > 0.$$

This condition is not stated in terms of the primitive parameters (it depends on $\Sigma_{i \neq j} Q_i$), so to be quite proper I ought to solve it out further, but I will not do that here.

The result does depend on linear demand. This can be shown by counterexample. Suppose $P = \alpha - \beta Q^2$. Then, attempting the construction above,

$$\pi_j = (\alpha - \beta(\Sigma_i Q_i)^2 - C_j)Q_j,$$

so

$$\frac{d\pi_j}{dQ_j} = \alpha - 3\beta Q_j^2 - 2\beta \Sigma_{i \neq j} Q_i Q_j - C_j = 0.$$

Solving this for Q_j will involve taking a square root of C_j. But if Q_j is a function of the square root of C_j, then increasing C_j by a given amount and decreasing C_l by the same amount will *not* keep the sum of Q_j and Q_l the same, unlike before, where Q_j was a linear function of C_j. So the proposition fails for quadratic demand, and, more generally, whenever demand is nonlinear.

4 Dynamic Games with Symmetric Information

4.1. Politician Smith has pictures of politician Jones dressed in frilly underwear, while Jones has tapes of Smith promising a woman a government job in exchange for her favors. The harm from public exposure is 20 for Smith and 50 for Jones. Smith threatens to show the pictures of Jones if Jones votes against a tariff. Smith receives an extra utility of 3 if Jones votes for the tariff, but Jones loses 4 in utility from that vote. Smith would get utility of 5 from showing the pictures of Jones, and Jones would get utility of 7 from playing the tapes of Smith. The vote is one month from today, but you may assume that both politicians live forever.

(4.1a) What is a perfect equilibrium for this game?

ANSWER: One answer is: *Smith shows the pictures. Jones votes against the tariff and plays the tapes.* Given that Jones's behavior is unconditional, Smith should show the pictures to get the extra utility of 5 from that action. Given that Smith's behavior is unconditional, Jones should show the pictures to get the extra utility of 7 from that action, and should vote against the tariff to avoiding losing 4 in utility. The order of actions is immaterial to all this.

(4.1b) What is an equilibrium for this game that is Nash but not perfect?

ANSWER: One answer is: *Smith shows the pictures iff Jones votes against the tariff or Jones plays the tapes first. Jones votes for the tariff and plays the tapes iff Smith shows the pictures first.* The equilibrium OUTCOME (distinct from the equilibrium, which is a STRATEGY PROFILE) is that Jones votes for the tariff and nobody exposes anybody.

This is Nash. If Smith deviates and shows the pictures of Jones, Jones will retaliate by exposing him, for a net loss of 15 (=20-5) to Smith. If Jones deviates and plays the tape of Smith, Smith will retaliate by showing the pictures, for a net loss of 43 (=50-7) for Jones. If Jones deviates and votes against the tariff, both politicians will will have their secrets exposed, and Jones will have a net loss of 39 (=50-7-4).

This is not perfect. Suppose we start the game with Jones having deviated by voting against the tariff. If Smith follows his assigned strategy of showing the pictures, Jones will play the tapes in retaliation, so Smith will suffer a net loss of 15 (=20-5). Thus, Smith should deviate by not showing the pictures if Jones votes against the tariff.

In this game, the moves do not follow a neat sequence. The first move is clearly that Jones votes for or against the tariff, but thereafter, either player has the option to expose the other's secrets at any time. Thus, we need to consider both Smith showing the pictures first and Jones playing the tapes first. Both of them have a threat available, but know that there is a counterthreat.

4.2. It would seem that all human males must have the same strength of sex drive, because a more motivated male would be more successful in his reproduction, mating with more or better females. In fact, sex drives seem to differ. Use the idea behind the Hawk-Dove model to explain this.

ANSWER. Males with strong sex drives are like Hawks. They more aggressively pursue females, but this means they use up more of their resources in the pursuit, without any corresponding gain if they must compete with other males with strong sex drives. Males with weak sex drives are like Doves. They devote little energy to reproduction, and hence do badly in competition with highly sexed males, but they do fine in competition with each other and can more easily survive.

In equilibrium, both types would persist. If there were too many males with strong sex drives, it would be more advantageous to have a weak sex drive, and waste less energy in fruitless pursuit while winning the occasional female by luck or lack of any competition at all. If there were too many males with weak sex drives, it would be more advantageous to have a strong sex drive, and devote more energy to snapping up the females against the feeble competition.

5 Reputation and Repeated Games with Symmetric Information

5.1. Suppose that the Battles of the Sexes is repeated an infinite number of times, without discounting. Find a subgame perfect equilibrium strategy profile in which the two players go to different events for the first three repetitions, and thereafter go to the Ballet.

ANSWER: One such strategy profile is: *Man: Ballet, Ballet, Ballet, and thenceforth go to the Ballet unless someone deviated in the first three repetitions, in which case mix between Fight and Ballet in the one-shot mixed strategy equilibrium proportions every time after the deviation occurs. Woman: Fight, Fight, Fight, and thenceforth go to the Ballet unless someone deviated in the first three repetitions, in which case mix between Fight and Ballet in the one-shot mixed strategy equilibrium proportions every time after the deviation occurs.*

If either player deviates in the first three repetitions, the long-term behavior follows the mixed strategy, which has a much lower payoff than BB does for either the Man or the Woman.

5.2. A company has profits of 10 per year, (paid at the end of the year) but will go bankrupt with probability .2. The interest rate is 5 percent. How much is the company worth?

ANSWER.

$$V = (1/1.05)(.8)(V + 10) = .8(V + 10)/1.05,$$

so

$$\frac{1.05 - .8}{.1.05} V = 10 + V$$

, and $V = 10(.80/.25) = 32$.

Note: I also gave credit for another answer, based on the 10 being paid out even if the firm went bankrupt. Also note that even though the value with 0 percent chance of bankruptcy is 200 (=10/.05), the value with a 20 percent chance is not 160, but only 32. The amount 160 would be the value of a profit stream each year of 10 with probability .8, 0 with probability .2. Bankruptcy, however, ends not only next year's profits, but *all* future years.

5.3. Each of N "Premium" firms has quality cost $c = \underline{c}$ and each of D "Discount" firms has quality cost $c = \overline{c}$, where $\underline{c} < \overline{c}$. Each firm chooses quality to be $q = 0$, which costs 0 or $q = 1$, which costs c. Consumers do not observe quality. Each firm also chooses a one-time level of conspicuous spending at time zero, $S \geq 0$, and a price, P. Each of N consumers decides whether to buy, and from which firm, and after buying, the consumer discovers quality for that period. The discount rate per period is r. Consumers will pay up to $\overline{P}$ for a good of known high quality and 0 for known low quality, and they maximize consumer surplus. Unless otherwise specified, all payments are made at the end of a period.

(a) If this game is not repeated, so there is only one period, what is the equilibrium? Be sure to specify the complete strategy for each player.

ANSWER: In the last period, $q = 0$ is a dominant strategy for every firm of each type. Consumers will therefore pay no more than 0, and that will be the equilibrium price. The firms have no reason to engage in conspicuous spending, so $S = 0$.

Consumers: Buy if the price is 0; do not buy otherwise. Out of equilibrium beliefs are that firms produce low quality.

Firms: $S = 0$ and low quality. Prices can take any of a variety of levels.

This and part (b) are good examples of the importance of understanding Nash equilibrium and backwards induction. It seems like every test of mine, people lose points for not using these simple concepts.

(b) If this game is repeated three times, what is the equilibrium?

ANSWER: The same as in one period, repeated three times, using backwards induction.

(c) If this game is repeated an infinite number of times, describe an equilibrium in which the N consumers buy from the N Premium firms.

ANSWER: With an infinite number of periods, there are lots of equilibria. Here is one. (7c) is by far the hardest question on the test. I did not expect anyone to get it completely correct, and nobody did.

Consumers: Consumer i starts by buying from Firm i for $i = 1, ...N$. Buy if the price is p^* and the firm spent S^*; switch to another firm which has been charging that price and spent S^* otherwise; don't buy if no firm satisfies those conditions. Out-of-equilibrium beliefs: passive conjectures, where needed. [1]

Premium firms: Quality is $q = 1$, spending is $S = S^*$, and price is $p = p^*$. If a firm deviates in any way, it switches to $q = 0$ and $p = 23$ thereafter.

[1] Out-of-equilibrium beliefs are not crucial here, since any firm of any type that deviates from equilibrium is expected to produce low quality thereafter.

Discount firms: Quality is $q = 0$, spending is $S = 0$, and price is $p = 23$.

To find S^* and p^* we need to do some calculations. A premium firm's continuation payoff from deviating to low quality is $\frac{p^*-0}{1+r}$ for the one period and zero thereafter. Its continuation payoff from high quality is $\frac{p^*-\underline{c}}{r}$. Equating these requires that $rp^* = p^* + rp^* - (1+r)\underline{c}$, so $p^* \geq (1+r)\underline{c}$ will work as far as this kind of deviation goes. Call this Condition (*).

The premium firm's overall equilibrium payoff is $-S^* + \frac{p^*-\underline{c}}{r}$, so we need $S^* \leq \frac{p^*-\underline{c}}{r}$, or the premium firms will deviate to behaving like discount firms, to earn zero payoffs.

How about the discount firm? It must spend S^* to fool consumes into thinking it is a premium, for a payoff of $-S^* + p^*$ if it then chooses low quality. $S^* \geq p^*$ will prevent this. Combining the results of these two paragraphs, we need $S^* \in [p^*, \frac{p^*-\underline{c}}{r}]$. For this range to exist, though, we need $p^* \leq \frac{p^*-\underline{c}}{r}$, or $p^* \geq \frac{\underline{c}}{1-r}$. This is more binding than Condition (*); any price which satisfies this condition will satisfy (*) too. So let us take $p^* = \frac{\underline{c}}{1-r}$ for our equilibrium, and let $S^* = \frac{\underline{c}}{1-r}$ too.

Another possibility is that the discount firm will deviate by choosing $S = S^*$ and then producing high quality once or forever (if it is profitable for one period, it is profitable forever). Given that $S^* = \frac{\underline{c}}{1-r}$, however, the premium firms will be making zero profits producing high quality, and the discount firms would make negative profits trying to do so with their higher production costs.

6. Dynamic Games with Asymmetric Information

6.1. Bigfirm Inc., is thinking about taking over the Target Co., which is still controlled by its founder, Mr. Target, even though he does not own much of the stock any more. He is uncomfortable about the takeover, because he loses 120 in utility when he loses his control. With probability .8, Target is in BAD financial shape, and has value 10 for Bigfirm. With probability .2, Target is in GOOD financial shape, and has value 200 for Bigfirm.

Bigfirm only observes the accounting numbers, though. If Target is in BAD shape, the accounting earnings are LOW. If Target is in good shape, Mr. Target gets to choose between HIGH and LOW earnings. If the earnings are HIGH, Bigfirm must pay 100 for the stock; if they are LOW, Bigfirm must pay 60.

Mr. Target receives a utility of 60 from HIGH earnings and 20 from LOW earnings, in addition to whatever else is going on in his utility function.

(a) Draw the game tree for this situation, including the payoffs and information sets.

ANSWER. Natures chooses Bad or Good. If Nature chooses Bad, Target chooses LOW. If Bigfirm then chooses Takeover, Target's payoff is -100 (-120 +20) and Bigfirm's is -50 (10-60). If Bigfirm instead chose Don't Takeover, Target's payoff is 20 (20) and Bigfirm's is 0 (0).

If Nature chooses Good, Target can choose HIGH. If Bigfirm then chooses Takeover, Target's payoff is -60 (-120 +60) and Bigfirm's is 100 (200-100). If Bigfirm instead chose Don't Takeover, Target's payoff is 60 (60) and Bigfirm's is 0 (0). Or Target can choose LOW. If Bigfirm then chooses Takeover, Target's payoff is -100 (-100 +20) and Bigfirm's is 140 (200-60). If Bigfirm instead chose Don't Takeover, Target's payoff is 20 (20) and Bigfirm's is 0 (0).

(b) What is the equilibrium? (hard)

ANSWER. The equilibrium is (LOW, TAKEOVER|HIGH, DON'T|LOW). No out-of-equilibrium beliefs are needed. If Target deviates to HIGH, a takeover will occur, and his payoff falls from 20 to -60. If Bigfirm deviates to DON'T|HIGH, his payoff falls from 100 to 0. If Bigfirm deviates to TAKEOVER|LOW, his payoff falls from 0 to .8(-50) + .2(140) =-40+28 = -12 .

Mistakes to watch out for: (i) Not specifying a strategy for each player, (ii) Forgetting the TAKEOVER|HIGH part of the equilibrium.

(c) What is the equilibrium outcome, in terms of actions taken and expected payoffs from playing this game? (hard)

ANSWER. Nature may choose either GOOD or BAD, Target will always choose LOW, and Bigfirm will always choose DON'T. Target's payoff is 20 and Bigfirm's is 0.

Mistakes to watch out for: (i) Not answering the question (that is, not specifying the actions taken or the expected payoffs).

7 Moral Hazard: Hidden Actions

7.1. In the hidden actions problem facing an employer, inefficiency arises because
@(a.) The worker is risk averse.
(b.) The worker is risk neutral.
(c.) No contract can induce high effort.
(d.) The type of the worker is unknown.
(e.) The level of risk aversion of the worker is unknown.

7.2. An agent's utility function is U =(log(wage) - effort). What should his compensation scheme be if different (output,effort) pairs have the probabilities in Table 7.1?
a. The agent should be paid exactly his output.
@b. The same wage should be paid for outputs of 1 and 100.
c. The agent should receive more for an output of 100 than of 1, but should receive still lower pay if output is 2.
d. None of the above.

Table 7.1: Output Probabilities

		Output		
		1	2	100
Effort	*High*	0.5	0	0.5
	Low	0.1	0.8	0.1

7.3. For the next few problems, use Table 7.2. The utility function of an agent is $U = w + \sqrt{w} - \alpha e$, and his reservation utility is 0. Principals compete for agents, and have reservation profits of zero. Principals are risk neutral. If $\alpha = 2$, then if the agent's action can be observed by the principal, his equilibrium utility is in the interval
(a) $[-\infty, 0.5]$
(b) $[0.5, 5]$

(c) $[5, 10]$
(d) $[10, 40]$
@(e) $[40, \infty]$

Table 7.2: Output Probabilities

		Effort	
		Low ($e = 0$)	High ($e = 5$)
Output	$y = 0$	0.9	0.5
	$y = 100$	0.1	0.5

7.4. If $\alpha = 10$, then if the agent's action can be observed by the principal, his equilibrium utility is in the interval
(a) $[-\infty, 0.5]$
(b) $[0.5, 5]$
(c) $[5, 10]$
@ (d) $[10, 40]$
(e) $[40, \infty]$

7.5. If $\alpha = 5$, then if the agent's action can be observed by the principal, his equilibrium effort level is
(a) Low
@ (b) High
(c) A mixed strategy effort, sometimes low and sometimes high

7.6. (2 points) If $\alpha = 2$, then if the agent's action cannot be observed by the principal, and he must be paid a flat wage, his wage will be in the interval
(a) $[-\infty, 2]$
(b) $[2, 5]$
(c) $[5, 8]$
@ (d) $[8, 12]$
(e) $[12, \infty]$

7.7. (2 points) If the agent owns the firm, and $\alpha = 2$, will his utility be higher or lower than in the case where he works for the principal and his action can be observed?
(a) Higher
@ (b) Lower
(c) Exactly the same.

7.8. If the agent owns the firm, and $\alpha = 2$, his equilibrium utility is in the interval
(a) $[-\infty, 0.5]$
(b) $[0.5, 5]$
(c) $[5, 10]$
(d) $[10, 40]$
@ (e) $[40, \infty]$

7.9. If the agent owns the firm, and $\alpha = 8$, his equilibrium utility is in the interval
(a) $[-\infty, 0.5]$
(b) $[0.5, 5]$
(c) $[5, 10]$

@ (d) $[10, 40]$
(e) $[40, \infty]$

7.10. A salesman must decide how hard to work on his own time on getting to know a potential customer. If he exerts effort X, he incurs a utility cost $X^2/2$. With probability X, he can then go to customer X and add V to his own earnings. With probability (1-X), he offends the customer, and on going to him would subtract L from his earnings. The boss will receive benefit B from the sale in either case. The ranking of these numbers is $V > L > B > 0$. The boss and the salesman have equal bargaining power, and are free to make side payments to each other.

(a) What is the first-best level of effort, X_a?

ANSWER. Total surplus is

$$-\frac{X^2}{2} + X(V+B),$$

so

$$-X + (V+B) = 0,$$

and

$$X_a = V + B$$

(b) If the boss has the authority to block the salesman from selling to this customer, but cannot force him to sell, what value will X take?

ANSWER. If the salesman is successful, the total benefit from the sale will be $V + B$, split between boss and salesman. The salesman therefore maximizes

$$-\frac{X^2}{2} + X\frac{V+B}{2},$$

so

$$-X + \frac{V+B}{2} = 0,$$

and

$$X = \frac{V+B}{2}.$$

(c) If the salesman has the authority over the decision on whether to sell to this customer, and can bargain for higher pay, what will his effort be?

ANSWER. If the salesman is successful, he will want to make the sale to get V for himself. He cannot bargain for more from his boss, because the boss knows the salesman will make the sale even if agreement is not reached; any threat not to make the sale is not credible. Thus, the salesman's payoff is therefore

$$-\frac{X^2}{2} + XV,$$

which when maximized yields

$$X = V.$$

(d) Rank the effort levels X_a, X_b, and X_c in the previous three sections.

ANSWER. $X_a > X_c > X_b$.

7.11. A one-man firm with concave utility function $U(X)$ hires a lawyer to sue a customer for breach of contract. The lawyer is risk-neutral and effort averse, with a convex disutility of effort.

What can you say about the optimal contract? What would be the practical problem with such a contract, if it were legal?

ANSWER. The contract should give the firm a lump-sum payment and let the lawyer collect whatever he can from the lawsuit. The problem is that the firm would not have any incentive to help win the case.

7.12. An agent has the utility function $U = log(w) - e$, where e can take the levels 0 and 4, and his reservation utility is $\overline{U} = 4$. His principal is risk-neutral. Denote the agent's wage conditioned on output as $\underline{w}$ if output is 0 and $\overline{w}$ if output is 10. Only the agent observes his effort. Principals compete for agents. Output is as shown in the table below:

	Probability of Outputs		
Effort	0	10	Total
$Low (e = 0)$	0.9	0.1	1
$High\ (e = 4)$	0.2	0.8	1

What are the incentive compatibility and participation constraints for obtaining high effort?

ANSWER. The incentive compatibility is $.2log(\underline{w}) + .8log(\overline{w}) - 4 \geq .9log(\underline{w}) + .1log(\overline{w}) - 4$.

The participation constraint is $.2log(\underline{w}) + .8log(\overline{w}) - 4 \geq 4$.

Finding these conditions is separate from the issue of whether the principal actually will want to induce high effort.

7.12. Suppose an agent has the utility function $U = log(w) - e$, where e can take the levels 1 or 3, and a reservation utility of $\overline{U}$. The principal is risk-neutral. Denote the agent's wage conditioned on output as $\underline{w}$ if output is 0 and $\overline{w}$ if output is 100. Only the agent observes his effort. Principals compete for agents, and outputs occur according to the table below.

	Probability of Outputs	
Effort	0	100
$Low (e = 1)$	0.9	0.1
$High\ (e = 3)$	0.5	0.5

What conditions must the optimal contract satisfy, given that the principal can only observe output, not effort? You do not need to solve out for the optimal contract– just provide the equations which would have to be true. Do not just provide inequalities– if the condition is a binding constraint, state it as an equation.

ANSWER: This is a tricky question because it turns out with these numbers that low effort $(e = 1)$ is optimal. In that case, the optimal contract is simple: a flat wage. Because principals compete, a zero-profit constraint must be satisfied, and $w = .9(0) + .1(100) = 10$. The incentive compatibility constraint is an inequality that is not binding: $U(e = 1) = log(10) - 1 \geq U(e = 3) = log(10) - 3$.

The problem was set up to make it look like high effort was optimal, though, and I did not have you solve out for the entire equilibrium, so I gave full credit for finding the optimal contract for when firms compete to offer high-effort contracts. This is not much more difficult. The contract must satisfy a zero-profit constraint for the principal, and an incentive compatibility constraint for the agent. The zero profit constraint is:

$$.5(0) + .5(100) = .5\underline{w} + .5\overline{w},$$

so $100 = \underline{w} + \overline{w}$.

The incentive compatibility constraint is

$$.5log(\underline{w}) + .5log(\overline{w}) - 3 = .9log(\underline{w}) + .1log(\overline{w}) - 1.$$

That is the constraint, which must be an equality since principals are competing to offer the highest-utility contract to the agent (subject to the zero-profit constraint). Solving out a bit further, $4log(\underline{w}) + 4log(\overline{w}) = 20$, so $log(\underline{w}/\overline{w}) = 5$, and $\underline{w}/\overline{w} = Exp(5) \approx 148$.

The participation constraint for the agent would not be binding.

8 Topics in Moral Hazard

8.1. Table 8.1 shows the payoffs in the following game. Sally has been hired by Rayco to do either Job 1, Job 2, or to be a Manager. Rayco believes that Tasks 1 and 2 have equal probabilities of being the efficient ones for Sally to perform. Sally knows which task is efficient, but what she would like best is a job as Manager that gives her the freedom to choose rather than have the job designed for the task. The CEO of Rayco asks Sally which task is efficient. She can either reply "Task 1," "Task 2," or be silent. Her statement, if she makes one, is an example of "cheap talk," because it has no direct effect on anybody's payoff.[2]

Table 8.1: The Right To Silence Game Payoffs

		Sally's Job		
		Job 1	Job 2	Manager
	Task 1 is efficient (.5)	2,5	$1,-2$	3,3
Sally knows:				
	Task 2 is efficient (.5)	$1,-2$	2,5	3,3

Payoffs to: (Sally, Rayco).

(a) If Sally did not have the option of speaking, what would happen?

ANSWER. Rayco would make her a Manager. Rayco's payoff is 3 then, but a deviation to either Job 1 or Job 2 would yield a payoff of .5(5) + .5(-2) = 1.5. Sally has no choices to make.

[2]Joseph Farrell and Matthew Rabin, "Cheap Talk," *Journal of Economic Perspectives*, 10:103-118, Summer 1996.

(b) There exist perfect Bayesian equilibria in which it does not matter how Sally replies. Find one of these in which Sally speaks at least some of the time, and explain why it is an equilibrium. You may assume that Sally is not morally or otherwise bound to speak the truth.

ANSWER. The key to answering this question and part (c) is to know what a perfect bayesian equilibrium is: a strategy for each player, plus any out-of-equilibrium beliefs that are needed. Someone who remembers that a strategy must specify what Sally does in each of the two states of the world and what Rayco does in response to each of Sally's three possible actions is a long ways towards answering the questions correctly. Here, try the following equilibrium:

Sally: Always say "Task 1." Rayco: Give Sally the job as Manager, regardless of her message. Out-of-equilibrium belief: Rayco thinks the probability that Task 1 is efficient is .5 if Sally says Task 2 or is silent.

Sally's payoff is 3, and she cannot change it by deviating. Rayco's payoff is 3, but a deviation to either Job 1 or Job 2 would yield a payoff of .5(5) + .5(-2) = 1.5.

This is an example of a "babbling equilibrium," so called because the uninformed player treats the informed player's cheap talk as meaningless babbling.

(c) There exists a perverse variety of equilibrium in which Sally always tells the truth and never is silent. Find an example of this equilibrium, and explain why neither player would have incentive to deviate to out-of-equilibrium behavior.

ANSWER. Sally: Say Task 1 if Task 1 is efficient. Say Task 2 if Task 2 is efficient. Rayco: If Sally says Task 1, give her Job 1. If Sally says Task 2, give her Job 2. If Sally is silent, give her Job 1. Out-of-equilibrium belief: If Sally is silent, then Task 1 is efficient.

Sally will tell the truth because if she deviates and the wrong task is assigned, her payoff will be 1 instead of 2. In particular, if she deviates and is silent, she will be given Job 1. Rayco has no incentive to deviate, because given that Sally always tells the truth, Rayco's payoff would fall from 5 to -2 from a deviation. If Sally is silent, which never happens in equilibrium, then Rayco's belief requires that Rayco give her Job 1 in order to maximize Rayco's payoff.

This out-of-equilibrium belief is not particularly plausible, and Farrell and Rabin use this as an example of an implausible equilibrium. It is good for learning how to describe equilibria, though!

8.2. Applying the Revelation Principle to a problem
(a.) Increases the welfare of all the players in the model.
(b.) Increases the welfare of just the player offering the contract.
(c.) Increases the welfare of just the player accepting the contract.
@ (d.) Makes the problem easier to model, but does not raise the welfare of the players.
(e.) Makes the problem easier to model and raises the welfare of some players, but not all.

8.3. Mr. Smith is thinking of buying a custom-designed machine from either Mr. Jones or Mr. Brown. This machine costs 5000 dollars to build, and it is useless to anyone but Smith. It is common knowledge that with 90 percent probability the machine will be worth 10,000 dollars to Smith at the time of delivery, one year from today, and with 10 percent probability it will only be worth 2,000 dollars. Smith owns assets of 1,000 dollars. At the time of contracting, Jones and Brown believe there is there is a 20

percent chance that Smith is actually acting as an "undisclosed agent" for Anderson, who has assets of 50,000 dollars.

Find the price be under the following two legal regimes: (a) An undisclosed principal is not responsible for the debts of his agent; and (b) even an undisclosed principal is responsible for the debts of his agent. Also, explain (as part [c]) which rule a moral hazard model like this would tend to support.

ANSWER. (a) The zero profit condition, arising from competition between Jones and Brown, is

$$-5000 + .9P + .1(1000) = 0, \tag{1}$$

because Smith will only pay for the machine with probability 0.9, and otherwise will default and only pay up to his wealth, which is 1. This yields $P \approx 5,444$.

(b) If Anderson is responsible for Smith's debts, then Smith will pay the 5,000 dollars. Hence, zero profits require

$$-5000 + .9P + .1(.2)P + .1(.8)(1000) = 0, \tag{2}$$

which yields $P \approx 5,348$.

(c) Moral hazard tends to support rule (b). This is because it reduces bankruptcy and the agent will be more reluctant to order the machine when there is a high chance it is unprofitable. In the model as constructed, this does not arise, because there is only one type of agent, but more generally it would, because there would be a continuum of types of agents, and some who would buy the machine under rule (b) would find it too expensive under rule (a).

Even in the model as it stands, rule (a) leads to the inefficient outcome that a machine worth 2,000 to Smith is not give to Smith. Rather, he pays his wealth and lets the seller keep the machine, which is inefficient since the machine really is worth 2000 to Smith.

Note: Nobody answered this question correctly, which surprised me. It basically is a question about zero-profit prices. Guessing would have been a good idea here: it is very intuitive that the price would always be above \$5,000, and that it would be higher if the principal never had to cover the agent's debts. You should be able to tell that $P > 10,000$ is impossible, because Smith would never pay it. Also, the sellers compete, so it is their profits that provide a participation constraint, not the benefit to the buyer.

9 Adverse Selection

No questions.

10 Signalling

10.1. If education is to be a good signal of ability,
(a.) Education must be inexpensive for all players.
(b.) Education must be more expensive for the high ability player.
@ (c.) Education must be more expensive for the low ability player.

(d.) Education must be equally expensive for all types of players.
(e.) Education must be costless for some small fraction of players.

10.2. Suppose that with equal probability a worker's ability is $a_L = 1$ or $a_H = 5$, and that the worker chooses any amount of education $y \in [0, \infty)$. Let $U_{worker} = w - \frac{8y}{a}$ and $\pi_{employer} = a - w$.

There is a continuum of pooling equilibria, with different levels of y^*, the amount of education necessary to obtain the high wage. What education levels, y^*, and wages, $w(y)$, are paid in the pooling equilibria, and what is a set of out-of-equilibrium beliefs that supports them? What are the self selection constraints?

ANSWER: A pooling equilibrium for any $y^* \in [0, 0.25]$ is

$$w = \left| \begin{array}{l} 1 \; if \; y \neq y^* \\ 3 \; if \; y = y^* \end{array} \right. \quad (3)$$

with the out-of-equilibrium belief that $Pr(L|(y \neq y^*)) = 1$, and with $y = y^*$ for both types.

The self selection constraints say that neither High nor Low workers want to deviate by acquiring other than y^* education. The most tempting deviation is to zero education, so the constraints are:

$$U_L(y^*) = w(y^*) - 8y^* \geq U_L(0) = w(y \neq y^*) \quad (4)$$

and

$$U_H(y^*) = w(y^*) - \frac{8y^*}{5} \geq U_H(0) = w(y \neq y^*). \quad (5)$$

The constraint on the Lows requires that $y^* \leq 0.25$ for a pooling equilibrium.

10.3. Suppose a salesman's ability might be either $x = 1$ (with probability θ) or $x = 4$, and that if he dresses well, his output is greater, so that his total output is $x + 2s$ where s equals 1 if he dresses well and 0 if he dresses badly. The utility of the salesman is $U = w - \frac{8s}{x}$, where w is his wage. Employers compete for salesmen.

(a) Under full information, what will the wage be for a salesman with low ability?

(b) Show the self selection contraints that must be satisfied in a separating equilibrium under incomplete information.

(c) Find all the equilibria for this game if information is incomplete.

ANSWER. (a) Salesmen with low ability would not dress well. Dressing well would raise their output to 3, but their utility at a wage of 3 would be -5, whereas if they dress poorly their utility is 1. Thus, the wage is 1.

(b) In a separating equilibrium, the low-ability salemen must be satisfied with a contract in which they dress poorly, so it must be true that

$$\pi_L(poorly) = w(poorly) \geq \pi_L(well) = w(well) - 8.$$

The high-ability salemen must be satisfied with a contract in which they dress well, so it must be true that

$$\pi_H(poorly) = w(poorly) \leq \pi_H(well) = w(well) - 2.$$

(c) In the separating equilibrium, $w(poorly) = 1$ and $w(well) = 6$. This satisfies the self selection constraints of part (b) and yield zero profits to the employers.

In one pooling equilibrium, $w(poorly) = \theta + 4(1-\theta)$ and $w(well) = 3$ and all salesmen dress poorly, where θ is the percentage of low-ability salesmen. This is supported by the out-of-equilibrium belief that anyone who dresses well has low ability.

There is no pooling equilibrium in which everyone dresses well. That would require that $w(poorly) = 1$ and $w(well) = \theta + 4(1-\theta) + 2$, and that

$$\pi_L(poorly) = w(poorly) \leq \pi_L(well) = w(well) - 8,$$

so

$$\pi_L(poorly) = 1 \leq \pi_L(well) = \theta + 4(1-\theta) + 2 - 8,$$

but regardless of how close θ is to 0, this is impossible.

10.4. Explain the difference between an "action" and a "strategy," using a signal jamming game as an example.

ANSWER. An action is a choice a player makes in a game. A strategy is a rule giving the player's choices contingent on each possible information set he might reach in the course of the game.

Consider a signal jamming game of entry deterrence in which the incumbent firm's revenue would ordinarily indicate the size of the market, but in which it can jam that signal by reducing quality and causing revenue to be low even if the market is actually large. The incumbent's *action* is his choice of quality— Q_1, for example. His *strategy* is his choice of quality as a function of the size of the market—- $(Q_1|Large, Q_2|Small)$, for example. It may be that the market is almost always small, but the incumbent's strategy must say what quality he will choose in the rare case when the market is large.

Note: This was a surprisingly difficult question; only two of eight students did a satisfactory job. It is very important to be able to explain the difference between actions and strategies, and a good exercise would be to do it using five or so different games as examples.

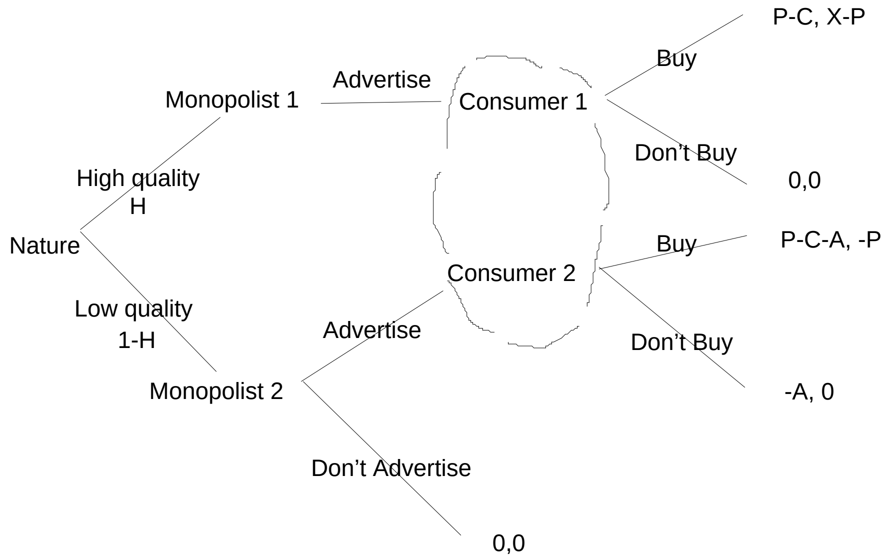
10.5. A consumer faces a monopoly. He initially believes that the probability that the monopoly has a high-quality product is H, and that a high-quality monopoly would be able to send him an advertisement at zero cost. With probability (1-H), though, the monopoly has low quality, and it would cost the firm A to send an ad. He does receive an ad, offering the product at price P. The consumer's utility from a high-quality product is $X > P$, but from a low quality product it is 0, and the production cost is C for the monopolist regardless of quality, where $C < P - A$.[3]

You may assume that the high-quality firm always sends an ad, that the consumer will not buy unless he receives an ad, and that P is exogenous.

[3] This question did not make it clear whether the cost C was incurred before or after the consumer made his decision, but that does not affect any of the answers except the diagram, where I gave credit either way.

(a) Draw the extensive form for this game.

ANSWER.



(b) What is the equilibrium if H is sufficiently high?

ANSWER. If H is high, then both types of monopoly will advertise, and the consumer will buy the product if he gets an advertisement.

(c) If H is low enough, the equilibrium is in mixed strategies. The high-quality firm always advertises, the low quality firm advertises with probability M, and the consumer buys with probability N. Show using Bayes Rule how the consumer's posterior belief R that the firm is high-quality changes once he receives an ad.

ANSWER. The prior is H. The posterior is

$$R = Prob(High|Advertise) = \frac{Prob(Advertise|High)Prob(High)}{Prob(Advertise)} = \frac{(1)(H)}{(1)(H) + (M)(1-H)}.$$

(d) Explain why the equilibrium is not in pure strategies if H is too low (but H is still positive).

ANSWER. If H is low, then it cannot be an equilibrium for the Low firm always to advertise. Suppose H is close to zero. Then if the Low firm always enters, almost all advertising firms will have low quality, and the consumer will not buy. This would result negative payoffs for the Low firms, so they would not want to advertise.

But neither can it be an equilibrium for no Low firm to advertise. In that case, the consumer would buy, which would make it profitable for the Low firm to advertise.

(e) Find the equilibrium probability of M. (You don't have to figure out N.)

ANSWER. The Low firm's mixing probability M must be such that the consumer is indifferent between buying and not buying. His expected payoff from not buying is 0. From buying, the payoff must be computed using his belief about the probability that the seller has high quality which is the posterior probability R. Thus,

$$R(X-P)+(1-R)(-P)=RX-P=\frac{(X)(H)}{(1)(H)+(M)(1-H)}-P$$

Equating this to the payoff of zero from not buying yields $HX=(H+M-MH)P$, so $HX-HP=MP-MHP$ and $M=\frac{H(X-P)}{P(1-H)}$.

10.56. Suppose the Federal Reserve Bank is deciding on May 2 whether or not to intervene to support the dollar. With probability .4 it does not expect the fundamental value of the dollar to be different by July 30. , but with probability .3 it expects the dollar to rise by amount X against the yen and with probability .3 it expects the dollar to fall by X. On May 2, a dollar costs 100 yen. The Fed's objective is to maximize the function $U=\sqrt{D}+F$, where D is the value of the dollar on May 5 and F is the amount of the Fed's profits from trading foreign exchange. On May 2 Fed announces a purchase of B dollars to be made on May 3 (where B can be a negative number). This trade will yield revenues of B, $B(1+X)$ or $B(1-X)$.

Develop a model of this situation and determine what values B might take.

ANSWER. For the model, use the following order of play.
(0) Nature chooses the dollar to rise to $100+X$ with probability .3 (good news), fall to $100-X$ with probability .3 (bad news), and remain unchanged at 100 with probability .4 (no news). The Fed observes Nature's move but the market does not.
(1) The Fed chooses a purchase size B, which could be negative.
(2) The market chooses a price D for the dollar which results in zero expected profits.
(3) The Fed purchase is completed.
(4) The true value of the dollar is revealed and the Fed makes profits or losses.

The Fed's payoff function is $U=\sqrt{D}+F$.

Possible strategies include (a) for the Fed to always choose $B=0$, (b) to choose $B>0$ if and only if the dollar is going to rise, (c) to choose $B>0$ always, and (d) to choose $B>0$ if the dollar is going to rise or if it is going to remain unchanged.

One equilibrium is for the Fed to choose $B=0$ always, supported by the out-of-equilibrium belief that $B>0$ implies good news with probability one and $B<0$ indicates bad news with probability one.

A second equilibrium is for the Fed to buy a very large amount if and only if the news is good. Let us call this amount B^*. In equilibrium, the Fed's trading profits will be zero because D will rise to $1+X$ immediately after it offers to buy B^*. If it refrains from buying, D will fall, because goods news is ruled out. In that case,

$$D=D^*=\frac{.3}{.3+.4}(100-X)+\frac{.4}{.3+.4}(100).$$

If there is no news, the Fed's equilibrium payoff is $\sqrt{D^*}$. If it deviates and buys B^*, its payoff is

$$\sqrt{100+X}+B^*(100-100+X).$$

The Fed will not deviate if

$$\sqrt{D^*}\geq\sqrt{100+X}+B^*(100-100+X).$$

This inequality puts a lower bound on B^* as a function of X. If B^* is too small, the Fed would deviate to make D rise, taking a loss from trading profits. In this equilibrium B^* can take any value larger than that defined by the last equation.

In a third equilibrium, the Fed buys a limited amount B^* if the news is good or there is no news, and does nothing otherwise. This is supported by the belief that if the Fed buys any other non-negative amount besides 0 or B^*, the news is certainly good.

This third equilibrium is fine for the Fed with good news or bad news, but we must check for incentive compatibility for when there is no news. B^* cannot be too large, or when there is no news, the Fed will give up and not trade. If the Fed buys B^*, the value of the dollars rises to

$$D = D^* = \frac{.4}{.4 + .3}(100) + \frac{.3}{.4 + .3}(100 + X).$$

The equilibrium payoff for the Fed with no news is

$$\sqrt{D^*} + B^*(100 - D^*).$$

If it deviates and buys zero, its payoff is $\sqrt{100 - X}$. Thus, the equilibrium requires

$$\sqrt{D^*} + B^*(100 - D^*) \geq \sqrt{100 - X},$$

which means that B^* must not be too large.

Thus, overall, B might take any of a wide range of values in equilibrium, supported by different expectations.

Note: Nobody answered this question correctly, and it is genuinely difficult to answer fully. This is a signalling model, and the basic idea is that the Fed will trade off trading profits against a strong dollar. It is also important to realize that when the Fed announces a trade, the market will change the exchange rate, just as the marketmaker changes the asset price in the Kyle model. From there, the first step is to sort out the order of play, and the second step is to figure out an equilibrium. The difficulty arises because there are multiple equilibria. In general, when you are confused, try either (a) writing out an order of play (*without* trying to think about the equilibrium) or (b) telling yourself a story, imagining yourself in the place of the players.

11 Bargaining

(11.1) Smith makes a take-it-or-leave-it offer to Jones of X for his car, which is worth 2000 dollars to Jones and 8000 dollars to Smith. What is X?

ANSWER. 2000 dollars. Jones will accept, in the only Nash equilibrium of this game. 2001 is not a Nash equilibrium offer, because 2000.5 dominates it.

11.2. Two parties, the Offeror and the Acceptor, are trying to agree to the clauses in a contract. They have already agreed to a basic contract, splitting a surplus 50-50, for a surplus of Z for each player. The offeror can at cost C offer an additional clause which the acceptor can accept outright, inspect carefully (at cost M), or reject outright. The additional clause is either "genuine," yielding the Offeror X_g and the Acceptor Y_g

if accepted, or "misleading," yielding the Offeror X_m (where $X_m > X_g > 0$) and the Acceptor $-Y_m < 0$.

What will happen in equilibrium?

ANSWER. One equilibrium is for the Offeror never to offer the additional clause and for the Acceptor to believe, out of equilibrium, than any clause offered is Misleading and hence to reject it.

A more interesting equilibrium is in mixed strategies. There is not a pure strategy equilibrium in which the Misleading clause is always offered, because it would always be rejected then. There is not a pure strategy equilibrium in which the Genuine clause is always offered, because there would never be any inspection. This is an auditing game. If the inspection cost, M, is not too high, there is a mixed strategy equilibrium.

Let the probability of offering a Genuine clause be θ and the probability of inspecting be γ. Equating the payoffs to the Offeror from offering Genuine or Misleading clauses gives us

$$\pi(genuine) = -C + X_g = \pi(misleading) = -C + (1-\gamma)X_m,$$

which solves to $\gamma = 1 - \frac{X_g}{X_m}$.

Equating the payoffs to the Acceptor from Accepting and Inspecting gives us

$$\pi(Accept) = \theta Y_g - (1-\theta)Y_m = \pi(Inspect) = -M + \theta Y_g,$$

which solves to $\theta = 1 - \frac{M}{Y_m}$.

Note: This was a moderately difficult question. The key here is to tell yourself a story about what will happen, taking the point of view of each player in turn.

12. Auctions

12.1. If I am bidding for a rare coin in a second-price sealed-bid auction, and the coin would be worth 1000 dollars to me, the most reasonable bid below is
(a.) 1100 dollars.
@(b.) 1000 dollars.
(c.) 950 dollars.
(d.) 100 dollars.
(e.) 0 dollars.

13. Pricing

13.1. Two firms submit bids to supply a goverment contract. Firm 1 has known cost c. Firm 2 has cost of either c, or, with probability θ, infinity. θ is a probability lying between 0 and 1, inclusive of both end points. The government will buy one unit at the lowest price, paying up to reservation price R.

What happens? Be as precise as possible about the prices the two firms charge. (I do not expect most people to analyze this completely correctly. Do the best you can, describing what happens verbally and mathematically.)

ANSWER. If theta is big enough, firm 2 charges R.

If θ is zero, both firms charge c.

If θ is low, both firms play mixed strategies.

13.2. A seller faces a large number of buyers whose market demand is given by $P = \alpha - \beta Q$. Production marginal cost is constant at c.

(a) What is the monopoly price and profit?

ANSWER: Profit is $PQ - cQ$ or $(\alpha - \beta Q - c)Q$. The first order condition is $\alpha - 2\beta Q - c = 0$, so $Q = \frac{\alpha - c}{2\beta}$. The price is then $P = \alpha - \beta\frac{\alpha-c}{2\beta} = \alpha - \frac{\alpha-c}{2} = \frac{\alpha+c}{2}$. The profit is $(P-c)Q = (\frac{\alpha+c}{2} - c)\frac{\alpha-c}{2\beta} = \frac{(\alpha-c)^2}{4\beta}$.

I was shocked at how badly people did on this undergraduate-level question, and the similarly easy (b).

(b) What are the prices under perfect price discrimination if the seller can make take-it-or-leave-it offers? What is the profit?

ANSWER: Under perfect price discrimination, there is a continuum of prices along the demand curve from α to c. The profit equals the area of the triangle under the demand curve and above the flat MC curve, which is $(1/2)(\alpha - c)Q(c) = (1/2)(\alpha - c)\frac{\alpha-c}{\beta} = \frac{(\alpha-c)^2}{2\beta}$. Notice how profit has doubled compared to the simple monopoly profit.

(c) What are the prices under perfect price discrimination if the buyer and sellers bargain over the price and split the surplus evenly? What is the profit?

ANSWER: If buyers and sellers split the surplus evenly, then instead of the seller getting the entire surplus, he only gets half, so profits are half those found in part (b). There is a continuum of prices between $c + \frac{\alpha-c}{2}$ and c. The profit is $\frac{(\alpha-c)^2}{4\beta}$, the same as the monopoly profit in this special case.

14. Entry

No questions.

15. The New Industrial Organization

15.1. Renting helps the durable monopolist because
(a) it permits him to produce a less durable product.
(b) it rescues him from a Prisoner's Dilemma.
(c) it reduces adverse selection.
@ (d) he is then not tempted to lower his future price.

16. Other Topics (Externalities, Miscellaneous, Opportunity Cost)

16.1. A monopolist faces a linear demand curve $q = a - bp$ and has constant marginal cost c.

(i) Show that the magnitude of the elasticity of demand is increasing in b.

(ii) Compute the welfare loss from monopoly pricing. How does the ratio of the deadweight loss to total welfare vary with b?

(iii) Suppose you were told outright that the deadweight loss was monotonic in the slope of the demand curve. Prove without algebra that it must be monotonically *decreasing* in the slope.

ANSWER. (i) The elasticity of demand is $\epsilon = -\frac{p}{q}\frac{dq}{dp}$. With linear demand, $\epsilon = \frac{pb}{a-pb}$. Differentiating with respect to b yields

$$\frac{d\epsilon}{db} = \frac{p}{a-bp} + \frac{bp^2}{(a-bp)^2},$$

which is positive. Thus, the elasticity increases in b.

(ii) Keeping the other parameters fixed, let's change b. The deadweight loss is the triangle,

$$D(b) = \frac{1}{2}[p_m(b) - c][a - bc - q_m(b)]$$

Differentiate to get

$$D'(b) = \frac{1}{2}\{[p'_m(b)[a - bc - q_m(b)] - [p_m(b) - c][c + q'_m(b)]\},$$

or

$$D'(b) = \frac{1}{2}[p_m(b) - c][p'_m(b)b - c - q'_m(b)].$$

The first two terms of this are positive, because price exceeds cost. To find the second term, we need the monopoly price and output. These are $q_m = (a - bc)/2$ and $p_m = (a + bc)/2b$. Thus, the second term is $-(a+bc)/2b$, which is negative and we can conclude that $D'(b) < 0$. The deadweight loss falls as the demand curve gets steeper.

(iii) The answer, and the intuition behind part (ii), is that as the the demand curve gets steeper, but the price-intercept stays the same, the market is shrinking. If it shrinks enough, the deadweight loss goes to zero, because the entire social surplus goes to zero.

16.2. Why does it not create inefficiency if I bid up the price of the Onassis diamond ring, hurting the other bidders in an auction, while it does create inefficiency if I smoke during the auction, hurting the other bidders?

ANSWER. Pecuniary vs. real externalities.

16.3. The idea behind Chapter 11 of the bankruptcy code is that some firms can avoid bankruptcy if given some temporary relief from creditors. They go to a judge, and persuade him that they have a good chance of ultimately repaying their creditors if they are allowed to delay repayment. It is argued this is socially useful, because it results in fewer firms going bankrupt and less waste of resources. Why does the concept of opportunity cost suggest that this reasoning is wrong, and that immediate bankruptcy is actually a good thing?

ANSWER. Keeping the firm alive has an opportunity cost—- its assets are tied up in that firm, so no other firm can use them. If it goes bankrupt, those resources are freed up.

16.4. A millionaire lives next to a small woods owned by a lumber company. The lumber company will make a good profit if it cuts down all the trees this year, but this would ruin the value of the millionaire's residence.

(a) Explain why it does not make any difference to the survival of the trees whether the law allows the lumber company to cut down the trees without the millionaire's permission or whether it requires the company to get the millionaire's permission first.

(b) What would happen if the law currently did not require permission, but lobbying efforts could change the law in time to be apply to this tree harvest?

ANSWER. Not recorded.

16.5. Explain, using an Edgeworth Box, how, if two agents have convex preferences, and two equilibrium prices are possible starting from a given endowment, it is necessary that the two agents prefer different equilibria.

ANSWER. Not recorded.

16.6. If I say that midterms reduce Craig's utility twice as much as they reduce Chris's, what assumptions am I making about interpersonal comparison, ordinality, and cardinality of utility functions?

ANSWER. The assumption is that utility can be compared interpersonally (Craig vs. Chris), that it is ordinal (we can say that utility falls because of a midterm), and that it is cardinal (it not only falls, but by an amount which can meaningfully be called "twice" that of someone else).

16.7. Construct a numerical example to show that a monopoly might produce a quality level greater than the social optimum.

ANSWER: Here is one example— the one I used in class. Let there be two consumers, each of whom buys up to one unit. The eager consumer will pay up to 102 for high quality or low quality. The reluctant consumer will pay up to 82 for low quality and 102 for high quality. Low quality costs 2 per unit to produce, while high quality costs 20. The seller cannot observe consumer type.

If he offers just low quality, he should charge a price of 82, for profit of 2(82-2) = 160 and social surplus of (102+82-2(2)) = 180. If he offers just high quality, he should charge a price of 102, for profit of 2(102-20) = 164 and social surplus of (102+102-2(20)) = 164. He can't do better by offering both qualities. Thus, he will offer high quality, but that is not socially optimal.

One thing to watch out for in constructing a two-customer example is whether the seller will choose to sell to both customers, or only one.

16.8. A comedian chooses the offensiveness level of his jokes. His payoff from offensiveness x is $6x - x^2$. His audience gets a payoff of zero because he is able to perfectly price discriminate. Certain other people are offended, and incur cost $2x$.

(a) What is the socially optimal level of offensiveness?

ANSWER: Total surplus is $6x - x^2 - 2x$, or $4x - x^2$. Thus, the first order condition is $4 - 2x = 0$, and $x^* = 2$.

(b) What is the laissez faire level of offensiveness?

ANSWER. 3. That maximizes $6x - x^2$, which has first order condition $6 - 3x = 0$.

(c) Explain in words why the level of offensiveness you found in part (a) is socially optimal.

ANSWER: If the offensiveness level were any higher, the extra benefit to the comedian would be less than the extra cost to the people who are offended. If the offensiveness level were any lower, it should be increased, because then the the extra benefit to the comedian would be greater than the extra cost to the people who are offended.

This proved to be a hard question. Its purpose was to illustrate that not just math, but words can be made precise. A number of bad answers are: "2 solves the social optimization problem," "2 both maximizes the comedian's payoff and minimizes the cost to other people," "2 gives weight to both the comedian's payoff and the cost to others," and "2 maximizes the social surplus." If you got this wrong, go back and read Rhoads again.