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Http://www.rasmusen/org/GI/chap10_mechanisms.pdf.

10 Mechanism Design and Post-Contractual Hidden Knowledge

For two 75 minute sessions. Price discrimination is in a separate section.

Post-Contractual Hidden Knowledge

Information is complete in moral hazard games, but in **moral hazard with hidden knowledge**, also called **post-contractual adverse selection**, the agent, but not the principal, observes a move of Nature after the game begins, but before he takes his action.

Information is symmetric at the time of contracting– thus the "moral hazard"— but becomes asymmetric later– thus the "hidden knowledge".

From the principal's point of view, agents are identical at the beginning of the game but develop private types midway through.

There is just ONE participation constraint even if there are eventually n possible types of agents.

There are still N incentive compatibility constraints.

Production Game VIII: Mechanism Design

1 The principal offers the agent a wage contract of the form $w(q,m)$, where q is output and m is a message to be sent by the agent.
2 The agent accepts or rejects the principal's offer.
3 Nature chooses the state of the world s, according to probability distribution $F(s)$, where the state s is *good* with probability 0.5 and *bad* with probability 0.5. The agent observes s, but the principal does not.
4 If the agent accepted, he exerts effort e unobserved by the principal, and sends message $m \in \{good, bad\}$ to him.
5 Output is $q(e,s)$, where $q(e, good) = 3e$ and $q(e, bad) = e$, and the wage is paid.

Payoffs: If the agent rejects the contract, $\pi_{agent} = \bar{U} = 0$ and $\pi_{principal} = 0$. If the agent accepts the contract, $\pi_{agent} = U(e,w,s) = w - e^2$ and $\pi_{principal} = V(q-w) = q - w$.

The optimal efforts and outputs s are $e_g = 1.5$ and $q_g = 4.5$ or $e_b = 0.5$ and $q_b = 0.5$. The principal solves the problem,

$$\underset{q_g, q_b, w_g, w_b}{\overset{Maximize}{}} \; [0.5(q_g - w_g) + 0.5(q_b - w_b)], \tag{1}$$

using two forcing contracts, (q_g, w_g) if he reports $m = good$ and (q_b, w_b) if he reports $m = bad$, where producing the wrong output for a given contract results in boiling in oil.

The participation constraints of Production Game VII now merge.

$$0.5\pi_{agent}(q_g, w_g|good) + 0.5\pi_{agent}(q_b, w_b|bad) \geq 0. \qquad (2)$$

$$0.5\left(w_g - \left(\frac{q_g}{3}\right)^2\right) + 0.5\left(w_b - q_b^2\right) \geq 0. \qquad (3)$$

The self-selection constraints are the same as in Production Game VII. In the good state, the agent must choose the good-state contract,so

$$\pi_{agent}(q_g, w_g|good) \geq \pi_{agent}(q_b, w_b|good) \qquad (4)$$

$$w_g - \left(\frac{q_g}{3}\right)^2 \geq w_b - \left(\frac{q_b}{3}\right)^2 \qquad (5)$$

and in the bad state he must choose the bad-state contract, so

$$\pi_{agent}(q_b, w_b|bad) = w_b - q_b^2 \geq \pi_{agent}(q_g, w_g|bad) = w_g - q_g^2. \qquad (6)$$

The single participation constraint is binding.

The good state's self-selection constraint will be binding.

Solving yields $w_b = \frac{5}{9}q_b^2$ and $w_g = \frac{1}{9}q_g^2 + \frac{4}{9}q_b^2$.

Returning to the principal's maximization problem in (1) and subsituting for w_b and w_g, we can rewrite it as

$$\underset{q_g, q_b}{Maximize} \quad \pi_{principal} = \left[0.5 \left(q_g - \frac{q_g^2}{9} - \frac{4q_b^2}{9} \right) + 0.5 \left(q_b - \frac{5q_b^2}{9} \right) \right] \tag{7}$$

with no constraints. The first-order conditions are

$$\frac{\partial \pi_{principal}}{\partial q_g} = 0.5 \left(1 - \left[\frac{2}{9} \right] q_g \right) = 0, \tag{8}$$

so $q_g = 4.5$, and

$$\frac{\partial \pi_{principal}}{\partial q_b} = 0.5 \left(-\frac{8q_b}{9} \right) + 0.5 \left(1 - \frac{10q_b}{9} \right) = 0, \tag{9}$$

so $q_b = \frac{9}{18} = .5$. We can then find the wages that satisfy the constraints, which are $w_g \approx 2.36$ and $w_b \approx 0.14$.

As in Production Game VII, in the good state the effort is at the first-best level while in the bad state it is less.

Unlike in Production Game VII the agent does not earn informational rents, because at the time of contracting he has no private information.

The principal in Production Game VIII is less constrained, and thus able to (a) come closer to the first-best when the state is bad, and (b) reduce the rents to the agent.

Observable but Nonverifiable Information and the Maskin Matching Scheme

If the courts cannot observe the state, a contract conditioning the wage on the state is unenforceable, no better than having no contract at all. We say that the variable s is **nonverifiable** if contracts based on it cannot be enforced.

Even if the courts will not enforce a contract based on a variable, if both the principal and the agent observe it they should be able to come up with a more efficient contract than if just the agent observes it.

MASKIN:

(1) Principal and agent simultaneously send messages m_p and m_a to the court saying whether the state is good or bad. If $m_p \neq m_a$, then no contract is chosen and both players earn zero payoffs. If $m_p = m_a$, the court enforced part (2) of the scheme.

(2) The agent is paid the wage $(w|q)$ with either the good-state forcing contract $(2.25|4.5)$ or the bad-state forcing contract $(0.25|0.5)$, depending on his report m_a, or is boiled in oil if he the output is inappropriate to his report.

Renegotiation is a problem as in the Holmstrom teams model.

Wise Guys Nicholas Pileggi quotes low- level gangster Henry Hill saying :

> "For instance, say I've got a fifty-thousand-dollar hijack load, and when I make my delivery, instead of getting paid, I get stuck up. What am I supposed to do? Go to the cops? Not likely. Shoot it out? I'm a hijacker, not a cowboy. No. The only way to guarantee that I'm not going ripped off by anybody is to be established with a member, like Paulie. Somebody who is a made man. A member of a crime family. A soldier. Then ... that's the end of the ball game. Goodbye. They're dead... Of course, problems can arise when the guys sticking you up are associated with wiseguys too. Then there has to be a sit-down between your wiseguys and their wiseguys. What usually happens then is that the wiseguys divide whatever you stole for their own pocket, send you and the guy who robbed you home with nothing. And if you complain, you're dead."

The low-level gangsters have a strong incentive to report the same story, or the higher-ups take away the property under dispute.

This may sound familiar to parents too— "If we can't resolve this, the toy is going in the closet for a whole week."

It is perhaps even the wisdom of Solomon.

The Revelation Principle *For every contract $w(q, m)$ that leads to lying (that is, to $m \neq s$), there is a contract $w^*(q, m)$ with the same outcome for every s but no incentive for the agent to lie.*

A **direct mechanism**, in which the agents tell the truth in equilibrium, can be found that is equivalent to any **indirect mechanism** in which they lie.

Suppose we are trying to design a mechanism to make people with higher incomes pay higher taxes, but anyone who makes $70,000 a year can claim he makes $50,000 and we do not have the resources to catch him.

We could design a mechanism in which higher reported incomes pay higher taxes, but reports of $50,000 would come from both people who truly have that income and people whose income is $70,000.

The revelation principle says that we can rewrite the tax code to set the tax to be the same for taxpayers earning $70,000 and for those earning $50,000, and the same amount of taxes will be collected without anyone having incentive to lie.

The Revelation Principle does depend heavily on an implicit assumption we have made: the principal cannot breach his contract.

10.4 The Groves Mechanism: The Streetlight Game

The Order of Play

0. Nature chooses the value v_i that each of 5 householders i places on having a streetlight installed, using distribution $f_i(v_i)$. Only Householder i observes v_i.

1. The mayor announces a mechanism, M, which requires a householder who reports m to pay $w(m)$ if the streetlight is installed and installs the streetlight if $g(m_1, ..., m_5) \geq 0$.

2. Householder i reports value m_i simultaneously with all other householders.

3. If $g(m_1, ..., m_5) \geq 0$, the streetlight is built and householder i pays $w(m_i)$.

Payoffs

The mayor tries to maximize social welfare, including the welfare of taxpayers besides the 5 householders. His payoff is zero if the streetlight is not built. Otherwise, it is

$$\pi_{mayor} = \left(\sum_{i=1}^{5} v_i \right) - 100, \tag{10}$$

subject to the constraint that $\sum_{i=1}^{n} w(m_i) \geq 100$ so he can raise the taxes to pay for the light.

The payoff of householder i is zero if the streetlight is not built. Otherwise it is

$$\pi_i(m_1, ..., m_5) = v_i - w(m_i). \tag{11}$$

Each of 5 citizens has value v_i from a streetlight, known only to himself. Each sends a message m_i of his value to the mayor. The streetlight costs 100. What mechanism should the mayor use? He will set tax $w(m)$ for message m if he installs the streetlight.

$$M_1: \quad \left(w = 20, Build\ iff\ \sum_{i=1}^{5} m_i \geq 100 \right);$$

$$M_2: \quad \left(w(m_i) = Max\{m_i, 0\}, Build\ iff\ \sum_{j=1}^{5} m_j \geq 100 \right).$$

$$M_3: \quad \left(w(m_i) = 100 - \sum_{j \neq i} m_j, Build\ iff\ \sum_{j=1}^{5} m_j \geq 100 \right).$$

Suppose the true values are $(10, 30, 30, 30, 80)$. What message will be sent under each mechanism?

M1: meaningless ones

M2: Nash equilibrium ones,e. g. (0, 25, 25,25,25)

M3: dominant strategy equilibrium ones, the truth

$$M_3: \quad \left(w(m_i) = 100 - \sum_{j \neq i} m_j, Build\ iff\ \sum_{j=1}^{5} m_j \geq 100 \right).$$

w1= 100- 170 = -70
w2 = 100-150 =-50
w3 = -50
w4= -50
w5 = 100- 100 =0

The sum of valuations is 180, so the streetlight is installed.

Each player is indifferent about his message in equilibrium, except that he does not want to reduce it so much that the streetlight would not be installed (it would have to be negative for that!)

What if player 2 thinks the other players will overreport a total of 200 instead of their true sum of 150? He's still happy for the project to go through (he gets an even bigger tax) so he won't underreport.

What if player 2 thinks the other players will UNDERrreport a total of 60 instead of their true sum of 150? He is sad. He could get the project to go through by reporting 40 instead of his true value of 30. But then his tax would be 100-60=40, and that's a little too high for him to want to lie. Instead, he'll tell the truth and kill the project.

What if player 2 thinks the other players will UNDER-rreport a total of 69 instead of their true sum of 150? He is sad. He could get the project to go through by reporting 31 instead of his true value of 30. But then his tax would be 100-69=31, and that's a little too high for him to want to lie. Instead, he'll tell the truth and kill the project.

A **dominant-strategy mechanism**, M_3.

$$M_3: \quad \left(w(m_i) = 100 - \sum_{j \neq i} m_j, Build\ iff\ \sum_{j=1}^{5} m_j \geq 100 \right). \tag{12}$$

Suppose Smith's valuation is 40 and the sum of the valuations is 110, so the project is indeed efficient. If the other players report their truthful sum of 70, Smith's payoff from truthful reporting is his valuation of 40 minus his tax of 30. Reporting more would not change his payoff, while reporting less than 30 would reduce it to 0.

What if the other players lie? If they underreported, announcing 50 instead of the truthful 70, then Smith could make up the difference by overreporting 60, but his payoff would be -10 $(= 40 + 50 - 100)$ so he would do better to report the truthful 40, killing the project and leaving himself with a payoff of 0.

If the other players overreported, announcing 80 instead of the truthful 70, then Smith would benefit if the project went through, and he should report at least 20.

Whether he reports 20, 21, 40, or 400, the streetlight is built and he pays a tax of 20 under mechanism M_3, leaving him with payoff of 20 (= 40-20).

In particular, he is willing to report exactly 40, so it is a weakly best response to the other players' lies.

The problem with a dominant-strategy mechanisms like M_3 is that it is not budget balancing.

This is not so bad if the budget had a surplus, as required in our game rules above, but it turns out to have a deficit except in special cases where it is perfectly balanced (e.g., $m_i = 20$ for all 5 householders).

Unravelling: Voluntary Statements, Lying Prohibited

Suppose the agent is prohibited from lying and only has a choice between telling the truth or remaining silent.

In Production Game VIII, this set-up would give the agent two possible message sets. If the state were good, the agent's message would be taken from $m \in \{good, silent\}$. If the state were *bad*, the agent's message would be taken from $m \in \{bad, silent\}$.

Suppose s is uniform on $[0, 10]$ and the agent's payoff is increasing in the principal's estimate of s.

If $s = 2$, he can send the uninformative message $m \geq 0$ (equivalent to no message), or the message $m \geq 1$, or $m = 2$, but not the lie that $m \geq 4.36$.

When $s = 2$ the agent might as well send a message that is the exact truth: "$m = 2$."

If he were to choose the message "$m \geq 1$" instead, the principal's first thought might be to estimate s as the average value in the interval $[1, 10]$, which is 5.5.

But the principal would realize that no agent with a value of s greater than 5.5 would want to send the message "$m \geq 1$" if 5.5 was the resulting deduction.

The Sender-Receiver Game of Crawford and Sobel: Coarse Information Transmission

Even if the informed and uninformed players have different incentives and can't commit to a mechanism, if their incentives are close enough, truthful if imperfect messages can be sent in equilibrium.

Let us call the informed player "the sender" (AGENT) and the uninformed player "the receiver" (PRINCIPAL).

Crawford & Sobel (1982) "Strategic Information Transmission."

The Order of Play
0 Nature chooses the sender's type to be $t \sim U[0, 10]$.
1 The sender chooses message $m \in [0, 10]$.
2 The receiver chooses action $a \in [0, 10]$.

Payoffs

$$\begin{aligned} \pi_{sender} &= \alpha - (a - [t+1])^2 \\ \pi_{receiver} &= \alpha - (a - t)^2 \end{aligned} \tag{13}$$

Perfect truthtelling cannot happen in equilibrium.

One equilibrium is the pooling equilibrium in which the sender's message is ignored and the receiver chooses $a = Et = 5$.

Partial Pooling Equilibrium 3
Sender: Send $m = 0$ if $t \in [0,3]$ or $m = 10$ if $t \in [3,10]$.
Receiver: Choose $a = 1.5$ if $m < 3$ and $a = 6.5$ if $m \geq 3$
Out-of-equilibrium belief: If m is something other than 0 or 10, then $t \sim U[0,3]$ if $m \in [0,3)$ and $t \sim U[3,10]$ if $a \in [3,10]$.

The Sender has reduced his message space to two messages, LOW (=0) and HIGH (=10).

The receiver's optimal strategy in a partially pooling equilibrium is to choose his action to equal the expected value of the type in the interval the sender has chosen. Thus, if $m = 0$, the receiver will choose $a = x/2$ and if $m = 10$ he will choose $a = (x + 10)/2$.

The receiver's equilibrium response determines the sender's payoffs from his two messages. The payoffs between which the sender chooses are:

$$\begin{aligned} \pi_{sender,m=0} &= \alpha - \left([t+1] - \frac{x}{2} \right)^2 \\ \pi_{sender,m=10} &= \alpha - \left(\frac{10+x}{2} - [t+1] \right)^2 \end{aligned} \tag{14}$$

The payoffs between which the sender chooses are:

$$\begin{aligned}\pi_{sender,m=0} &= \alpha - \left([t+1] - \frac{x}{2}\right)^2 \\ \pi_{sender,m=10} &= \alpha - \left(\frac{10+x}{2} - [t+1]\right)^2\end{aligned} \tag{15}$$

There exists a value x such that if $t = x$, the sender is indifferent between $m = 0$ and $m = 10$, but if t is lower he prefers $m = 0$ and if t is higher he prefers $m = 10$.

To find x, equate the two payoffs in expression (17) and simplify to obtain

$$[t+1] - \frac{x}{2} = \frac{10+x}{2} - [t+1]. \tag{16}$$

We set $t = x$ at the point of indifference, and solving for x then yields $x = 3$.

Thus, the divergence in preferences of the sender and receiver coarsens the message space, in effect.

Comments on the Crawford and Sobel

If instead of wanting $(t+1)$ to be the action, the preferences of sender and receiver diverged more—

say, to $(t+8)$— then there would only be the uninformative pooling equilibrium.

If they divereged less— say, to $(t+0.1)$— then there would exist other partially pooling equilibria that had more than just two effective messages and would distinguish between three or more intervals instead of between just two.

In the Crawford-Sobel Game, the receiver cannot commit to the way he reacts to the message, so this is not a mechanism design problem.

Nor is the sender punished for lying, so the unravelling argument for truthtelling does not apply.

Nor do the players' payoffs depend directly on the message, which might permit signalling.

Instead, this is a **cheap-talk game**, so called because of these very absences: m does not affect the payoff directly, the players cannot commit to future actions, and lying brings no direct penalty.

10.6 Rate-of-Return Regulation and Government Procurement (Baron & Myerson's 1982 "Regulating a Monopolist with Unknown Costs.")

Suppose the government wants a firm to provide cable television service to a city.

The firm knows more about its costs before agreeing to accept the franchise (adverse selection), discovers more after accepting it and beginning operations (moral hazard with hidden knowledge), and exerts greater or smaller effort to keep costs low (moral hazard with hidden actions).

The government cannot just cover the firm's costs, because the firm would always claim high costs and exert low effort.

Instead, the government might auction off the right to provide the service,

might allow the firm a maximum price (a **price cap**),

or might agree to compensate the firm to varying degrees for different levels of cost (**rate-of- return regulation**).

Regulatory franchises are like government procurement. If the government wants to purchase a missile, it also has the problem of how much to offer the firm. Flat price, or cost-plus?

The first version of the model will be one in which the government can observe the firm's type and so the first-best can be attained. It will be a benchmark for our later versions.

Procurement I: Full Information

Players: The government and the firm.

The Order of Play

0 Nature assigns the firm expensive problems with the project, which add costs of x, with probability θ. A firm is thus "normal", with type N and $s = 0$, or "expensive", with type X and $s = x$. The government and the firm both observe the type.

1 The government offers a contract $\{w(m) = c(m) + p(m), c(m)\}$ which pays the firm its observed cost c and a profit p if it announces its type to be m and incurs cost $c(m)$, and pays the firm zero otherwise.

2 The firm accepts or rejects the contract.

3 If the firm accepts, it chooses effort level e, unobserved by the government.

4 The firm finishes the missile at a cost of $c = \bar{c} + s - e$, which is observed by the government, plus an additional unobserved cost of $f(e - \bar{c})$. The government reimburses $c(m)$ and pays $p(m)$.

Payoffs. Both firm and government are risk-neutral and both receive payoffs of zero if the firm rejects the contract. If the firm accepts, its payoff is

$$\pi_{firm} = p - f(e - \bar{c}) \tag{17}$$

$$\tag{18}$$

where $f(e - \bar{c})$, the cost of effort, is increasing and convex, so $f' > 0$ and $f'' > 0$. Assume for technical convenience that f is increasingly convex, so $f''' > 0$. The government's payoff is

$$\pi_{government} = B - (1+t)c - tp - f, \tag{19}$$

where B is the benefit of the missile and t is the deadweight loss from the taxation needed for government spending.

This is substantial. Hausman & Poterba (1987) estimate the loss to be around \$0.30 for each \$1 of tax revenue raised at the margin for the United States.

Assume for the moment that B is large enough that the government definitely wishes to build the missile.

FIRST BEST: The government pays p_N to a normal firm with the cost c_N, p_X to an expensive firm with the cost c_X, and $p = 0$ to a firm that does not achieve its appropriate cost level.

The government thus maximizes its payoff, equation (22), by choice of p_X, p_N, c_X, and c_N, subject to participation and incentive compatibility constraints.

The expensive firm exerts effort $e = \bar{c} + x - c_X$, achieves $c = c_X$, generating unobserved effort disutility $f(e - \bar{c}) = f(x - c_X)$, so its participation constraint, that type X's payoff from reporting that it is type X, is:

$$\begin{aligned} \pi_X(X) &\geq 0 \\ p_X - f(x - c_X) &\geq 0. \end{aligned} \tag{20}$$

Similarly, in equilibrium the normal firm exerts effort $e = \bar{c} - c_N$, so its participation constraint is

$$\begin{aligned} \pi_N(N) &\geq 0 \\ p_N - f(-c_N) &\geq 0 \end{aligned} \tag{21}$$

The incentive compatibility constraints are trivial here: the government can use a forcing contract that pays a firm zero if it generates the wrong cost for its type, since types are observable.

To make a firm's payoff zero and reduce the deadweight loss from taxation, the government will provide prices that do no more than equal the firm's disutility of effort. Since there is no uncertainty, we can invert the cost equation and write it as $e = \bar{c} + x - c$ or $e = \bar{c} - c$. The prices will be $p_X = f(e - \bar{c}) = f(x - c_X)$ and $p_N = f(e - \bar{c}) = f(-c_N)$.

Suppose the government knows the firm has expensive problems. Substituting the price p_X into the government's payoff function, equation (22), yields

$$\pi_{government} = B - (1+t)c_X - tf(x - c_X) - f(x - c_X). \quad (22)$$

Since $f'' > 0$, the government's payoff function is concave, and standard optimization techniques can be used. The first-order condition for c_X is

$$\frac{\partial \pi_{government}}{\partial c_X} = -(1+t) + (1+t)f'(x - c_X) = 0, \quad (23)$$

so

$$f'(x - c_X) = 1. \quad (24)$$

Equation(27) is the crucial efficiency condition for effort. Since the argument of f is $(e - \bar{c})$, whenever $f' = 1$ the effort level is efficient. At the optimal effort level, the marginal disutility of effort equals the marginal reduction in cost because of effort. This is the first-best efficient effort level, which we will denote by $e^* \equiv e : \{f'(e - \bar{c}) = 1\}$.

If we derived the first-order condition for the normal firm we would find $f'(-c_N) = 1$ in the same way, so $c_N =$

$c_X - x$. Also, if the equilibrium disutility of effort is the same for both firms, then both must choose the same effort, e^*, though the normal firm can reach a lower cost target with that effort. The cost targets assigned to each firm are $c_X = \bar{c} + x - e^*$ and $c_N = \bar{c} - e^*$. Since both types must exert the same effort, e^*, to achieve their different targets, $p_X = f(e^* - \bar{c}) = p_N$. The two firms exert the same efficient effort level and are paid the same price to compensate for the disutility of effort. Let us call this price level p^*.

Procurement II: Incomplete Information (Adverse Selection)

In the second variant of the game, the existence of expensive problems is not observed by the government.

If the government offered the two contracts of Procurement I, both types of firm would accept the expensive-cost contract, which has a price of p^* for a cost of $c = \bar{c} + x - e^*$, enough to compensate the firm with expensive problems for its effort, and $p = 0$ for any other cost.

That is the cheapest pooling contract, since any contract that paid less would violate the expensive-cost firm's participation constraint.

It is inefficient, though, because the normal firm can reduce costs to $c = \bar{c} + x - e^*$ by exerting effort lower than e^*.

The government would still be willing to build the missile, since the social cost of having the normal firm build the missile inefficiently is still lower than of having the expensive-cost firm build it efficiently.

But it will turn out that separating contracts will yield higher welfare than the pooling contract.

A separating contract menu superior to the pooling contract would be a choice of

(1) the old pooling contract $(p^*, c = \bar{c} + x - e^*)$, and

(2) a new contract that offers a slightly higher price p but requires reimbursable costs c to be slightly lower. By definition of e^* in first-order condition (27),

$f'(e^* - \bar{c}) = 1$, so $f'(e' - \bar{c}) < 1$ for the effort the normal firm exerts in the old pooling contract. If the normal firm increased its effort from e' by some small amount Δe, costs would fall by $(1)\Delta e$ but the firm would only have to be paid $f'(e' - \bar{c})\Delta e$ more to compensate for its extra disutility.

Thus, there is a new contract that would draw the normal firm away from the old pooling contract and be preferred by the government.

We will proceed to find the optimal pair of contracts (c_N, p_N) (c_X, p_X) for firms that announce *Normal* or *Expensive* (with $p = 0$ for other cost levels).

Adapting the government's payoff in (22) to the probability θ of a expensive firm and probability $1 - \theta$ of a normal firm, the government's maximization problem under incomplete information is

$$\underset{c_N, c_X, p_N, p_X}{Maximize} \quad \theta \left[B - (1+t)c_X - tp_X - f(x - c_X)\right] + [1 - \theta] \left[B - \right. \tag{25}$$

A separating contract must satisfy participation constraints and incentive compatibility constraints for each type of firm. The participation constraints are the same as in Procurement I: inequalities (23) and (24):

$$\pi_X(X) = p_X - f(x - c_X) \geq 0 \quad (23)$$

and

$$\pi_N(N) = p_N - f(-c_N) \geq 0 \quad (24)$$

The incentive compatibility constraints are

$$\pi_X(X) = p_X - f(x - c_X) \geq \pi_X(N) = p_N - f(x - c_N), \tag{26}$$

$$\pi_N(N) = p_N - f(-c_N) \geq \pi_N(X) = p_X - f(-c_X). \tag{27}$$

Since the normal firm can achieve the same cost level as the expensive firm with less effort, inequality (30) tells us that if we are to have $c_N < c_X$, as is necessary for us to have a separating equilibrium, we need $p_N > p_X$. The second half of inequality (30) must be positive. If the expensive firm's participation constraint, inequality (23), is satisfied, then $p_X - f(-c_X) > 0$. This, in turn implies that (24) is a strong inequality; the normal firm's participation constraint is nonbinding.

The expensive firm's participation constraint, (23), will be binding (and therefore satisfied as an equality), because the government wishes to keep the price p low to reduce the deadweight loss of extra taxation, the $-tp_X$ term in problem (28).

The normal firm's incentive compatibility constraint must also be binding, because if the pair (c_N, p_N) were strictly more attractive for the normal firm, the government could reduce the price p_N and save on the $-tp_N$ term in problem (28).

Knowing that constraints (23) and (30) are binding, we can write, from constraint (23),

$$p_X = f(x - c_X) \tag{28}$$

$$p_N = f(-c_N) + f(x - c_X) - f(-c_X). \tag{29}$$

Substituting for p_X and p_N from (31) and (32) into the maximization problem, (28), reduces the problem to

$$\underset{c_N, c_X}{\mathit{Maximize}} \quad \theta[B - (1+t)c_X - tf(x - c_X) - f(x - c_X)]$$

$$+[1-\theta][B - (1+t)c_N - tf(-c_N) - tf(x - c_X) + tf(-c \tag{30}$$

(1) The first-order condition with respect to c_N is

$$(1-\theta)[-(1+t)+tf'(-c_N)+f'(-c_N)]=0, \tag{31}$$

which simplifies to

$$f'(-c_N)=1. \tag{32}$$

Thus, as in Procurement I, $f'_N(e-\bar{c})=1$. The normal firm chooses the efficient effort level e^* in equilibrium, and c_N takes the same value as it did in Procurement I. Equation (32) can be rewritten as

$$p_N=p^*+f(x-c_X)-f(-c_X). \tag{33}$$

Because $f(x-c_X) > f(-c_X)$, equation (36) shows that $p_N > p^*$. Incomplete information increases the price for the normal firm, which earns more than its reservation utility in the game with incomplete information. Since the expensive firm will earn exactly zero, this means that the government is on average providing its supplier with an above-market rate of return, not because of corruption or political influence, but because that is the way to induce normal suppliers to reveal that they do not have expensive problems.

(2) The first-order condition with respect to c_X is

$$\theta\left[-(1+t)+tf'(x-c_X)+f'(x-c_X)\right]+[1-\theta]\left[tf'(x-c_X)-t \tag{34}$$

This can be rewritten as

$$f'(x-c_X)=1-\left(\frac{1-\theta}{\theta(1+t)}\right)\left[tf'(x-c_X)-tf'(-c_X)\right]. \tag{35}$$

Since the right-hand-side of equation (38) is less than one, the expensive firm has a lower level of f' than the normal firm, and if f' is lower and $f''>0$, effort must be less than the optimum, e^*.

Since, however, the expensive firm's participation constraint, (23), is satisfied as an equality, it must also be true that $p_X<p^*$. The expensive firm's price is lower than under full information, although since its effort is lower its payoff stays the same.

To summarize, the government's optimal contract will

induce the normal firm to exert the first-best efficient effort level and achieve the first-best cost level,

but will yield that firm a positive profit.

The contract will induce the expensive firm to exert something less than the first-best effort level

and result in a cost level higher than the first-best,

but its profit will be zero.

There is a tradeoff between the government's two objectives of inducing the correct amount of effort and minimizing the subsidy to the firm.

Under incomplete information, not only must the subsidies be positive but the normal firm earns **informational rents**; the government offers a contract that pays the normal firm more than under complete information to prevent it from mimicking an expensive firm and choosing an inefficiently low effort.

The expensive firm, however, does choose an inefficiently low effort, because if it were assigned greater effort it would have to be paid a greater subsidy, which would tempt the normal firm to imitate it. In equilibrium, the government has compromised by having some probability of an inefficiently high subsidy ex post, and some probability of inefficiently low effort.