11 Signalling

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This is for two 75-minute sessions. I skip section 11.5 (two-signal models)

In the first session, I covered 11.1-11.4, but didn't have time for the separating equilibrium of the continuous-signal model.

I did that in the second session, but then didnt' have time to do more than lay out the idea and model of Countersignalling (I didn't solve out the model).

Interesting digressions that came up:

1. What if there is a monopoly screener?

2. What if signalling productive? How would you model that in Education I?

In the first printing of the 4th edition, I messed up in section 11.6 on the value of v*. It is 4F/q, not 2F/q. I fix that here.

Education I

The Order of Play

0 Nature chooses the worker's ability $a \in \{2, 5.5\}$, the *Low* and *High* ability each having probability 0.5. The variable a is observed by the worker, but not by the employers.
1 The worker chooses education level $s \in \{0, 1\}$.
2 The employers each offer a wage contract $w(s)$.
3 The worker accepts a contract, or rejects both of them.
4 Output equals a.

Payoffs

The worker's payoff is his wage minus his cost of education, and the employer's is his profit.

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w. \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{for the accepted-contract employer,} \\ 0 & \text{for the other employer.} \end{cases}$$

Equilibria

Pooling Equilibrium 1.1 $\begin{cases} s(Low) = s(High) = 0 \\ w(0) = w(1) = 3.75 \\ Prob(a = Low|s = 1) = 0.5 \end{cases}$

If $Prob(a = Low|s = 1) = 0$ the pooling equilibrium breaks down.

Separating Equilibrium 1.2 $\begin{cases} s(Low) = 0, s(High) = 1 \\ w(0) = 2, w(1) = 5.5 \end{cases}$

The Intuitive Criterion rules out the beliefs of the pooling equilibrium.

The Constraints

The participation constraints for the employers require that

$$w(0) \leq a_L = 2 \text{ and } w(1) \leq a_H = 5.5. \tag{1}$$

Competition between the employers makes the expressions in (1) hold as equalities.

The self-selection constraint of the *Lows* is

$$U_L(s=0) \geq U_L(s=1), \tag{2}$$

which in Education I is

$$w(0) - 0 \geq w(1) - \frac{8(1)}{2}. \tag{3}$$

Since in Separating Equilibrium 1.2 the separating wage of the *Low*s is 2 and the separating wage of the *High*s is 5.5 from (1), the self-selection constraint (3) is satisfied.

The self-selection constraint of the *High*s is

$$U_H(s=1) \geq U_H(s=0), \tag{4}$$

which in Education I is

$$w(1) - \frac{8(1)}{5.5} \geq w(0) - 0. \tag{5}$$

Another Conceivable Equilibrium

There is another conceivable pooling equilibrium for Education I, in which $s(Low) = s(High) = 1$, but this turns out not to be an equilibrium, because the *Low*s would deviate to zero education.

Even if such a deviation caused the employer to believe they were low - ability with probability 1 and reduce their wage to 2, the low - ability workers would still prefer to deviate, because

$$U_L(s=0) = 2 \geq U_L(s=1) = 3.75 - \frac{8(1)}{2}. \tag{6}$$

The Single Crossing Property

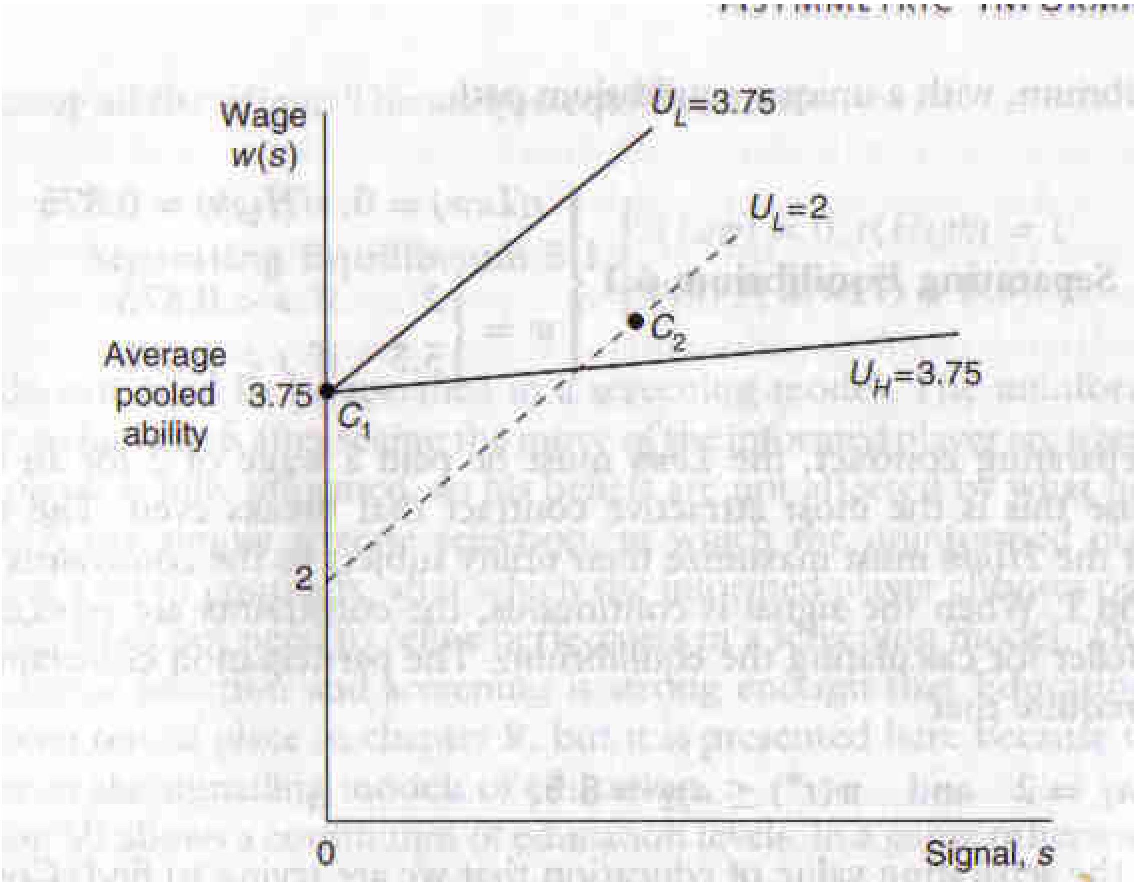


Figure 1: Education VI: No Pooling Equilibrium in a Screening Game

The indifference curves in Education I are like this too. They cross a single time— the single crossing property.

Education II: Modelling Trembles so Nothing Is Out of Equilibrium

The Order of Play

0 Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5. (a is observed by the worker, but not by the employer.) With probability 0.001, Nature endows a worker with free education of $s = 1$.

...

Payoffs

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{Accepts contract } w \text{ (ordinarily)} \\ w & \text{Accepts contract } w \text{ (with free education)} \\ 0 & \text{Refuses contract} \end{cases}$$

Out-of-equilibrium, if the employer observed $s = 1$, he could use Bayes's Rule:

$$\begin{aligned} Prob(a = Low|s = 1) &= \frac{Prob(s=1|a=L)Prob(L)}{Prob(s=1|a=L)Prob(L)+Prob(s=1|a=H)Prob(H)} \\ &= \frac{(0.001)(0.5)}{(0.001)(0.5)+(0.001)(0.5)} \\ &= 0.5 \end{aligned} \tag{7}$$

Normal workers would not deviate from $s = 0$ because it would not increase the employer's estimate of their ability.

Education III: No Separating Equilibrium, Two Pooling Equilibria

the worker abilities from $\{2, 5.5\}$ to $\{2, 12\}$.

Pooling Equilibrium 3.1

$$\begin{cases} s(Low) = s(High) = 0 \\ w(0) = w(1) = 7 \\ Prob(a = Low|s = 1) = 0.5 \end{cases}$$

Pooling Equilibrium 3.2
$$\begin{cases} s(Low) = s(High) = 1 \\ w(0) = 2, w(1) = 7 \\ Prob(a = Low|s = 0) = 1 \end{cases}$$

Education IV: Continuous Signals and a Continua of Equilibria

The Order of Play

0 Nature chooses the worker's ability $a \in \{2, 5.5\}$, the *Low* and *High* ability each having probability 0.5. The variable a is observed by the worker, but not by the employers.
1 The worker chooses education level $s \in [0, \infty$.
2 The employers each offer a wage contract $w(s)$.
3 The worker accepts a contract, or rejects both of them.
4 Output equals a.

Payoffs

The worker's payoff is his wage minus his cost of education, and the employer's is his profit.

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{Accepts contract } w. \\ 0 & \text{Rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{Contract is accepted.} \\ 0 & \text{Contract rejected.} \end{cases}$$

The Continua of Equilibria

Each s^* in the interval $[0, \bar{s}]$ supports a different equilibrium.

$$\textbf{Pooling Equilibrium 4.1} \quad \begin{cases} s(Low) = s(High) = s^* \\ w(s^*) = 3.75 \\ w(s \neq s^*) = 2 \\ Prob(a = Low | s \neq s^*) = 1 \end{cases}$$

The critical value $\bar{s}$ can be discovered from the incentive compatibility constraint of the *Low* type, which is binding if $s^* = \bar{s}$.

The most tempting deviation is to zero education, so that is the deviation that appears in the constraint.

$$U_L(s = 0) = 2 \leq U_L(s = \bar{s}) = 3.75 - \frac{8\bar{s}}{2}. \tag{8}$$

Equation (8) yields $\bar{s} = \frac{7}{16}$. Any value of s^* less than $\frac{7}{16}$ will also support a pooling equilibrium.

Note that the incentive-compatibility constraint of the *High* type is not binding. If a *High* deviates to $s = 0$, he, too, will be thought to be a *Low*, so

$$U_H(s = 0) = 2 \leq U_H(s = \frac{7}{16}) = 3.75 - \frac{8\bar{s}}{5.5} \approx 3.1. \tag{9}$$

Separating Equilibrium 4.2

$$\begin{cases} s(Low) = 0, \quad s(High) = s^* \\ w(s^*) = 5.5 \\ w(s \neq s^*) = 2 \\ Prob(a = Low | s \notin \{0, s^*\}) = 1 \end{cases}$$

The critical value $\overline{s}$ can be discovered from the incentive-compatibility constraint of the *Low*, which is binding if $s^* = \overline{s}$.

$$U_L(s = 0) = 2 \geq U_L(s = \overline{s}) = 5.5 - \frac{8\overline{s}}{2}. \tag{10}$$

Equation (10) yields $\overline{s} = \frac{7}{8}$.

If the education needed for the wage of 5.5 is too great, the *High* workers will give up on education too. Their incentive compatibility constraint requires that

$$U_H(s = 0) = 2 \leq U_H(s = \overline{\overline{s}}) = 5.5 - \frac{8\overline{\overline{s}}}{5.5}. \tag{11}$$

Equation (11) yields $\overline{\overline{s}} = \frac{77}{32}$.

Problems in Applying Signalling to Education

On the empirical level, the first question to ask of a signalling model of education is, "What is education?".

For operational purposes this means, "In what units is education measured?".

Two possible answers are "years of education" and "grade point average."

Layard & Psacharopoulos (1974) give three rationales for rejecting signalling as an important motive for education.

1. Dropouts get as high a rate of return on education as those who complete degrees, so the signal is not the diploma, although it might be the years of education.

2. Wage differentials between different education levels rise with age, although one would expect the signal to be less important after the employer has acquired more observations on the worker's output.

3. Testing is not widely used for hiring, despite its low cost relative to education.

Efficient Uses of Signalling

1. it allows the employer to match workers with jobs suited to their talents.

2. Signalling keeps talented workers from moving to jobs where their productivity is lower but their talent is known.

3. If ability is endogenous — moral hazard rather than adverse selection — signalling encourages workers to acquire ability.

Education V: Screening with a Discrete Signal

The Order of Play

0 Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5. Employers do not observe ability, but the worker does.

1 Each employer offers a wage contract $w(s)$.

2 The worker chooses education level $s \in \{0, 1\}$.

3 The worker accepts a contract, or rejects both of them.

4 Output equals a.

Payoffs

$$\pi_{worker} = \begin{cases} w - \frac{8s}{a} & \text{if the worker accepts contract } w. \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{Contract is accepted.} \\ 0 & \text{Contract is rejected.} \end{cases}$$

Separating Equilibrium 5.1

Education V has no pooling equilibrium, because if one employer tried to offer the zero profit pooling contract, $w(0) = 3.75$, the other employer would offer $w(1) = 5.5$ and draw away all the *High*s. The unique equilibrium is

$$\begin{cases} s(Low) = 0, s(High) = 1 \\ w(0) = 2, w(1) = 5.5 \end{cases}$$

Beliefs do not need to be specified.

Education VI: Screening with a Continuous Signal

Players

A worker and two employers.

The Order of Play

0 Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5. Employers do not observe ability, but the worker does.

1 Each employer offers a wage contract $w(s)$.

2 The worker choose education level $s \in [0, 1]$.

3 The worker chooses a contract, or rejects both of them.

4 Output equals a.

Payoffs

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w. \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{Contract is accepted.} \\ 0 & \text{Contract is rejected.} \end{cases}$$

Separating Equilibrium 6.1

$$\begin{cases} s(Low) = 0, s(High) = 0.875 \\ \\ w = \begin{cases} 2 & \text{if } s < 0.875 \\ 5.5 & \text{if } s \geq 0.875 \end{cases} \end{cases}$$

The participation constraints for the employers require that

$$w(0) \leq a_L = 2 \text{ and } w(s^*) \leq a_H = 5.5, \qquad (12)$$

where s^* is the separating value of education that we are trying to find.

The self selection constraint for the low-ability workers is

$$U_L(s = 0) \geq U_L(s = s^*), \qquad (13)$$

which in Education VI is

$$w(0) - 0 \geq w(s^*) - \frac{8s^*}{2}. \qquad (14)$$

Since the separating wage is 2 for the *Low*s and 5.5 for the *High*s, constraint (14) is satisfied as an equality if $s^* = 0.875$, which is the crucial education level in Separating Equilibrium 6.1.

$$U_H(s = 0) = w(0) \leq U_H(s = s^*) = w(s^*) - \frac{8s^*}{5.5}. \qquad (15)$$

If $s^* = 0.875$, inequality (15) is true, and it would also be true for higher values of s^*. But competition makes $s^* = 0.875$.

Competition between Firms

Competition in offering attractive contracts rules out pooling contracts. The nonpooling constraint, required by competition between employers, is

$$U_H(s = s^*) \geq U_H(pooling), \tag{16}$$

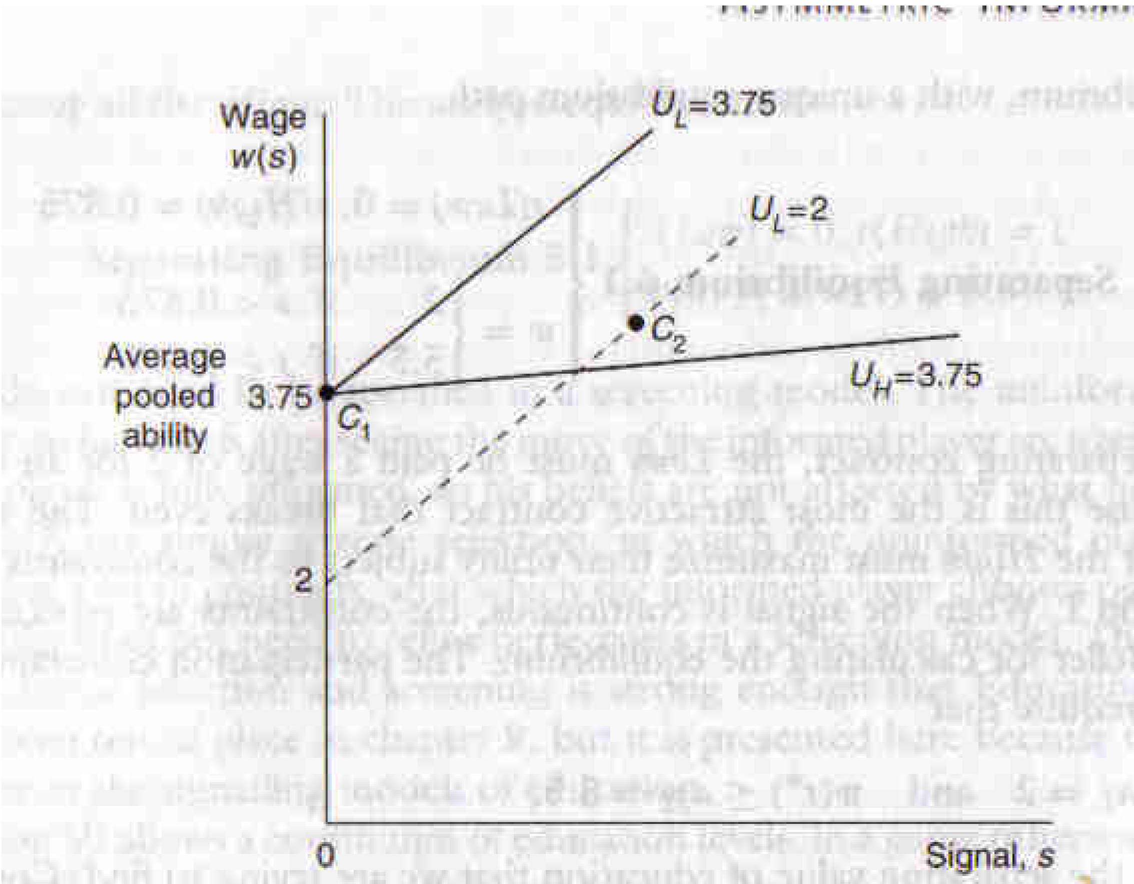
which, for Education VI, is, using the most attractive possible pooling contract,

$$w(s^*) - \frac{8s^*}{5.5} \geq 3.75. \tag{17}$$

Since the payoff of *High*s in the separating contract is 4.23 ($= 5.5 - 8 \cdot 0.875/5.5$, rounded), the nonpooling constraint is satisfied.

No Pooling Equilibrium in Screening: Education VI

The screening game Education VI lacks a pooling equilibrium, which would require the outcome $\{s = 0, w(0) = 3.75\}$, shown as C_1 here.



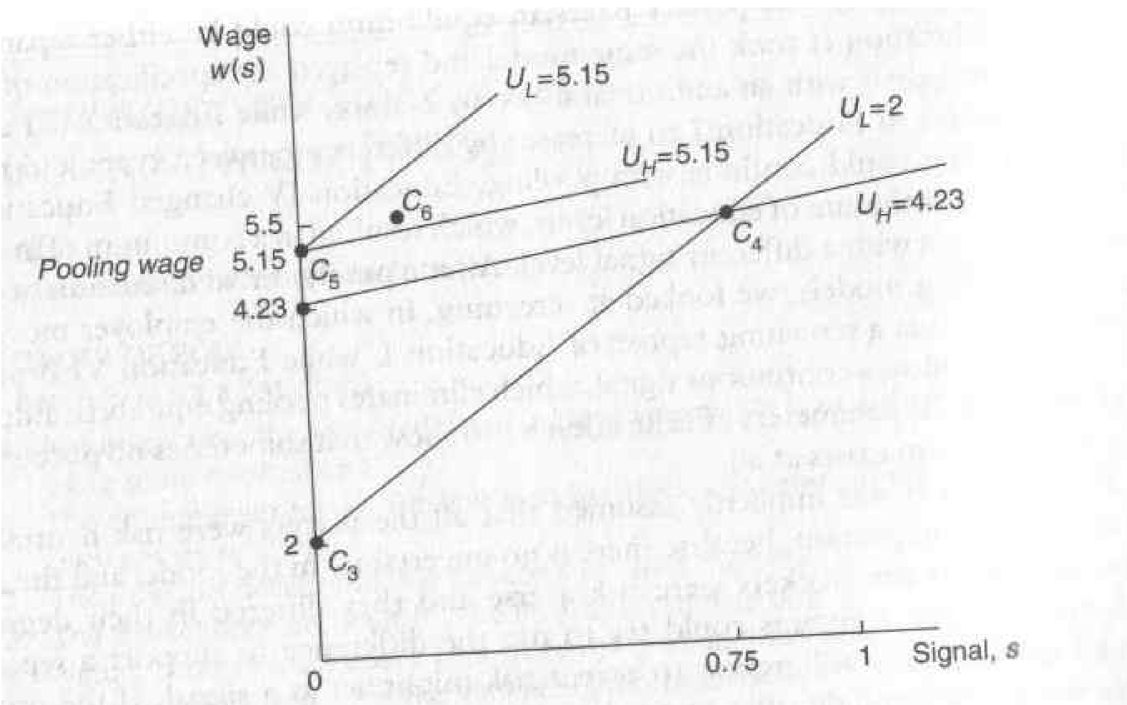
No employer can offer a pooling contract, because the other employer could always profit by offering a separating contract paying more to the educated, e.g., C_2.

C_2: Pay 5 to workers with an education of $s = 0.5$, a payoff of 4.89 to the *High*s ($= 5 - [8 \cdot 0.5]/5.5$, rounded) and 3 to the *Low*s ($= 5 - 8 \cdot 0.5/2$). Positive payoff for employer.

Nonexistence of a Pure-Strategy Equilibrium

Let the proportion of *High*s be 0.9 instead of 0.5

The pair (C_3, C_4) is the most attractive pair of contracts that separates *High*s from *Low*s. *Low* workers accept contract C_3, obtain $s = 0$, and receive a wage of 2, their ability. *High*s accept contract C_4, obtain $s = 0.875$, and receive a wage of 5.5, their ability. Education is not attractive to *Low*s.



The wage of the pooling contract C_5 is 5.15, so even *High*s prefer C_5 to (C_3, C_4). But our reasoning that no pooling equilibrium exists is still valid. No pure-strategy Nash equilibrium exists.

Ways to Communicate

1. Cheap Talk Games. The Sender's message is costless and there is no penalty for lying.

2. Truthful Announcement Games. The Sender's message is costless. He may be silent instead of sending a message, but if he sends a message it must be truthful.

3. Auditing Games. The Sender's message might or might not be costly. The Receiver may audit the message at some cost and discover if the Sender was lying.

4. Mechanism Games. The Sender's message might or might not be costly. Before he sends it, he commits to a contract with the Receiver, with their decisions based on what they can observe and enforcement based on what can be verified by the courts.

5. Signalling Games. The Sender's message is costly when he lies, and more costly when he lies than when he tells the truth. He sends it before the Receiver takes any action.

6. Expensive-Talk Games. The Sender's message is costly, but the cost is the same regardless of his type. There is no penalty for lying.

7. Screening Games. The Sender's message is costly when he lies, and more costly when he lies than when he tells the truth. He sends it in response to an offer by the Receiver.

Limit Pricing as Signal Jamming

The Order of Play

0 Nature chooses the reservation price v using the continuous density $h(v)$ on the support $[c, d]$, observed only by the incumbent. (The parameter c is marginal cost.)

1 The incumbent chooses the first-period price p_1, generating sales of q if $p_1 \leq v$ and 0 otherwise.The variables p_1 and q are observed by both players.

2 The rival decides whether to enter at cost F or to stay out.

3 If the rival did not enter, the incumbent chooses the second period price, $p_{2,monopoly}$, generating sales of q if $p_{2m} \leq v$ and 0 otherwise.

If the rival did enter, the duopoly price is $p_2(v)$, with $p_2 \geq c$ and $\frac{dp_2}{dv} > 0$.

Payoffs

If the rival does not enter and p_1 and p_{2m} are no greater than v, the payoffs are

$$\begin{aligned} \pi_{incumbent} &= (p_1 - c)q + (p_2 - c)q \\ \pi_{rival} &= 0. \end{aligned} \tag{18}$$

If the rival does enter and p_1 are no greater than v, the payoffs are

$$\begin{aligned} \pi_{incumbent} &= (p_1 - c)q + (p_2(v) - c)\left(\frac{q}{2}\right) \\ \pi_{rival} &= -F + (p_2(v) - c)\left(\frac{q}{2}\right). \end{aligned} \tag{19}$$

Assumptions on Functional Form

Assume that the second-period duopoly price is halfway between marginal cost and the monopoly price, so

$$p_2(v) = \frac{c+v}{2}, \tag{20}$$

the distribution of reservation prices is uniform, so

$$h(v) = \frac{1}{d-c}, \tag{21}$$

that

$$c = 0$$

and that $d > 6.4F/q$.

Finding the Equilibrium

Work back from the end.

The second period price is $p_{2,monopoly} = v$, since there is no threat of future entry.

The rival's expected payoff from entry depends on his beliefs about v, which in turn depend upon which of the multiple equilibria of this game is being played out.

If the incumbent were nonstrategic, he would charge $p_1 = v$, maximizing his first-period profit.

The rival would deduce the value v and would enter in the second period if p_1 exceeded the critical value $v^* = c + \frac{4F}{q}$, a value which yields zero profits because under our special assumptions,

$$p_2(v) = \frac{v}{2}$$

and

$$\begin{aligned} \pi_{rival,enter} &= -F + \left[(p_2(v^*) - c) \left(\frac{q}{2}\right) \right] \\ &= -F + \left[\left(\frac{v^*}{2}\right) \left(\frac{q}{2}\right) \right] \\ &= -F + \left[\left(\frac{4F/q}{2}\right) \left(\frac{q}{2}\right) \right] \\ &= 0 \end{aligned} \tag{22}$$

If the incumbent is willing to charge a lower p_1, though, and accept lower first-period profits, he may be able to deter entry.

A Limit Pricing Equilibrium

Incumbent: $p_2 = v$.

$$p_1 = \begin{cases} v \ if \ \ v < a \\ a \ if \ \ v \in [a, b] \\ v \ if \ \ v > b \end{cases}$$

Rival: Enter if $p_1 > a$. Otherwise, stay out.

If the rival observes $p_1 \in (a, b]$, his out-of-equilibrium belief is that $v = \frac{h(v)}{\int_{p_1}^{b} h(v) dv}$, the expected value of v if it lies between p_1 and b.

$$a = \left(\frac{2}{5}\right)\left(\frac{4F}{q}\right)$$

$$b = \left(\frac{8}{5}\right)\left(\frac{4F}{q}\right)$$

Will the Incumbent Deviate?

If the incumbent deters entry, his payoff is

$$\pi_{incumbent}(no\ entry) = (p_1 - c)q + (v - c)q \tag{23}$$

and if he does not it is

$$\pi_{incumbent}(entry) = (v - c)q + (p_2(v) - c)\left(\frac{q}{2}\right). \tag{24}$$

The incumbent's advantage from limit pricing is the difference between these when $p_1 = a$, which is

$$[(a - c)q + (v - c)q] - \left[(v - c)q + (p_2(v) - c)\left(\frac{q}{2}\right)\right]. \tag{25}$$

This advantage is declining in v, and b is defined as the value at which it equals zero. Choosing $v = b$ in expression (25), equating to zero, and solving for a yields

$$a = \frac{c + p_2(b)}{2} = \frac{b}{4} \tag{26}$$

using our specific functional forms.

Will the Rival Deviate?

The rival's payoff if he enters is

$$\begin{aligned}\pi_{rival,enter} &= -F + (p_2(v) - c)\left(\frac{q}{2}\right) \\ &= -F + \left(\int_a^b p_2(v)\left(\frac{h(v)}{\int_a^b h(v)dv}\right)dv - c\right)\left(\frac{q}{2}\right),\end{aligned} \tag{27}$$

where the density for the expectation $Ep_2(v)$ is $\frac{h(v)}{\int_a^b h(v)dv}$ instead of just $h(v)$ because it is conditional on v being between a and b, rather than v taking any of its possible values.

Deriving the value of a and b

The entry payoff of the rival equals zero in equilibrium for the entry-deterring values of a and b. That is how we derive those values.

Given our specific assumptions on $h(v)$ and $p_2(v)$, $p_2(b) = b/2$ and $h(v) = 1/d$. Recall that $a = b/4$ Since $\int_a^b h(v)dv = \int_a^b (1/d)dv = \frac{b}{d} - \frac{a}{d}$,

$$\begin{aligned}
\pi_{rival} &= -F + \left(\int_a^b \left(\frac{v}{2}\right) \left(\frac{\frac{1}{d}}{\frac{b}{d} - \frac{a}{d}} \right) dv \right) \left(\frac{q}{2}\right) \\
&= -F + \Big|_{v=a}^{b} \left(\frac{v^2}{4(b-a)} \right) \left(\frac{q}{2}\right) \\
&= -F + \left(\frac{b^2}{4(b-a)} - \frac{a^2}{4(b-a)} \right) \left(\frac{q}{2}\right) \\
&= -F + \left(b^2 - \frac{b^2}{16} \right) \left(\frac{q}{8(b - \frac{b}{4})} \right)
\end{aligned} \tag{28}$$

so

$$b = \left(\frac{8}{5}\right) \left(\frac{4F}{q}\right), \tag{29}$$

where $b < d$ because of our special assumption that $d > 6.4F/q$.

The Values in the Signal-Jamming Range

$$b = \left(\frac{8}{5}\right)\left(\frac{4F}{q}\right),$$

Since $a = b/4$,

$$a = \left(\frac{2}{5}\right)\left(\frac{4F}{q}\right). \tag{30}$$

It is useful to compare the limits a and b with v^*, the value of v for which entry yields a payoff of zero to the rival. That value, which we found above using equation (22), is

$$v^* = \frac{4F}{q}. \tag{31}$$

General Comments on Limit Pricing

The values of a and b were chosen so that, in effect, v^* was the expected value of v on $[a, b]$, and the rival would be deterred even though he did not know the exact value of v.

If $a \geq v^*$, then the expected value of v on $[a, b]$ would be greater than v^*, and the rival would feel safe in entering if the incumbent charged $p_1 = a$.

Thus, to use limit pricing, the incumbent must charge strictly less than the monopoly price appropriate if the duopoly market would yield zero profits to an rival.

This kind of signal jamming reduces the information that reaches the rival compared to nonstrategic behavior, since the rival learns the precise value of v only if v is less than a or greater than b.

Countersignalling

This idea is modelled in:

Feltovich, Nick, Richmond Harbaugh & Ted To (2002) "Too Cool for School? Signalling and Countersignalling," *RAND Journal of Economics*, 33(4): 630-649 (Winter 2002)

This is the idea that only mid-quality people signal. The highest quality do not. Why?

Think about restaurants. Would a restaurant of very high quality benefit from posting a notice of its high hygiene level? See:

Jin, Ginger & Philip Leslie (2003) "The Effect of Information on Product Quality: Evidence from Restaurant Hygiene Grade Cards," *Quarterly Journal of Economics*, 118(2): 409-451 (2003).

Countersignalling: The Bank Game

Players
Banks and depositors.

The Order of Play

0 Nature chooses the solvency θ_i of each bank on a continuum of length 1, using cumulative distribution $F(\theta)$ on the support $[-10, 10]$ and $F(0) = 0.2$.

1 Bank i chooses to spend s_i on its building, where it must spend at least $\bar{s} = 9$ to operate at all, and otherwise must exit.

2 Depositors on a continuum of length $D = 1$ observe $\hat{\theta} = \theta_i + u_i$, where the u_i values are chosen independently and take values of $-5, 0$, and $+5$ with equal probability.

3 Each depositor chooses a bank.

Payoffs in the Bank Game

Payoffs

Bank i's payoff if it attracts depositors is

$$\pi_i = \frac{D}{B} - \left(\frac{s_i}{10+\theta_i}\right), \tag{32}$$

where B is the interval of banks that attract depositors. If the bank attracts no depositors, its payoff is $-\left(\frac{s_i}{10+\theta_i}\right)$.

A depositor's payoff is 1 if he picks a bank with quality at or above 0, and 0 if he picks quality below 0.

An Equilibrium

Banks with solvency $\theta \in [-10, 0]$ choose $s = 0$. They do not signal, and attract no depositors.

Banks with solvency $\theta \in [0, 5)$ choose

$$s = s^* = 12.5.$$

They attract depositors.

Banks with solvency $\theta \in [5, 10]$ choose $s = 9$. They attract depositors.

Depositors choose banks with either $\hat{\theta} \geq 5$ or $s \geq s^*$ or both.

In this equilibrium, $B = 1 - F(0) = 0.8$ because depositors choose the banks which are solvent.

If $\hat{\theta} \geq 5$, a depositor can be sure the bank is solvent, because the minimum possible true level of θ is 0, with $u = +5$. Otherwise, they rely on the signal.

Note: setting the minimum level of s equal to 9, and the fraction of insolvent potential banks to 0.2 are just for concreteness.

The Self Selection Constraints

The self-selection constraint requires that banks with $\theta < 0$ not signal. Thus, the crucial signal level, s^*, requires that $\pi_0(s = 0) = \pi_0(s = s^*)$, so

$$0 = \frac{D}{B} - \left(\frac{s^*}{10 + 0} \right) \tag{33}$$

Solving equation (33) yields

$$s^* = \frac{10D}{1 - F(0)} = 12.5. \tag{34}$$

Bank's Payoffs

The payoff of a bank in $\theta = [0, 5)$ if it enters and signals s^* is

$$\pi(s = s^*) = \frac{D}{B} - \left(\frac{s^*}{10+\theta}\right). \tag{35}$$

This is at least as high as the zero payoff from not entering, because substituting in for B and s^* yields

$$\pi(s = s^*) = \frac{D}{1-F(0)} - \frac{\frac{10D}{1-F(0)}}{10+\theta} \geq \frac{D}{1-F(0)} - \frac{10D}{10[1-F(0)]} = 0. \tag{36}$$

Mid-Quality Banks

The mid-quality banks could also pretend to be high-quality banks by entering, not signalling and hoping to have a positive measurement error u. The payoff from that would be composed of a $2/3$ probability of attracting no customers and a $1/3$ probability of attracting D/B:

$$\pi(s = 9) = \left(\tfrac{2}{3}\right)(0) + \left(\tfrac{1}{3}\right)\left(\tfrac{D}{B}\right) - \left(\tfrac{4}{10+\theta}\right) \tag{37}$$

The payoff of the high-quality banks in $\theta = [5, 10]$ is highest from entry without signalling, when it is $\pi(\theta) = D/B - 9/(10 + \theta)$, which is positive. Signalling s^* merely adds costs, while not entering yields a payoff of zero.

Other equilibria exist with higher levels of signal, and fewer banks entering. Consumers might believe that the mid-range of bank quality signals with $s = \dot{s} > s^*$. In that case, the mid-range shrinks, since for $\theta = 0$ signalling no longer yields positive profits.

Countersignalling where Lack of a Signal Can Be Good News

In some countersignalling models the absence of a signal can have a positive, additional, effect on the uninformed player's estimate of the informed player's quality.

In our banking game, that can't happen because if the noisy observation is $\hat{\theta} \geq 5$, depositors have the highest possible opinion of the bank's safety.

Thus, the high-quality banks abstain from signalling simply to save money, not because the countersignal actually helps them.

Suppose, though, that we added new depositors to the model, an amount small enough to be measure zero so they would not affect the equilibrium, and these new depositors had the special features that:

(a) they could not observe even $\hat{\theta}$, and

(b) their payoffs were 0 not just for insolvent banks but for any bank with $\theta < 7$.

They would observe only that some banks have $s = 9$ and some have $s = s^*$. Since they would be looking for especially high-quality banks, they would actually prefer to choose a bank with the lower signal value of $s = 9$, because they could be sure that if $s = s^*$ then $\theta < 7$.