TIROLE, CHAPTER 2, p. 102: SWAN (1970)

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Kelvin Lancaster's "characteristics" approach to price theory looked at consumers as demanding not goods, but characteristics of goods. A consumer doesn't really want a car; he wants transportation, speed, privacy, and music, safety. When buying a car and riding a bus compete, it is because they provide different amounts of these characteristics, and we can think of each characteristic as having a separate value.

This is also the principle of "hedonic regression," where the value of a car is seen as the sum of the values of its characteristics, e.g. for car i,

$$p_i = \alpha + \beta_1 Speed_i + \beta_2 MP3Player_i + \beta_3 CrashProtection_i \tag{1}$$

In the Lancastrian spirit, suppose demand for light bulbs is really demand for hours of light. It doesn't matter to Tom whether he buys $q = 3$ bulbs that last $s = 2$ hours each or $q = 1$ bulb that lasts $s = 6$ hours. He is willing to pay \$6 for either combination of bulbs and durability: three low-quality bulbs at $p(q, s) = p(3, 2) = \$2$ per bulb or one high-quality bulb at $p(1, 6) = \$6$ per bulb for high-quality bulbs. (Frequency of the bother of changing lightbulbs is thus a characteristic Tom does not care about.)[1]

The general way to write a demand equation says that Tom is willing to pay $p(q, s)$ per bulb for q bulbs each of which lasts s hours. Since Tom only cares about hours of light, though, we can find some function $\tilde{p}(q \cdot s)$ and write his demand function as

$$p(q, s) = \tilde{p}(q \cdot s)s, \tag{2}$$

where $p(q, s)$ is the price per bulb and $\tilde{p}(q \cdot s)$ is the price per hour of light. In Tom's case we know that if $(q \cdot s) = 6$ he is willing to pay \$1 per hour of light, so $\tilde{p}(6) = \$1$. If the number of hours of light purchased were more than 6, he presumably would only be willing to pay something less than \$1 per hour of light for that greater quantity; demand curves slope down.

We have denoted $\tilde{p}(q \cdot s)$ as the amount Tom is willing to pay per hour of light if he is buying $(q \cdot s)$ hours. From our last equation,

$$\tilde{p}(q \cdot s) = \frac{p(q, s)}{s} \tag{3}$$

Let us also define $\tilde{q} = q \cdot s$ as the number of hours of light Tom is buying. Thus,

$$\tilde{p}(q \cdot s) = \tilde{p}(\tilde{q}), \tag{4}$$

and we have a new demand equation, in terms of price per hour of light given that Tom is buying a total of $\tilde{q}$ hours.

[1]Hubert asked about brightness— that you can burn 3 bulbs all at once and get more brightness. We ignore that too. Note that we could construct a similar model in which the quality is brightness in lumens and Tom did not care whether he had 1 bulb with 450 lumens or three with 150 each.

Social welfare is

$$
\begin{aligned}
W(q,s) &= \int_0^q p(x,s)dx - c(s)q \\
W(q,s) &= \int_0^q p(x,s)dx - c(s)\frac{qs}{s} \\
W(\tilde{q},s) &= \int_0^{\tilde{q}} \tilde{p}(x)dx - \frac{c(s)}{s}\tilde{q}
\end{aligned} \tag{5}
$$

The first order conditions are

$$\frac{\partial W(\tilde{q},s)}{\partial \tilde{q}} = \tilde{p}(\tilde{q}) - \frac{c(s)}{s} = 0 \tag{6}$$

$$\frac{\partial W(\tilde{q},s)}{\partial s} = 0 - \left(\frac{c'}{s} - \frac{c(s)}{s^2}\right)\tilde{q} = 0 \tag{7}$$

The first order condition for $\tilde{q}$ says that the price of an hour of light should equal the marginal cost per hours of light. The expression $\frac{c(s)}{s}$ is the marginal cost per hour because it is the cost of one light bulb divided by the number of hours a bulb gives light.

The first order condition for s says that $\frac{c(s)}{s}$, the marginal cost of an hour light, should be minimized (by setting $\frac{c'}{s} - \frac{c(s)}{s^2} = 0$). the price of an hour of light should equal the marginal cost per hours of light. The expression $\frac{c(s)}{s}$ is the marginal cost per hour because it is the cost of a light bulb divided by the number of hours it gives light.

Monopoly profit is

$$
\begin{aligned}
\pi(q,s) &= qp - c(s)q \\
&= q[s\tilde{p}] - \tfrac{c(s)\cdot q\cdot s}{s} \\
&= qs[\tilde{p} - \tfrac{c(s)}{s}] \\
\pi(\tilde{q},s) &= \tilde{q}[\tilde{p} - \tfrac{c(s)}{s}]
\end{aligned} \tag{8}
$$

The first order conditions are

$$\frac{\partial \pi(\tilde{q},s)}{\partial \tilde{q}} = \tilde{p} + \tilde{q}\left(\frac{d\tilde{p}}{d\tilde{q}}\right) - \frac{c(s)}{s} = 0 \tag{9}$$

$$\frac{\partial \pi(\tilde{q},s)}{\partial s} = 0 - \left(\frac{c'}{s} - \frac{c(s)}{s^2}\right)\tilde{q} \tag{10}$$

The first order condition for q says that the marginal revenue from an extra hour of light should equal its marginal cost. This is the standard MR=MC condition.

The first order condition for s is identical to that for maximizing social welfare. The monopolist, like the social planner, desires that $\frac{c(s)}{s}$, the marginal cost of an hour light, be minimized.