Notes on Present Discounted Value

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Time is money. That statement has many meanings, but one of them is that money now is worth more than money later. If you have a choice between 800 dollars in 1998 and 800 dollars in 2098, you will prefer the money you get 100 years in advance. This is so for many reasons. Three of them are:

1. Impatience, or time preference. You would prefer to be able to spend the money and consume things now simply because that is part of your utility function, part of your psychological preferences.

2. Option value. If you have the money earlier, you have the choice of spending it earlier or waiting and spending it later.

3. Investment value. If you get the money in 1998, you can invest it and have more than the original amount to spend in 2098.

We'll concentrate on Reason 3. Suppose that money can be invested risklessly (say, in U.S. government bonds) for an annual interest rate of $r = .10$, ten percent. It is easy to see that with a choice between \$800 now and \$800 in 100 years, you would prefer your money now. Also, there must exist some value V such that you would be indifferent between \$800 now and \$V in 100 years. What is the value of V?

Let's start with an easier problem. You have a choice between 1 dollar today and D dollars delivered a year from now. What is the value of D?

Well, if you had the 1 dollar today and invested it at interest rate r, you would end up with $1(1+r)$ dollars tomorrow. So the value of D is

$$D = 1(1+r) = 1(1+.05) = 1.05. \tag{1}$$

What about if you had a choice between 1 dollar today and D dollars to be delivered in two years? With one more year of interest, the value would be

$$D = 1(1+r)(1+r) = 1(1+.05)(1+.05) = 1.1025. \tag{2}$$

What about 1 dollar today versus D dollars in 100 years? The value would be

$$D = 1(1+r)^{100} = 1(1+.05)^{100} \approx 131.50. \tag{3}$$

With this information, we can solve the original problem. 800 dollars today is equivalent to $(800)(1+r)^{100}$ dollars in 100 years, which is \$105,200.

More often, we want to do a different form of the calculation. What value of D dollars today is equivalent to 1 dollar to be delivered in one year? If I had D dollars today, I could invest it and have D(1+r) dollar in one year, so D must solve the equation

$$D(1+r) = 1, \tag{4}$$

yielding

$$D = \frac{1}{1+r}. \tag{5}$$

That is the basic equation for **present discounted value**: the value, in terms of present dollars, of a future stream of money.

What would D be if the 1 dollar were to be delivered in 2 years, not 1? Then it must be that

$$D(1+r)(1+r) = 1, \tag{6}$$

so

$$D = \frac{1}{(1+r)^2}. \tag{7}$$

If the interest rate is 5 percent, this means 1 dollar to be delivered in 2 years is equivalent to about 91 cents delivered immediately.

It is easy to see that this means that 1 dollar to be delivered in 100 years is worth only

$$D = \frac{1}{(1+r)^{100}}, \tag{8}$$

so 800 dollars to be delivered in 100 years is worth only \$6.08 in present dollars.

None of this, by the way necessarily involves **inflation**. You should do the discounting using the nominal, not the real interest rate, if the future dollars are not adjusted for inflation. If the inflation rate is zero, however, the interest rate will still be positive, and a dollar in the present is still better than a dollar in the future.

A **zero coupon bond** is a bond that pays no interest. People buy such bonds only if they sell at a **discount**. For example, a company might issue a bond with a face value of 1,000 dollars maturing in 30 years. If the bond is riskless and the market interest rate is 5 percent, a person would pay $P = \frac{1000}{(1+r)^{30}} = \frac{1000}{(1+.05)^{30}} \approx 231$ dollars to the company for that bond. That is fine with the company– it has obtained 231 dollars to use as capital for 30 years. Over time, the value of the bond rises as maturity gets closer. On the day of maturity, the market value of the bond is 1,000 dollars.

You will find yourself using an equation like (8) for most of your discounting calculations, figuring out the present value of money rather than the future value. That is because the idea is helpful in calculating the value of an entire stream of future payments, positive and negative, arriving in different years. Suppose that your company makes an investment that you think will cost 200 thousand dollars now and 30 thousand dollars a year from now, but can be sold for 500 thousand dollars nine years from now, and that the discount rate is 8 percent. The present value is

$$P = -200 - \frac{30}{(1+r)} + \frac{500}{(1+r)^9} \approx -200 - 27.78 + 250.12 = 22.34. \tag{9}$$

This means that you would only pay \$22,340 for the opportunity to make this investment, even though the undiscounted profit is \$270,000. To put the same idea differently: if the company had a sudden windfall of \$22,340 in cash and bought a bond yielding 8 percent annually, at the end of 9 years the company would end up just as well off as if it made this investment. Companies do this kind of calculation all the time to figure out whether to undertake a project or use their funds in some other way.

One final point: it is often convenient to know the value of a **perpetuity** or **consol**, a stream of payments of 1 dollar each year starting in one year and going on forever. The present value of this stream is *not* infinite, even though the total number of dollars is. Using the formula above, the value is

$$P = \frac{1}{(1+r)^1} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + ... \tag{10}$$

It turns out that this value equals the very simple expression,

$$P = \frac{1}{r}. \tag{11}$$

Thus, if the interest rate is $r = .05$, you would pay $1/.05$ for the perpetuity, which is 20 dollars. That's a lot less than infinity.

This makes sense, though. Suppose you had a choice between 20 dollars or the perpetuity. The perpetuity yields one dollar per year forever. If you invest the 20 dollars at the interest rate of $4 = .05$, what do you get? – 1 dollar per year forever. The values are the same.

If interest rates go up, though, your perpetuity falls in value. If the interest rate rises to $r = .10$, then the value of the perpetuity falls to $p = 1/.10 = 10$. That is because now the stream of 1 dollar per year could be achieved even if you had only 10 dollars, not 20, to invest.

This is a way to explain why bond prices fall when interest rates rise.