PROBLEM SET 2 ANSWERS

14 September 2006.

Problems: 3.4, 3.5, 3.8.

3.4. Mixed Strategies in the Battle of the Sexes

Refer back to the Battle of the Sexes and Ranked Coordination in Section 1.4. Denote the probabilities that the man and woman pick *Prize Fight* by γ and θ.

(a) Find an expression for the man's expected payoff.

Answer. $\pi_m = \gamma(2\theta + (0)[1-\theta]) + (1-\gamma)((0)\theta + 1[1-\theta])$.

(b) What are the equilibrium values of γ and θ, and the expected payoffs?

Answer. $\theta^* = 1/3, \gamma^* = 2/3$, $\pi_m = \frac{2}{3}, \pi_w = \frac{2}{3}$.

(c) Find the most likely outcome and its probability.

Answer. *(Prize Fight, Ballet)* for (M, W), which has probability 4/9, about 0.444.

(d) What is the equilibrium payoff in the mixed-strategy equilibrium for Ranked Coordination ?

Answer. The probability is found by solving $2\theta + (1-\theta)(-1) = (-1)\theta + (1-\theta)$. Therefore, $Prob(Large) = 2/5$, $\pi = 1/5$ for each player.

(e) Why is the mixed-strategy equilibrium a better focal point in the Battle of the Sexes than in Ranked Coordination?

Answer. Ranked Coordination has a pareto-dominant Nash equilibrium, so if players are optimistic, they will focus on that equilibrium. In the Battle of the Sexes, neither Nash equilibrium is pareto-dominant.

3.5. A Voting Paradox

Adam, Karl, and Vladimir are the only three voters in Podunk. Only Adam owns property. There is a proposition on the ballot to tax property-holders 120 dollars and distribute the proceeds equally among all citizens who do not own property. Each citizen dislikes having to go to the polling place and vote (despite the short lines), and would pay 20 dollars to avoid voting. They all must decide whether to vote before going to work. The proposition fails if the vote is tied. Assume that in equilibrium Adam votes with probability θ and Karl and Vladimir each vote with the same probability γ, but they decide to vote independently of each other.

(a) What is the probability that the proposition will pass, as a function of θ and γ?

Answer. The probability that Adam loses can be decomposed into three probabilities— that all three vote, that Adam does not vote but one other does, and that Adam does not vote but both others do. These sum to $\theta\gamma^2 + (1-\theta)2\gamma(1-\gamma) + (1-\theta)\gamma^2$, which is, rearranged, $\gamma^2 + 2\gamma(1-\gamma)(1-\theta)$ or $\gamma(2\gamma\theta - 2\theta + 2 - \gamma)$.

(b) What are the two possible equilibrium probabilities γ_1 and γ_2 with which Karl might vote? Why, intuitively, are there two symmetric equilibria?

Answer. The equilibrium is in mixed strategies, so each player must have equal payoffs from his pure strategies. Let us start with Adam's payoffs. If he votes, he loses 20 immediately, and 120 more if both Karl and Vladimir have voted.

$$\pi_a(Vote) = -20 + \gamma^2(-120). \tag{1}$$

If Adam does not vote, then he loses 120 if either Karl or Vladimir vote, or if both vote:

$$\pi_a(Not\ Vote) = (2\gamma(1-\gamma) + \gamma^2)(-120) \tag{2}$$

Equating $\pi_a(Vote)$ and $\pi_a(Not\ Vote)$ gives

$$0 = 20 - 240\gamma + 240\gamma^2. \tag{3}$$

The quadratic formula solves for γ:

$$\gamma = \frac{12 \pm \sqrt{144 - 4 \cdot 1 \cdot 12}}{24}. \tag{4}$$

This equations has two solutions, $\gamma_1 = 0.09$ (rounded) and $\gamma_2 = 0.91$(rounded).

Why are there two solutions? If Karl and Vladimir are sure not to vote, Adam will not vote, because if he does not vote he will win, 0-0. If Karl and Vladimir are sure to vote, Adam will not vote, because if he does not vote he will lose, 2-0, but if he does vote, he will lose anyway, 2-1. Adam only wants to vote if Karl and Vladimir vote with moderate probabilities. Thus, for him to be indifferent between voting and not voting, it suffices either for γ to be low or to be high– it just cannot be moderate.

(c) What is the probability θ that Adam will vote in each of the two symmetric equilibria?

Answer. Now use the payoffs for Karl, which depend on whether Adam and Vladimir vote.

$$\pi_c(Vote) = -20 + 60[\gamma + (1-\gamma)(1-\theta)] \tag{5}$$

$$\pi_c(Not\ Vote) = 60\gamma(1-\theta). \tag{6}$$

Equating these and using $\gamma^* = 0.09$ gives $\theta =$ 0.70 (rounded). Equating these and using $\gamma^* = 0.91$ gives $\theta = 0.30$ (rounded).

(d) What is the probability that the proposition will pass?

Answer. The probability that Adam will lose his property is, using the equation in part (a) and the values already discovered, either 0.06 (rounded) $(= (0.7)(0.09)^2 + (0.3)(2(0.09)(0.91) + (0.09)^2))$ or 0.94 (rounded $(= (0.3)(0.91)^2 + (0.7)(2(0.91)(0.09) + (0.91)^2))$.

3.8. Triopoly

Three companies provide tires to the Australian market. The total cost curve for a firm making Q tires is $TC = 5 + 20Q$, and the demand equation is $P = 100 - N$, where N is the total number of tires on the market.

According to the Cournot model, in which the firms simultaneously choose quantities, what will the total industry output be?

Answer. Marginal cost is 20 for each firm. For firm 1, revenue is

$$R_1 = PQ_1 = (100 - Q_1 - Q_2 - Q_3)Q_1,$$

so marginal revenue is $100 - 2Q_1 - Q_2 - Q_3$. Setting this equal to marginal cost yields $20 = 100 - 2Q_1 - Q_2 - Q_3$. Since each firm produces the same quantity in equilibrium, $4Q_1 = 80$, and $Q_1 = 20$. Total industry output is therefore 60.