G601 Final, December 19, 2003: ANSWERS

This is a closed-book test, except that you may use one double-sided page of notes. If you do, please attach it to your answers. Answer each question as best you can. If you get lost in solving equations, write down in words what you are trying to do and what you think would come out of the mathematical analysis. Note that the last question, with its many sections, has 55 out of the 100 points on the test.

Comment: After a certain point, I decided to write up these detailed answers rather than commenting on individual tests as to whether an answer was right or wrong. Thus, you may get low points on an answer without any comment as why your answer is wrong.

Comment: A general problem was not answering the question. If a question asks whether quality is socially optimal, don't forget to answer that. If a question asks for an equilibrium, don't just say this and that about the payoffs: say what the equilibrium is. If the question asks for a demand function for good 1, don't answer by giving the equilibrium price of good 1.

Scores ranged from 17 to 80, with most scores being in the 50-60 interval.

1. (10 points) Explain, in a sentence or two for each, some idea that can be found in the selections we read by two of the following three economists: (a) Viner, (b) Harberger, and (c) Hart and Holmstrom.

ANSWER. (a) Viner shows how supply curves arise from cost curves, and, in turn how cost curves arise from the conditions of production. He also shows how this can differ from the short run to the long run. The focus of the article is on the U-shaped cost curve, and how that might or might not result in a U-shaped long run supply curve.

(b) Harberger's big point is that if we take a conservative measurement of the possible triangle loss from monopoly in manufacturing, the loss is surprisingly small.

(c) Hart and Holmstrom use coordination and prisoner's dilemma games to show the advantages and disadvantages of letting two production units be independent as opposed to being controlled by a central boss.

2. Explain what the following terms mean.

(a) Resale Price Maintenance (3 points)

(b) Double Marginalization (4 points)

(c) The Hold-up Problem (3 points)

ANSWER. Resale Price Maintenance occurs when a seller puts restrictions on the price at which the buyer can resell the product.

Double Marginalization occurs when a seller with market power sells to another firm that resells with market power. The resulting quantity sold to the consumer is too small to maximize either industry profits or social surplus.

The Hold-up Problem is less well defined than the first two concepts, but the general idea is that sunk investments by one player make that player vulnerable to contract changes by another player. The examples we looked at in class had one player making a sunk investment that generated a surplus which then had to be bargained over with another player. Anticipating the hold-up problem, the investing player invests too little.

3. (15 points) Mr. and Mrs. Rasmusen were driving to church one snowy morning and saw a pickup truck with a plow attachment clearing a house's driveway.

"That would be a good way for a lawnmowing company to make money," Mr. Rasmusen said. "A plow attachment doesn't cost much and people would be willing to pay quite a bit to not have to clear their driveway by hand."

"But won't competition drive down the price?" said Mrs. Rasmusen.

Answer Mrs. Rasmusen's question, discussing the various possibilities in light of what you have learned in this class and any other ideas you may have.

ANSWER. This is a good problem for you to use to determine whether you have learned anything from your training in economics, or whether you are worse off than when you had never heard of the subject.

Mrs. Rasmusen is thinking that this is a perfectly competitive market. There are grounds for thinking that. Entry is easy. All the entrant needs is a pickup truck and a snow plow attachment. Many people have pickup trucks for their personal use, and a snowplow attachment is not a big capital investment, as Mr. Rasmusen said. And there is no location advantage. The seller of plowing services must drive to each customer, and it does not matter much whether he is located 2.1 miles from the average customer or 2.09 miles. So the Bertrand model has some plausibility.

On the other hand, there may be informational problems. The Butter model might be an apt one. The problem for an entrant is not entry deterrence by the incumbents, but in notifying the potential customers that he exists. This would require advertising of some kind, and the market is so small that advertising might be too expensive to be profitable.

Also, there is a problem of product quality. The quality would not be how well the driveway was cleared—that is a homogeneous product– but whether it is cleared the same day as the snow arrives, and with what amount of noise, and with what damage to the driveway from scraping by the plow.

The lawnmowing companies have an advantage with both of these information problems. They already talk to the customers, and they have reputations to maintain, and could lose their lawnmowing business if they offend customers (think of the Klein-Leffler model). That is one reason why snowplowing would be a complementary good to lawnmowing (the other is that the lawnmowing companies already have trucks and sources of irregular labor).

Also, there may be a problem of collusion. If the information problems reduce the important competitors to two or three firms, then it would be easy for them to collude. They would know each other's identity, and the FTC and Justice Department are not going to prosecute such small firms, so the only legal problem is that they could not actually enforce a collusion contract in court.

The key to answering this kind of question is to combine your knowledge of industrial organization theory with your common sense. You need to tell yourself a story about what might happen, and your I.O. training helps suggest stories and tell you which stories make sense.

4. (10 points) Andrew and Bob are bidding for an oil lease whose value is either 0 or 100, with equal probability. Andrew knows the true value of the lease with probability 0.30. Bids must be in integer values and equal at least 1 to win. If both players bid 0, nobody wins and the seller keeps the lease.

The auction rule is that each player announces a bid in sequence, starting with Andrew, and the bidding continues until one player decides not to bid any higher. The bidder with the highest bid then wins the least and pays the amount he bid. Find a perfect Bayesian equilibrium.

ANSWER: This is really a question about Nash equilibrium more than about auction theory. You don't need to know any auction theory to answer it, but you do need to know what a strategy is and what a Nash equilibrium is.

In any equilibrium, if Andrew knows the true value is 0, he will not bid at all, and if he knows the true value is 100 he will be willing to bid up to 100 as necessary.

Here is one equilibrium.

Andrew: If Andrew knows the true value is 0, he will not bid at all.
If Andrew knows the true value is 100, he will bid 50 initially, and then bid up to 100 as necessary.
If Andrew does not know the true value at all, he will bid 50.

Bob: If Andrew does not make the first bid, neither will Bob. If Andrew bids 50, Bob will bid no higher. Out of equilibrium, if Andrew bids 1-49 or 51-100, Bob will bid as necessary up to 100.

Out of equilibrium, let Bob believe that if Andrew bids 1-49 or 51-100 then Andrew knows that the value is 100.

Here are a few wrong answers.

ANSWER 1.

Andrew: Bid 100 if v=100. Bid 0 if v=0. Bid 1 with probability .5 if he does not know the value.

Bob: If Andrew bids 100, do not bid. If Andrew bids 0, bid 1 with probability .35.

Out of equilibrium belief: Prob (v=0|Andrew bids 100) =0 and Prob (1st bid (1, 100)) =0.

Comments:
1. Why should Andrew bid 100 when he could win with a bid of 2?
2. Bob's strategy says he never bids more than 1. This is clearly absurd.
3. Prob (v=0|Andrew bids 100) =0 is not an out-of=equilibrium belief–it is an equilibrium belief, required by Bayes Rule and not up to the modeller. 4. Why use mixed strategies?

ANSWER 2.

Andrew bids 35 first period. Bob won't overbid. Out of equilibrium beliefs: Prob(don't know—Bid 35) = 0.7.

Comments:
1. Would Bob really not overbid if Andrew deviated and bid 1?
2. Andrew should deviate and bid 1 instead of 35. 35 is an arbitrary number. It has no importance in this problem. 3. Prob(don't know—Bid 35) = 0.7. is not an out-of=equilibrium belief–it is an equilibrium belief, required by Bayes Rule and not up to the modeller.

ANSWER 3.

Bob bids 0 if Andrew bids 0. Otherwise, Bob bids 50.

Comments:
1. This equilibrium omits any strategy for Andrew, so it is hard to evaluate.

ANSWER 4.

Andrew bids 0.

Bob bids 50. Out of equilibrium, if Andrew bids then bid up to 50 as necessary.

If, out of equilibrium, Andrew bids more than 50, both players will bid up to 100 as necessary

Comments:
1. What happens if Bob deviates to bid 1 instead? His payoff will rise by 49. so this is a profitable deviation.
2. If Andrew deviates to bid 50 if he knows v=100, he can increase his payoff.

ANSWER 5.

Andrew bids 15. Bob bids 16. Bob wins the auction.

Comments:
1. This makes no sense at all.

5. Buyers are distributed uniformly by quality desire θ on a continuum of length 1. Each buyer buys one unit of a good of quality $s \in [0, \overline{s}]$. The good is produced at a constant marginal cost of c, regardless of quality. The payoff of a buyer of type θ who pays p for quality s is $(\alpha + \theta)s - p$, and his payoff is zero if he does not buy at all.

(to add: You may assume $\alpha > 0$)

(a) (10 points) What single quality level s_1 and price p_1 would a monopolist choose? How does this compare to the social optimum?

ANSWER. A consumer will buy if his payoff is nonnegative. This is true if $(\alpha + \theta)s_1 - p_1 \geq 0$; i.e., if $\alpha + \theta \geq \frac{p_1}{s_1}$ so $\theta \geq \frac{p_1}{s_1} - alpha$. Thus, quantity demanded is $1 - (\frac{p_1}{s_1} - \alpha)$.

The monopolist maximizes $(1 - \frac{p_1}{s_1} + alpha)(p_1 - c)$. He should choose s_1 to be as large as possible, since that will increase demand for a given price. Thus, $s_1 = \overline{s}$.

Maximizing $(1 - \frac{p_1}{\overline{s}} + \alpha)(p_1 - c)$ generates the first order condition

$$1 + \alpha - 2\frac{p_1}{\overline{s}} + \frac{c}{\overline{s}} = 0.$$

Thus, $(1 + \alpha)\overline{s} + c = 2p_1$ and

$$p_1 = \frac{(1 + \alpha)\overline{s} + c}{2}.$$

The social optimum would have the same quality, but $p_1 = c$, to maximize gains from trade.

(b) (5 points) What qualities and prices would a perfectly price discriminating monopolist choose? How does this compare to the social optimum?

ANSWER. A perfectly price discriminating monopolist would get the entire gains from trade, and so would choose the socially optimal quantity and quality. Thus, the quality would be $s_1 = \overline{s}$ and the lowest price charged would be $p = c$. The participation constraint for the highest-valuing consumer, who has $\theta = 1$, is $(\alpha + 1)s - p \geq 0$, so for that consumer, $(\alpha + 1)\overline{s} - p = 0$ and $p = (\alpha + 1)\overline{s}$. Prices would range from c to $(\alpha + 1)\overline{s}$.

(c) (10 points) Suppose the seller can choose two quality levels and prices, selling good 1 with s_1, p_1 and good 2 with higher quality and price s_2, p_2, and that consumers with θ in $[0, q_1]$ all buy Good 1 and consumers with θ in $[q_1, 1]$ all buy Good 2. What participation constraints must be satisfied in equilibrium? For which type is the participation constraint going to be binding? Explain.

ANSWER. Consumers who buy good 1 must satisfy

$$(\alpha + \theta)s_1 - p_1 \geq 0.$$

This will be binding for the highest-valuing type willing to buy good 1, which is type $\theta = q_1$.

Consumers who buy good 2 must satisfy

$$(\alpha + \theta)s_2 - p_2 \geq 0.$$

This will be binding for the lowest-valuing type willing to buys good 2, which is type $\theta = q_1$.

(d) (10 points) In the model of part (c) what self selection constraints must be satisfied?

ANSWER. Consumers who buy good 1 must satisfy

$$(\alpha + \theta)s_1 - p_1 \geq (\alpha + \theta)s_2 - p_2.$$

Consumers who buy good 2 must satisfy

$$(\alpha + \theta)s_2 - p_2 \geq (\alpha + \theta)s_1 - p_1.$$

(e) (10 points) Now suppose there are two competing sellers. First, they simultaneously pick s_1 and s_2, and then, after seeing what they have chosen, they simultaneously pick prices p_1 and p_2. What will be the demand curve for Good 1?

ANSWER. Let us assume that the firms have chosen qualities such that each has some sales. Then there is an indifferent consumer type θ such that the consumer's payoff from each firm's good is equal. That consumer's type is q_1, since all consumers in $[0, q_1]$ will buy Good 1. For the $\theta = q_1$,

$$(\alpha + q_1)s_1 - p_1 = (\alpha + q_1)s_2 - p_2, \tag{1}$$

so

$$\alpha(s_1 - s_2) - p_1 + p_2 = q_1(s_2 - s_1) \tag{2}$$

The demand will thus be

$$q_1 = \frac{p_2 - p_1}{s_2 - s_1} - \alpha. \tag{3}$$

There is no need to compute equilibrium prices or qualities to answer this question– it is about the demand curve, not about the particular point on a particular demand curve which is the equilibrium.

(f) (10 points) In the model of part (e) it turns out that the reaction curves are

$$p_1 = \frac{p_2 + c - \alpha(s_2 - s_1)}{2} \tag{4}$$

and

$$p_2 = \frac{p_1 + c + (\alpha + 1)(s_2 - s_1)}{2} \tag{5}$$

and the equilibrium profits are

$$\pi_1 = \frac{(1 - \alpha)^2(s_2 - s_1)}{9} \tag{6}$$

and

$$\pi_2 = \frac{(\alpha + 2)^2(s_2 - s_1)}{9} \tag{7}$$

Are prices strategic complements, or substitutes? Are qualities strategic complements, or substitutes? What qualities will be chosen in the first stage of the game?

Prices are strategic complements, because when p_2 goes up, p_1 goes up too.

Qualities are neither strategic complements nor substitutes, if those concepts are defined in terms of the marginal benefit one's own strategy when the other player increases his strategy. That is because a player's quality is a corner solution:

$$\frac{\partial \pi_1}{\partial s_1} = \frac{-(1-\alpha)^2}{9} < 0. \tag{8}$$

It is nonetheless true that

$$\frac{\partial \pi_1}{\partial s_2} = \frac{(1-\alpha)^2}{9} > 0, \tag{9}$$

which is to say that Firm 1 is helped if Firm 2 chooses a higher quality level. This is a bit unusual: ordinarily this is the case only when the strategies are complements.

In any case, the two firms will pick the extreme qualities, so $s_1 = 0$ and $s_2 = \overline{s}$.

Here is a more complete exposition of the model used in the last part of this question:

The Order of Play
0 There is a continuum of buyers of length 1 parametrized by quality desire θ_i distributed by Nature uniformly on $[0, 1]$.
1 Sellers 1 and 2 simultaneously qualities s_1 and s_2 from the interval $[0, \overline{s}]$.
2 Sellers 1 and 2 simultaneously pick prices p_1 and p_2 from the interval $[0, \infty)$.
3 Buyer i chooses one unit of a good, or refrains from buying. The sellers produces at constant marginal cost c, which does not vary with quality.

Payoffs
Seller j's payoff is

$$(p_j - c)q_j. \tag{10}$$

Buyer i's payoff is zero if he does not buy. If he does buy, from seller j, it is

$$(\alpha + \theta_i)s_j - p_j, \tag{11}$$

where the parameter $\alpha \in (0, 1)$ is the same for all buyers.

Work back from the end of the game. Let us assume that the firms have chosen qualities so that each has some sales. Then there is an indifferent consumer type θ such that the consumer's payoff from each firm's good is equal. That consumer's type is q_1, since all consumers in $[0, q_1]$ will buy Good 1. For $\theta = q_1$,

$$(\alpha + q_1)s_1 - p_1 = (\alpha + q_1)s_2 - p_2, \tag{12}$$

so

$$\alpha(s_1 - s_2) - p_1 + p_2 = q_1(s_2 - s_1) \tag{13}$$

The demands will thus be [1]

$$q_1 = \frac{p_2 - p_1}{s_2 - s_1} - \alpha \tag{14}$$

and

$$q_2 = 1 - q_1 = 1 - \frac{p_2 - p_1}{s_2 - s_1} + \alpha \tag{15}$$

When firms maximize their payoffs by choice of price, the reaction curves turn out to be

$$p_1 = \frac{p_2 + c - \alpha(s_2 - s_1)}{2} \tag{16}$$

and

$$p_2 = \frac{p_1 + c + (\alpha + 1)(s_2 - s_1)}{2} \tag{17}$$

From the reaction curves we can see that prices are strategic complements.

When solved for equilibrium prices, it turns out that $p_2 > p_1$. Substituting from the last two equations,

$$\begin{aligned} p_1 &= \frac{\frac{p_1+c+(\alpha+1)(s_2-s_1)}{2}+c-\alpha(s_2-s_1)}{2} \\ &= \frac{p_1}{4} + \frac{c+(\alpha+1)(s_2-s_1)}{4} + \frac{c-\alpha(s_2-s_1)}{2} \\ &= \left(\frac{4}{3}\right)\left(\frac{c+(\alpha+1)(s_2-s_1)}{4} + \frac{c-\alpha(s_2-s_1)}{2}\right) \\ &= \frac{c+(\alpha+1)(s_2-s_1)+2c-2\alpha(s_2-s_1)}{3} \\ &= c + \frac{(1-\alpha)(s_2-s_1)}{3} \end{aligned} \tag{18}$$

[1] xxx This should be the same as in the two-good monopoly case.

Substituting into the reaction curve for Firm 2, this yields

$$
\begin{aligned}
p_2 &= \frac{c + \frac{(1-\alpha)(s_2-s_1)}{3} + c + (\alpha+1)(s_2-s_1)}{2} \\
&= \frac{3c + (1-\alpha)(s_2-s_1) + 3c + 3(\alpha+1)(s_2-s_1)}{6} \\
&= c + \frac{(1-\alpha+3\alpha+3)(s_2-s_1)}{6} \\
&= c + \frac{(4+2\alpha)(s_2-s_1)}{6} \\
&= c + \frac{(2+\alpha)(s_2-s_1)}{3}
\end{aligned}
\tag{19}
$$

The price p_2 is larger than p_1: $p_2 - p_1 = \frac{1+2\alpha(s_2-s_1)}{3} > 0$.

The quantities are then

$$
q_1 = \frac{p_2 - p_1}{s_2 - s_1} - \alpha = \frac{1+2\alpha}{3} - \alpha = \frac{1-\alpha}{3}, \tag{20}
$$

and

$$
q_2 = 1 - q_1 = \frac{2+\alpha}{3}. \tag{21}
$$

With a greater price and quantity than Firm 1, and the same average cost, Firm 2 has higher profits. The profits are

$$
\pi_1 = \frac{(1-\alpha)^2(s_2-s_1)}{9} \tag{22}
$$

and

$$
\pi_2 = \frac{(\alpha+2)^2(s_2-s_1)}{9} \tag{23}
$$

Note that for both firms, profits are increasing in $(s_2 - s_1)$.

How about quality choice,in the first stage? Well, both firms benefit from having more different qualities. So in equilibrium, they will be separated as far as possible–

$$
s_1 = 0 \quad s_2 = \overline{s}. \tag{24}
$$

Qualities are neither strategic complements nor substitutes, if those concepts are defined in terms of the marginal benefit one's own strategy when the other player increases his strategy. That is because a player's quality is a corner solution:

$$
\frac{\partial \pi_1}{\partial s_1} = \frac{-(1-\alpha)^2}{9} < 0. \tag{25}
$$

It is nonetheless true that

$$
\frac{\partial \pi_1}{\partial s_2} = \frac{(1-\alpha)^2}{9} > 0, \tag{26}
$$

which is to say that Firm 1 is helped if Firm 2 chooses a higher quality level. This is a bit unusual: ordinarily this is the case only when the strategies are complements.

This game is a battle of the sexes. There are two Nash equilibria in pure strategies, in each of which a differerent one of the two firms gets to be "Firm 2" with its higher profits.[2]

[2]xxx What is the mixed-strategy equilibrium in which the firm mix over qualities– so they would ompete head to head sometiems? Probably htye mix over lots of qualities.