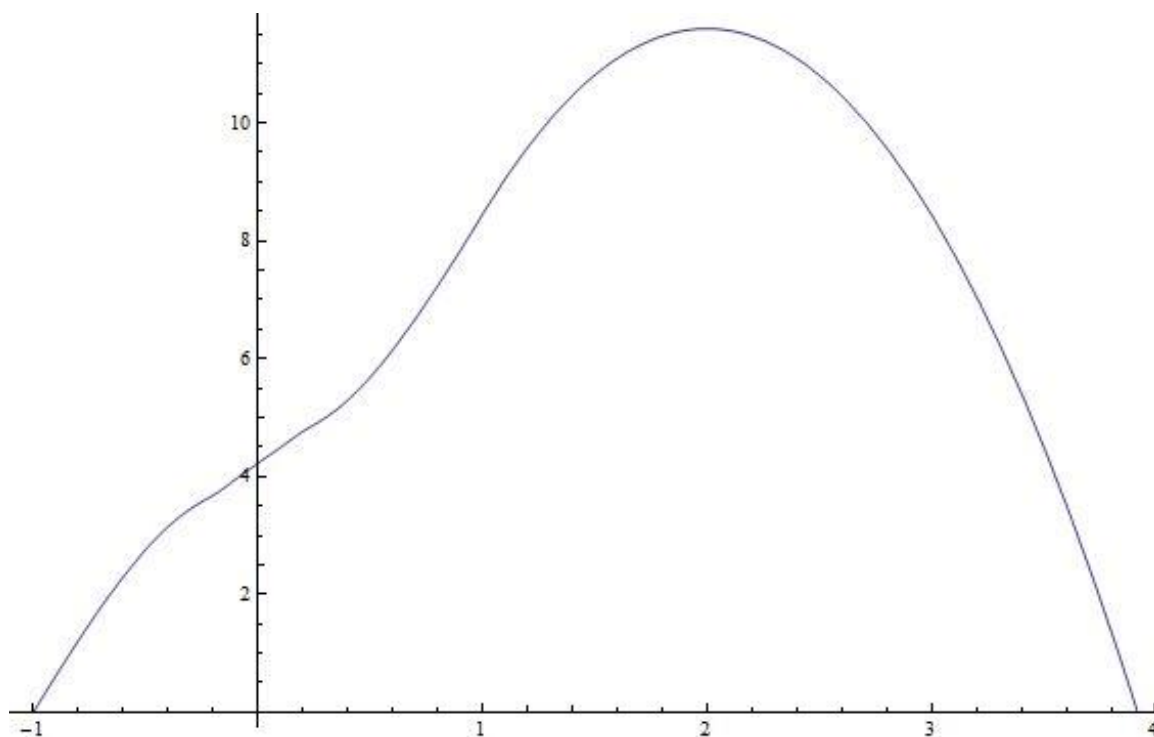


Life and Time

July 26, 2010

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Figure 1: **Temp diagram holder**

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### Discounting

Bloomberg inflation-indexed federal borrowing rates are useful: <http://www.bloomberg.com/markets/inflation-index.html>

Bond A pays out \$10,000 plus inflation to be received 1 year from today.

Bond B pays out \$10,000 plus inflation to be received 50 years from today.

Bond C pays out \$1,000 plus inflation each year forever, with the first payment one year from today.

If I gave each bond to you right now, how much would you sell it back for, if I'm the only possible buyer?

A: \$10,000 plus inflation in 1 year.

B: \$10,000 plus inflation in 50 years.

C: \$1,000 plus inflation each year forever.

If I gave each bond to you right now, how much would you sell it back for, if I'm the only possible buyer? Write down some considerations (e. g., alternative borrowings, investments, jobs, needs).

$1/(1+r) = \$9,100$  at 10%.  $\$9,300$  at 7%.  $\$9,500$  at 5%.  $\$9,700$  at 3%.  
 $\$9,800$  at 2%.  $\$9,900$  at 1%.  $\$10,000$  at 0.01%.

$1/(1+r)^{50} = \$100$  at 10%.  $\$300$  at 7%.  $\$900$  at 5%.  $\$2,300$  at 3%.  
 $\$3,700$  at 2%.  $\$6,100$  at 1%.  $\$10,000$  at 0.01%.

$1/r = \$10,000$  at 10%.  $\$14,300$  at 7%.  $\$20,000$  at 5%.  $\$33,300$  at 3%.  
 $\$50,000$  at 2%.  $\$100,000$  at 1%.  $\$10,000,000$  at 0.01%.

#### Government Discount Rates

The federal government used to use 10% till 1994. Then it switched to 7%— the average real return on private investment. That's in the Office of Management and the Budget (OMB) memo A-94 reading.

In 2004, it started asking for calculations too. That's an estimate of the average consumers rate of time preference.

Is 3% a good estimate of the rate most people would invest at?

What rate do they get?

What rate could they get?

What kind of investment does the average person make?— a house, and borrowing with credit cards. Many (most?) people have negative savings.

Investing in roads is different from investing in a new cost-saving heating system.

The road requires extra government spending, and takes money away from the private sector. Thus, use 7%, and apply the .25 cost of tax distortions (OMB numbers).

The heating system reduces government spending. Thus, use the government cost of capital, and no adjustment for tax distortion is needed.

What role should risk play?

What is the value of a human life?

The OMB says to use numbers between 1 and 10 million dollars.

A Global Warming Example <http://rasmusen.org/g406/discounting.xls>

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The Viscusi Approach to Valuing Life

(1) Suppose a person has wealth  $W$  and faces probability  $p$  of dying. The person's utility of wealth is 0 if he is dead, we will assume (what is crucial is not the normalization to zero, but that if he is dead, his utility does not depend on his remaining wealth). What is the maximum fraction of  $W$ ,  $p^*$ , that he willing to pay to eliminate risk  $p$ ?

(2) Suppose first that the risk is  $p=1$ . Then  $p^*$  will solve

$$U([1-p^*]W) = U(0),$$

so  $p^*=1$ . The person will give up his entire wealth  $W$  to remain alive.

(3) Now suppose the risk is  $p=.5$ . Then  $p^*$  will solve

$$U([1-p^*]W) = .5 U(0) + .5 U(W),$$

because the person will have  $[1-p^*]W$  for sure if he pays to eliminate the risk, but if he does not pay, he has a gamble between dying (for payoff 0) or being alive with his fortune intact (for payoff  $U(W)$ ).

If the person is risk averse, his utility function  $U(W)$  is concave. In that case, for it to be true that

$$U([1-p^*]W) = .5 U(W),$$

it must be true that

$$1-p^* \text{ is less than } .5$$

For example, if the person's lifetime wealth is 1 million dollars, and the risk is  $p=.5$ , the person might be willing to pay \$700,000 to eliminate the risk, because he is indifferent between \$300,000 and no risk and \$1 million with a 50% risk of dying.

Note that if we start from the amount the person is willing to pay—\$700,000 here— then to find the value of the person's life, doubling the \$700,000 is an *upper* bound to the value.

(4) Now look more generally at risk  $p$ . It must be true that

$$U([1-p^*]W) = pU(0) + (1-p)U(W),$$

so

$$U([1-p^*]W) = (1-p)U(W),$$

so quite generally,

$1-p^*$  is less than  $1-p$ ,

and thus

$p^*$  is greater than  $p$ ,

so to avoid a risk of death of 1%, the person will be willing to pay more than 1% of his wealth.

(5) The difference between  $p$  and  $p^*$  gets smaller as  $p$  gets smaller, though, because locally the utility curve is closer to linear.

I'm not sure how to show this, but I think it involves a Taylor series something like this:

$$U([1 - p^*]W) = U(W) + U'(W)(W - [1 - p^*]W) + (1/2)U''(W)([1 - p^*]W - W)^2 + \dots$$

$$U([1 - p^*]W) = U(W) - U'(W)(p^*W) + (1/2)U''(W)(p^*W)^2 + \dots$$

so, dropping second and higher-order terms,

$$U([1-p^*]W) \text{ is approximately } U(W) - U'(W)(p^*W),$$

which is approximately

$$U(W) - p^*U'(W)W = (1-p^*)U(W).$$

This is related to the idea that is behind portfolio diversification and behind the Rabin Paradox: that if a risk is small enough, any rational person is approximately risk neutral.

Thus, for a small risk,  $p$  and  $p^*$  are very close. This means that if someone is willing to pay  $Z$  to avoid a risk of death of  $p$ , we can conclude they would pay  $Z/p$  to avoid the certainty of death.

That is the use of this idea in the Valuation of Life literature and in chapter 20 of Viscusi, Vernon, and Harrington.

(6) But let us think of another implication. If we start with someone's lifetime wealth,  $X$ , then they should be willing to pay at most  $pX$  to avoid a risk of death of  $p$ , for a small risk. Thus, we don't need to do estimation through data from surveys, actual purchases of safety equipment, or job choice, if we know  $p$  and  $X$ . Instead, we just use  $p$  and  $X$ .

This makes the chief use of the estimates from surveys, actual purchases

and job choice \*not\* to find out anything about a person's utility function, but to discover their beliefs about their lifetime wealth.

Or, if we find that the resulting "value of life" is not close to what we know the person's lifetime wealth is using other data available to us, we know that the person is confused or irrational.

(7) This can be related to total wealth. But what is total wealth?

(a) U.S. GDP in 2004 is about 11 trillion dollars. If the discount rate is .05 (a critical assumption), then this represents a total U.S. wealth of something over  $11/.05 = 220$  trillion dollars. This is an underestimate, because GDP will rise over time with economic growth.

(b) The BEA says fixed assets, public and private, were \$23 trillion in 2001. <http://www.bea.doc.gov/>. If capital income is 1/3 of total income, and this is all of capital, then that would make total wealth, including human capital,  $3(23) = 69$  trillion dollars. That is a very different estimate.

Also, is what matters the paper value of wealth, or actual tangible assets that could be liquidated to pay to prevent risks? If, for example, we were trying to stop an asteroid from destroying America, all we could rely on would be the current GDP plus whatever is the liquidation value of tangible assets.

(c) Since I can't resolve those questions now, let's take another approach. Suppose the value of life is 5 million dollars per person. What value of wealth justifies that?

We have about 300 million people in the U.S., and a million millions is a trillion, so the wealth needed is  $5 \times 300 = 1500$  trillion. That is too high.

How about making the value of life 1 million dollars per person? Then the wealth of the US would be 150 trillion, which is in the correct range.

How about looking at it from the point of view of an individual? 40 years of undiscounted earnings of \$30,000/year equals 1.2 million dollars. So that puts us in the correct range also.

Note that in one sense it is easier to do these calculations in aggregate than for individuals. Looking at an individual, we would have to worry

about how to value babies, knowing that their parents and other relatives would be willing to pay to save them. But in aggregate, my test is making the relatives pay for their own lives too, preventing double counting and removing the question of just how altruistic they are.

(d) We can think about this for an individual too. What is your lifetime wealth? To make it easy, assume that your salary will grow at the discount rate, which is reasonable for some kinds of jobs, at least. (Taxes, bequests?)

(7) All of this is predicated on wealth being irrelevant to the person if he dies. That is a bad assumption for many people—especially for wealthy ones—since many people purposely leave bequests. The effect of wealth after death still being valuable will be to reduce the willingness to pay to avoid risks. The effect will still be important even as the risk  $p$  becomes very small, but I haven't figure out which way it goes, as far as value-of-life calculations go.

The Value of Risks to Life and Health W. Kip Viscusi *Journal of Economic Literature*, Vol. 31, No. 4. (Dec., 1993), pp. 1912-1946.

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#### Valuing Life I (Viscusi, p. 662)

Suppose this evening you will be crossing the street and you have one chance in 10,000 of being hit by a bus and killed instantly.

You may buy out of this risk for a cash payment now.

You may borrow to make the payment, at the t-bill rate.

This risk is about the same as the average yearly fatality rate for construction workers.

How much would you pay?

If you would pay  $X$ , your value for life is  $10,000X$ .

#### Valuing Life II

A group of 10,000 people know that one of them, picked randomly, will die next year unless we each pay amount  $X$  now.

What is the maximum  $X$  you would pay?

If each person pays \$1,000, the total payment is 10,000 times that, which is 10 million dollars. We can say that is the value of a life.

Viscusi's estimates for the value of the life of a blue-collar worker are from 3 to 6 million dollars.

How much would you be willing to \*accept\* to take on an extra risk of this size?

Suppose someone is only willing to pay \$2 to prevent the bus fatality risk. Should we require them to pay more anyway?

Suppose someone is only willing to pay \$2 to prevent the bus fatality risk. Should other citizens pay \$50 on their behalf?

Differing Attitudes towards Risk

The average worker valued a typical lost-workday injury at \$47,900.

Smokers valued it at \$26,100.

Workers who used seat belts valued it at \$78,200.

What value should the government use?

What value should the business use?

Table 20.4: Cost of Various Risk-Reducing Regulations per Life Saved

Regulation	Year	Agency	Cost per life saved (millions of 1995 \$)	Cost per year of life saved (millions of 1995 \$)
Unvented space heater ban	1980	CPSC	0.1	0.0
Steering column protection standards	1967	NHTSA	0.1	0.0
Children's sleepware flammability ban	1973	CPSC	1.0	0.1
Rear lap/shoulder belts	1989	NHTSA	3.8	0.2
Ethylene dibromide in drinking water	1991	EPAA	6.8	0.8
Benzene occupational exposure	1987	OSHA	10.6	1.3
Asbestos ban	1989	EPA	131.8	15.8
Atrazine/Alachor in drinking water	1991	EPA	109,608	13,126

A Thorough Explanation of the Viscusi Approach

(1) Suppose a person has wealth  $W$  and faces probability  $p$  of dying. The person's utility of wealth is 0 if he is dead, we will assume (what is crucial is not the normalization to zero, but that if he is dead, his utility does not depend on his remaining wealth). What is the maximum fraction of  $W$ ,  $p^*$ , that he willing to pay to eliminate risk  $p$ ?

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because the person will have  $[1-p^*]W$  for sure if he pays to eliminate the risk, but if he does not pay, he has a gamble between dying (for payoff 0) or being alive with his fortune intact (for payoff  $U(W)$ ).

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For example, if the person's lifetime wealth is 1 million dollars, and the risk is  $p= .5$ , the person might be willing to pay \$700,000 to eliminate the risk, because he is indifferent between \$300,000 and no risk and \$1 million with a 50% risk of dying.

Note that if we start from the amount the person is willing to pay—\$700,000 here— then to find the value of the person's life, doubling the \$700,000 is an *upper* bound to the value.

A Thorough Explanation of the Viscusi Approach: General Probability  
p

(4) Now look more generally at risk  $p$ . It must be true that

$$U([1-p^*]W) = p(0) + (1-p) U(W),$$

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and thus

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so to avoid a risk of death of 1%, the person will be willing to pay more than 1% of his wealth.

Small Probabilities

(5) The difference between  $p$  and  $p^*$  gets smaller as  $p$  gets smaller, though,

because locally the utility curve is closer to linear.

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so, dropping second and higher-order terms,

$$U([1 - p^*]W) \text{ is approximately } U(W) - U'(W)(p^*W),$$

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That is the use of this idea in the Valuation of Life literature and in chapter 20 of Viscusi, Vernon, and Harrington.

Backing This Up To Get Wealth Estimates

(6) But let us think of another implication. If we start with someone's lifetime wealth,  $X$ , then they should be willing to pay at most  $pX$  to avoid a risk of death of  $p$ , for a small risk. Thus, we don't need to do estimation through data from surveys, actual purchases of safety equipment, or job choice, if we know  $p$  and  $X$ . Instead, we just use  $p$  and  $X$ .

This makes the chief use of the estimates from surveys, actual purchases and job choice *\*not\** to find out anything about a person's utility function, but to discover their beliefs about their lifetime wealth.

Or, if we find that the resulting "value of life" is not close to what we know the person's lifetime wealth is using other data available to us, we know that the person is confused or irrational.

Measuring Wealth

(7) This can be related to total wealth. But what is total wealth?

(a) U.S. GDP in 2004 is about 10 trillion dollars. Suppose we assume that this will grow at 2% per year (per capita GDP growth has averaged 2.1% over the past 10 years), and suppose we discount it at 5% (a reasonable figure for a risky asset). That gives us a net discount rate of 3%, so the present value is  $10/.03 = 333$  trillion dollars.

(b) The BEA says Non-labor wealth equals 39 trillion. Add 5 trillion for government wealth. Then multiply by 4, because labor's share of GNP is about 75% (see Krueger article). This yields  $44 \times 4 = 176$  trillion.

Dividing these figures by 300 million yields an average value for a life of .586 to 1.11 million dollars.

Perhaps, though, we should be using median wealth rather than mean wealth. Mean income is 57 thousand dollars per household and the median is 41 thousand, so median wealth— if proportional to income— is  $41/57 = .71$  times as high as mean wealth. This would give us a value for a life of \$416,000 to \$788,000.

We might also want to subtract out bequests, which people clearly do value. Perhaps 1/4 of wealth is bequeathed, since 1/4 of it is physical rather than human capital and our amount of capital is steady or growing over time.

We might or might not want to subtract out the portion of wealth owned by the government—18.9%, if proportional to income. Sources

Statistical Abstract, 2003. Table 680. Money income of households: median was 41,000. Mean is  $5978,107/(104539) = 57,000$  in 2000. Table 709: 2001 median family net worth was 86K, mean was 395K. Table 710. Household and nonprofit net worth in 2002 was 39 trillion. Table 659. GDP in 2002 was 10.4 trillion dollars. 18.9% of that is government. Table 666. Per capita GDP rose 2.1% annually over the past 10 years.

Measuring Labor's Share, Alan B. Krueger, *The American Economic Review*, Vol. 89, No. 2, Papers and Proceedings of the One Hundred Eleventh Annual Meeting of the American Economic Association. (May, 1999), pp. 45-51.

Stock Market Wealth and Consumption, James M. Poterba, *The Journal of Economic Perspectives*, Vol. 14, No. 2. (Spring, 2000), pp. 99-118.

Recent Trends in the Size Distribution of Household Wealth, Edward N. Wolff, *The Journal of Economic Perspectives*, Vol. 12, No. 3. (Summer, 1998), pp. 131-150.

The Value of Risks to Life and Health W. Kip Viscusi *Journal of Economic Literature*, Vol. 31, No. 4. (Dec., 1993), pp. 1912-1946.

Actual Wealth: Other Approaches

(c) Here's another approach. Suppose the value of life is 5 million dollars per person. What value of wealth justifies that?

We have about 300 million people in the U.S., and a million millions is a trillion, so the wealth needed is  $5 \times 300 = 1500$  trillion. That is too high.

How about making the value of life 1 million dollars per person? Then the wealth of the US would be 150 trillion, which is in the correct range.

How about looking at it from the point of view of an individual? 40 years of undiscounted earnings of \$30,000/year equals 1.2 million dollars. So that puts us in the correct range also. (Suppose someone starts at \$30,000 per year and then their salary increases at a rate of 5% per year, In 40 years their salary would be about \$210,000. So this is an overestimate.)

In one sense it is easier to do these calculations in aggregate than for individuals. Looking at an individual, we would have to worry about how to value babies, knowing that their parents and other relatives would be willing to pay to save them. But in aggregate, my test is making the relatives pay for their own lives too, preventing double counting and removing the question of just how altruistic they are.

(d) We can think about this for an individual too. What is your lifetime wealth? To make it easy, assume that your salary will grow at the discount rate, which is reasonable for some kinds of jobs, at least. (Taxes, bequests?)

A More Complete Model, Model II

(1) Suppose a person has wealth  $W$  and faces probability  $p$  of dying from a certain risk that can, at a cost, be eliminated. If the person lives and

has wealth  $W$ , his utility from consumption and bequests at the end of his natural life is  $U(W)$ . If he dies of the risk and leaves amount  $W$  to his heirs, his utility is  $-D + V(W)$ . What is the maximum fraction of  $W$ ,  $p^*W$ , that the person would be willing to pay to eliminate risk  $p$ ? If there is an interior solution,

$$U(W - p^*W) = p [-D + V(W)] + [1-p] U(W).$$

In our special case above,  $D=0$  and  $V(W)=0$ .

What would it mean to assume that  $-D + V(W) \geq 0$ ? I'm not sure. I haven't done a normalization of  $U(0) = 0$ , which would give that meaning.

But it might happen that there is a corner solution at  $p^* = 1$ , because

$$U(0) \geq p [-D + V(W)] + [1-p] U(W)$$

Model II: Small Probabilities

In this model, I think maybe  $p$  and  $p^*$  do *not* converge as  $p$  gets small, even though locally the utility curve is close to linear. But I'm not sure. Start from the interior solution defining  $p^*$ :

$$U(W - p^*W) = p [-D + V(W)] + [1-p] U(W)$$

and rewrite it as

$Z = U(W - p^*W) = p[-D + V(W)] + [1 - p]U(W)$  Now take a Taylor series:

$$U(W - p^*W) = U(W) + U'(W)(W - [1 - p^*]W) + (1/2)U''(W)([1 - p^*]W - W)^2 + \dots$$

$$U(W - p^*W) = U(W) - U'(W)(p^*W) + (1/2)U''(W)(p^*W)^2 + \dots$$

so, dropping second and higher-order terms,

$$U(W - p^*W) \text{ is approximately } U(W) - U'(W) (p^*W),$$

which is approximately

$$U(W) - p^*U'(W) = (1-p^*)U(W).$$

But if these converge, what about the term  $p [-D + V(W)]$ ? It is still hanging there. I'll have to figure this out some day.

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**Questions You Should Be Able to Answer**

**Terms to Know**

**Homework Questions**

HERE PUT EXAMPLES WITH DIFFERENT NUMBERS THAN IN THE TEXT