The Learning Curve in a Competitive Industry

(Forthcoming, *RAND Journal of Economics*)

May 23, 1996

Emmanuel Petrakis, Eric Rasmusen, and Santanu Roy

Abstract

We consider the learning curve in an industry with free entry and exit, and price-taking firms. A unique equilibrium exists if the fixed cost is positive. Although equilibrium profits are zero, mature firms earn rents on their learning, and, if costs are convex, no firm can profitably enter after the date the industry begins. Under some cost and demand conditions, however, firms may have to exit the market despite their experience gained earlier. Furthermore, identical firms facing the same prices may produce different quantities. The market outcome is always socially efficient, even if it dictates that firms exit after learning. Finally, actual and optimal industry concentration does not always increase in the intensity of learning.

Petrakis: Departamento de Economia, Universidad Carlos III de Madrid, Calle Madrid 126 28903 Getafe (Madrid) Spain. Fax: 341-624-9875. Phone: 341-624-9652. Internet: Petrakis@eco.uc3m.es.

Rasmusen: Indiana University School of Business, 1309 East 10th Street , Bloomington, Indiana, U.S.A 47405-1701. Phone: (812) 855-9219. Fax: 812-855-3354. Internet: Erasmuse@indiana.edu.
Web: http://ezinfo.ucs.indiana.edu/∼erasmuse.

Roy: Department of Economics, Microeconomics Section, H09-20, Erasmus University, 3000 DR Rotterdam, The Netherlands; Fax: (+31)-10-212-1724, Tel: (+31)-10-408-1420, Internet: roy@micro.few.eur.nl.
Keywords: Learning curve, industry evolution, perfect competition.

We would like to thank Tai-Yeong Chung and seminar participants at Notre Dame, Tilburg University, the University of Southern California, the University of Western Ontario, the 1992 European Econometric Society Meetings, the Seventh Annual Congress of the European Economic Association, and the ASSET Meetings in Toulouse for their comments. We would especially like to thank the referees for their care in reading this paper. The current version has benefitted considerably from their insight. John Spence provided research assistance. This work was begun at the University of California, Los Angeles.

I. Introduction

Economists have long been aware that a firm's cost curve for producing a given item may shift down over time as learning occurs. The plot of the cost level against cumulative output is known as the learning curve or experience curve. Our subject here is learning in an industry of price-taking firms with free entry and exit– an arbitrarily large measure of firms with identical technologies producing homogeneous good. Each firm's cost curve shifts down with its own accumulated experience in production, measured by its cumulative output.

The assumption of a perfectly competitive market structure distinguishes our model from much of the existing literature on learning-by-doing, which has focussed on monopoly and oligopoly.[1] If the average cost at any point of time is constant in current output, then learning introduces an intertemporal economy of scale that creates a natural monopoly. This need not be the case, however, if the technology displays sufficient decreasing returns. In that case, learning does not lead to a natural monopoly and is, in fact, compatible with perfect competition. Learning-by-doing is distinct from increasing returns to scale in this sense.[2]

Our model is not part of that branch of the learning literature in which an individual firm's experience spills over to other firms in the industry[3] While the market structure in these models is competitive, the presence of learning spillovers gives rise to decreasing-cost industries as distinct from decreasing-cost firms. We exclude such spillovers, and consider only firm-

[1] See Spence (1981), Clarke, Darrough & Heineke (1982), Fudenberg & Tirole (1983), Smiley & Ravid (1983), Bhattacharya (1984), Dasgupta & Stiglitz (1988), Jovanovic & Lach (1989), Mookherjee & Ray (1991), and Cabral & Riordan (1991).

[2] See Mookherjee & Ray (1992) for a discussion of other differences between increasing returns to scale and learning-by-doing.

[3] The learning spillover literature includes Arrow (1962), Ghemawat & Spence (1985), Romer (1986), Lucas (1988), and Stokey (1986, 1988).

specific learning-by-doing.

Our point of departure is Fudenberg & Tirole (1983), which considers learning-by-doing in a competitive industry with constant instantaneous marginal cost. In their setting, learning-by-doing is incompatible with perfect competition, but we will come to a different conclusion, because we analyze an industry with the usual textbook assumption of *increasing* marginal cost, not constant marginal cost. We will show that in a two-period model with a fixed cost, a unique perfectly competitive equilibrium exists.

If we make the stronger assumption that costs are convex, then no firm can profitably enter after the date at which the industry begins. This reflects the entry-deterring aspect of learning-by-doing; however, in our model entry is not prevented by any strategic action of incumbent firms, but is a direct consequence of dynamic competitive equilibrium with atomistic firms. The unique equilibrium takes one of two forms, depending on the demand and cost parameters of the economic environment.

In the first type of environment, all firms that enter remain in the industry permanently. The equilibrium discounted stream of profits is zero, but mature firms earn quasi-rents on their learning, compensating for their losses in the first period.

In the second type of environment, some firms exit, because the mature industry cannot sustain the original number of firms with non-negative profits. Relatively inelastic demand coupled with a strong learning effect gives rise to this outcome. Initially identical firms, facing the same prices, produce different quantities of the homogeneous good in the first period, and some of them will exit in the second period. Firms which exit produce less than the staying firms in the first period.

This is a new explanation for the endogenous differentiation of ex ante identical firms and *shakeout* of small firms, which does not rely on either aggregate or firm-specific uncertainty.

There is widespread empirical support for shakeout (e.g., Gort and Klepper [1982]) and for higher exit rates among smaller firms (Dunne, Roberts & Samuelson [1989], Davis & Haltiwanger [1992]). The existing literature has explained shakeout as the process of "market selection" in the presence of firm-level uncertainty. In this explanation, firms that are identical ex ante become heterogeneous over time through idiosyncratic shocks, uncertain learning, innovation and diffusion, and so forth, which lead to exit of the ex post inefficient firms.[4] In our model, shakeout is a direct result of endogenous technical change at the firm level, through deterministic learning and in the presence of stationary market demand. All firms have perfect foresight at their point of entry, and some enter fully intending to exit later.

The equilibrium outcome is socially efficient whether it is characterized by exit or not. Even if exit occurs in equilibrium, a social planner would choose the same number of firms of each type, the same quantity produced by each firm in each period, and the same prices as in the competitive equilibrium. Thus, the presence of learning-by-doing implies neither the usefulness of a government industrial policy to ensure optimal learning, nor the useful effects of large, innovative monopolies so often attributed to Schumpeter (1950). Our model will uncover a pitfall that may exist for antitrust and regulatory authorities. Although all firms in our model are price-takers, one possible feature of equilibrium is that prices are sometimes below marginal cost, sometimes above marginal cost, that profits rise over time, small firms drop out of the market and large firms expand even further, and that the large firms increase their profits from negative to positive levels without any new entry occuring. Government intervention is not only unnecessary, but

[4]See, in a large literature, Lippman & Rumelt (1982), Jovanovic (1982), Jovanovic & Lach (1989), Hopenhayn (1992a,b), 1993), and Jovanovic & MacDonald (1994a, b).

possibly harmful.

Section II describes the model and discusses its assumptions. Section III presents theorems on existence and efficiency of the competitive equilibrium, and discusses the pattern of entry and exit. Section IV characterizes the equilibrium under the assumption of convex costs. Section V contains a numerical example and looks into special cases where (a) learning reduces only the fixed cost, not the variable cost of production, and (b) learning reduces just the marginal cost, not the fixed cost. Section VI concludes.

II. The Model

An arbitrarily large measure of initially identical firms compete to enter a homogeneous-good industry. The measure of firms actually operating is determined by free entry and exit. Each firm is a price taker, since it is infinitesimal compared to the industry.[5] Firms are indexed by i. Time is discrete, and the market lasts for two periods. Firm i produces output $q_t(i)$ in period $t = 1, 2$.

Each firm i faces the same current total cost at time t as a function of its current output $q_t(i)$ and its experience $x_t(i)$:

$$C(q_t(i), x_t(i)),$$

where $x_t(i)$ is firm i's cumulative output before time t, so that $x_1(i) = 0$ and $x_2(i) = q_1(i)$. Let

$$, (q_1, q_2) = C(q_1, 0) + \delta C(q_2, q_1),$$

where $\delta \in [0, 1]$ is the discount factor, so , represents a firm's discounted sum of production costs across the two periods.

[5]Other models of learning in which firms are price takers include Fudenberg & Tirole (1983), Boldrin & Scheinkman (1988), and Majd & Pindyck (1989).

If amount n of firms are active, industry output is $Q_t = \int_0^n q_t(i)di$. The market demand function, $D(p)$, is the same in both periods and is separable across time. Let $P(Q)$ be the inverse demand function. Define p_m as the minimum average cost at zero experience.

$$p_m = \min_{q \geq 0} [C(q,0)/q],$$

and let q_m be the corresponding minimum efficient scale, so

$$q_m \in \operatorname{argmin}_{q \geq 0} [C(q,0)/q]$$

We impose the following six assumptions on costs and demand:

(A1) $C(q,x)$ is continuously differentiable on $\mathbf{R}_+^2$.

(A2) $C_q(q,x) > 0$ for $q > 0$ and $x \geq 0$; $C_x(q,x) \leq 0$ for all $(q,x) \in \mathbf{R}_+^2$ and $C_x(q,x) < 0$ for all $(q,x) \in (0,K] \times [0,K]$, where K is defined in (A5).

(A3) For any $q > 0$ and $x \geq 0, C(q,x) > 0$; Also, $C(0,0) > 0$.

(A4) P is continuous and strictly decreasing; $P(Q) \to 0$ as $Q \to +\infty$; $P(Q)$ is integrable on any closed interval of $\mathbf{R}_+$.

(A5) [Eventual Strong Decreasing Returns] There exists $K > 0$ such that the following holds: if either $q_1 > K$ or $q_2 > K$ (or both), then there exist α and $\beta \in [0,1]$ such that

$$, (q_1, q_2) > , (\alpha q_1, \beta q_2) + , ((1-\alpha)q_1, (1-\beta)q_2).$$

(A6) $P(0) > p_m$.

Assumption (A1) guarantees the continuity of the marginal cost and marginal benefit functions.

Assumption (A2) says that the marginal cost is always positive, that greater experience never increases the total cost, and that greater experience strictly reduces the total cost of producing output in any period.

The total production cost is nonincreasing in the amount of accumulated experience. Figure 1 shows one cost function that satisfies the assumptions—the cost function which will be used in Example 2 later in the article. Note the increasing marginal costs for any level of learning, and the decreasing returns to learning for any level of output.

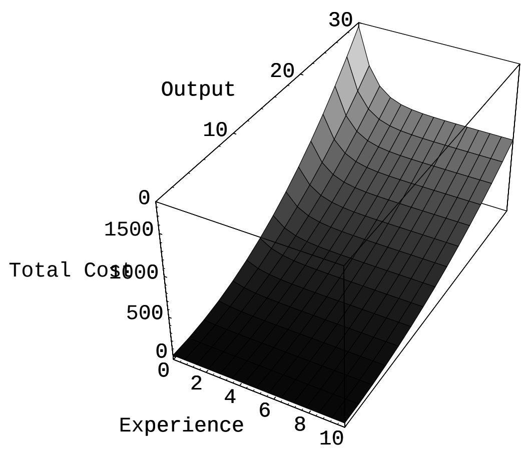

Figure 1: A Firm's Total Cost as a Function of Output and Experience

Assumption (A3) says that a firm with no experience incurs a fixed cost of production, a cost which must be incurred even if output is zero.

Assumption (A4) is a standard assumption on the demand curve, and allows the social planner's problem to be well-defined.

Assumption (A5) says that if output produced by a firm is too large in any period , it is possible to have two firms produce the same output vector at a lower total cost. This prevents the industry from being a natural monopoly.[6]

Assumption (A6) places restrictions on the demand and cost functions jointly to ensure existence of a nondegenerate equilibrium. If $P(0)$ were allowed to take any value, no matter how small, then the equilibrium might be at zero output for every firm.

Only the six assumptions listed above are needed for our main results, but with a little more structure we can strengthen the results further. We will do that in Section IV by adding the following assumption, which is not implied by (A1)-(A6):

(A7) [Convex Costs] C is convex; for all $x \geq 0$, $C(q, x)$ is strictly convex in q, and for all $q \geq 0$, if $x_1 > x_2$, then $C_q(q, x_1) \leq C_q(q, x_2)$.

Assumption (A7) requires that the total cost function be convex in x and q, and that marginal cost be non-increasing in experience. For any level of experience, the current marginal cost is strictly increasing in current output. Assumption (A7) is sufficient to ensure strict convexity of , on $\mathbf{R}^2_+$. Part of this assumption is that C_x is nondecreasing in x; that is, there are decreasing returns to learning at any given level of current output. Note

[6] If one thinks in terms of multiproduct firms, (A5) requires that the joint cost of production is no longer subadditive if the firm produces an excessive amount of the two goods (see Panzar [1989]). Note, incidentally, that the crucial difference between a learning model and a static model of joint production is time consistency: in our learning model we will require that second-period profits be non-negative (or no firms would operate in the second period), whereas in static joint production, profits on either one of the goods can be negative.

that this assumption allows for a positive fixed cost, incurred even when no output is produced.

Assumption (A7) is not necessary for existence, uniqueness, and optimality of equilibrium prices and so will not be used for Propositions 1 and 2.

Our specification of the cost function allows a firm to accumulate experience on both its fixed and marginal costs. Each firm maximizes its discounted stream of profits, taking prices as given. An active firm exits the industry in the second period if its profits from that time on would be negative. A firm with no experience enters the industry in the second period only if it can make positive profits in that period.

III. Properties of the Competitive Equilibrium

Let p_t be the market price in period t. Denote firms that stay in the market for both periods as **staying or S-type firms**, with output q_t and flow profit π_t in period t. Denote firms that exit at the end of the first period as **exiting or E-type firms**, with output q_E and profit π_E. Denote firms that enter the industry at the beginning of the second period as **late-entering or L-type firms**, with output q_L and profit π_L in the second period (*not* discounted back to the first period). Finally, let n_S, n_E, and n_L be the measures of active firms of each type.

A firm staying in the industry for both periods maximizes its discounted sum of profits. The first order conditions for the firm's profit maximization problem are:

$$p_1 = C_q(q_1, 0) + \delta C_x(q_2, q_1) \tag{1}$$

$$p_2 = C_q(q_2, q_1). \tag{2}$$

Equation (1) says that as long as learning still occurs, a staying firm will choose output in the first period so that its marginal cost is greater than the market price, since $C_x(q_2, q_1)$ is negative. In other words, a staying firm overproduces in the first period in order to reduce its cost in the second period. As a result, a staying firm makes losses initially in equilibrium, which are counterbalanced by positive gains later. Equation (2) is the standard "price equals marginal cost" condition. If the market ends in the second period, any further learning is of no use to a staying firm, so a firm maximizing profits from that time on chooses output to equate price to its marginal cost.

Marginal cost in the standard model is here replaced by what we will call the *Net Marginal Cost*: the increase in lifetime discounted costs when current output increases, which for the first period is $C_q(q_1, 0) + \delta C_x(q_2, q_1)$. Note that the net marginal cost comes arbitrarily close to the marginal cost of a non-experienced firm for sufficiently large q_1, because if a firm produces too much today, its marginal benefit from learning becomes almost zero.

An equilibrium should be characterized by rational price-taking behavior on the part of firms, but rationality and price taking do not necessarily result in identical behavior by all firms. Let $\pi_S(p_1, p_2)$ be the profit function of S-type firms. Then,

$$\pi_S(p_1, p_2) = \overset{Maximum}{\{q_1, q_2 \geq 0\}} \ [p_1 q_1 + \delta p_2 q_2 - C(q_1, 0) - \delta C(q_2, q_1)]. \qquad (3)$$

Similarly, let $\pi_E(p_1, p_2)$ and $\pi_L(p_1, p_2)$ denote the profit functions of E and L-type firms, so

$$\pi_E(p_1, p_2) = \overset{Maximum}{\{q \geq 0\}} \ [p_1 q - C(q, 0)] \qquad (4)$$

and

$$\pi_L(p_1, p_2) = \overset{Maximum}{\{q \geq 0\}} \ [p_2 q - C(q, 0)]. \qquad (5)$$

For types $k = S, E, L$, let $J_k(p_1, p_2)$ denote the sets of solutions to the maximization problems in equations (3), (4), and (4).

We define equilibrium as follows.

Definition: An equilibrium is defined by:

(a) Measures (n_S, n_E, n_L) of type S, E and L firms who enter the market.

(b) Functions $q_1(i)$ and $q_2(i)$, where $q_t : [0, n_S] \to \mathbf{R}_+, \; j = 1, 2, q_t(.)$ integrable (with respect to Lebesgue measure); $q_t(i)$ is the output produced by firm i of S-type in period t.

(c) Functions $q_E : [0, n_E] \to \mathbf{R}_+$ and $q_L : [0, n_L] \to \mathbf{R}_+$, integrable, where $q_E(i)$ and $q_L(j)$ are the output produced by the i-th E and j-th L type firms in their periods in the market.

(d) Prices $p_1 \geq 0$ and $p_2 \geq 0$.

The variables defined in (a) - (d) must satisfy the following conditions to constitute an equilibrium:

$(i) p_1 = P(Q_1 + Q_E), Q_1 = \int_0^{n_S} q_1(i)di, Q_E = \int_0^{n_E} q_E(i)di$ (Markets clear in the first period.)

$(ii) p_2 = P(Q_2 + Q_L), Q_2 = \int_0^{n_S} q_2(i)di, Q_L = \int_0^{n_L} q_L(i)di$ (Markets clear in the second period.)

(iii) If $n_S > 0, (q_1(i), q_2(i)) \in J_S(p_1, p_2)$; if $n_E > 0$, then $q_E(i) \in J_E(p_1, p_2)$; if $n_L > 0$, then $q_L(i) \in J_L(p_1, p_2)$ (Active firms of all types maximize profit.)

(iv) For $k = S, E, L$,

$$\begin{aligned} \pi_k(p_1, p_2) \quad &= 0 \; if \; n_k > 0 \\ &\leq 0 \; if \; n_k = 0 \end{aligned}$$

(Every active firm earns zero total profit over its period of stay, and further entry is not strictly profitable.)

In equilibrium, no firm can make positive profit by behaving like some other type. No S-type firm can do better by exiting at the end of period 1 nor can an E-type firm make positive profit by staying on till period 2 (even if there are no S-type firms in the market) and so forth. This ensures sequential rationality on the part of the E-type and S-type firms, who might otherwise find it advantageous to change their second-period behavior halfway through the evolution of the industry.

In an equilibrium with exit, an exiting firm makes zero profits in the first period, and in an equilibrium with late entry, a late entrant makes zero profits in the second period. A firm with no experience behaving optimally during the single period in which it remains in the market makes zero profits if and only if the market price equals its minimum average cost. In any equilibrium, no firm produces output in the range of strongly diminishing returns.

These requirements for rational and competitive behavior on the part of the firms imply a number of restrictions on equilibrium outcomes, which are summarized in Proposition 1.

PROPOSITION 1. *1. In any equilibrium, the price in each period is at most the minimum average cost for a firm with zero experience. 2. A strictly positive measure of staying firms produce in both periods. 3. All staying firms earn strictly negative profits in the first period and strictly positive profits in the second period. 4. If there is exit in the equilibrium, the first-period price exactly equals the minimum average cost, and all exiting firms earn zero profits in the first period. 5. If there exist late-entering firms, the second-period price is the minimum average cost for a firm with zero experience. 6. The output of every active firm in any time period is bounded above by K. 7. Late entry and exit cannot simultaneously occur in a single equilibrium.*

Proof. The proof of Proposition 1 is accomplished by showing the following:

1. $p_j \leq p_m < P(0), j = 1, 2$.
2. $n_S > 0, Q_1 > 0$ and $Q_2 > 0$.
3. For all $i \in [0, n_S], [p_1 q_1(i) - C(q_1(i), 0)] < 0$ and $[p_2 q_2(i) - C(q_2(i), q_1(i))] > 0$.
4. If $n_E > 0$, then $Q_E > 0$, $p_1 = p_m$ and for $i \in [0, n_E], q_E(i) \in \{q : [C(q, 0)/q] = p_m\}$.
5. If $n_L > 0$, then $Q_L > 0, p_2 = p_m$ and for $i \in [0, n_L], q_L(i) \in \{q : [C(q, 0)/q] = p_m\}$.
6. $q_1, q_2, q_E, q_L \leq K$.
7. Either $n_E > 0$ or $n_L > 0$, but not both.

Recall that $p_m = Min\{[C(q, 0)/q] : q \geq 0\}$. Conditions (iii) and (iv) imply that if in an equilibrium we have $n_E > 0$, then $p_1 = p_m$ and $q_E(i) \in \{q : C(q, 0)/q = p_m\}$. Similarly, if $n_L > 0$ then $p_2 = p_m$ and $q_L(i) \in \{q : C(q, 0)/q = p_m\}$.

Condition (iv) also implies that $p_i \leq p_m, i = 1, 2$. From assumption (A6), we have $P(0) > p_m$ and so in any equilibrium it must be true that $p_i < P(0)$. It follows that $Q_1 + Q_E = D(p_1) > 0$ and $Q_2 + Q_L = D(p_2) > 0$.

To prove part (6), it is sufficient to consider the case of the staying firms. Suppose $q_t > K$ for some t. In equilibrium, a firm's lifetime profit is zero, so

$$0 = p_1 q_1 + \delta p_2 q_2 - , (q_1, q_2) < p_1 q_1 + p_2 q_2 - , (\alpha q_1, \beta q_2) - , ((1-\alpha) q_1, (1-\beta) q_2), \quad (6)$$

for some α, β in $[0, 1]$, using assumption (A5). The rightmost expression, can be rewritten as

$$[p_1 \alpha q_1 + p_2 \beta q_2 - , (\alpha q_1, \beta q_2)] + [p_1(1-\alpha) q_1 + \delta p_2 (1-\beta) q_2 - , ((1-\alpha) q_1, (1-\beta) q_2)], \quad (7)$$

which is either zero or negative. In combination with the strong inequality in (6), this yields a contradiction, so it must be false that $q_t > K$ for some t.

Suppose $n_E > 0$ and $n_L > 0$. Then, $p_1 = p_2 = p_m$. This violates condition (iv) of equilibrium, since by part (6) of Proposition 1, $C_x < 0$, and facing those prices a firm could produce q_m in each period and earn $\pi_1 = 0$ and $\pi_2 > 0$. Thus, $n_E > 0$ and $n_L > 0$ is impossible.

Now, suppose there is an equilibrium where $n_S = 0$. Then, since $D(p_t) > 0, t = 1, 2$ in equilibrium implies that $n_E > 0, n_L > 0$, a contradiction. So, in equilibrium, we must have $n_S > 0$. This, in turn, can be used to show that $Q_1 > 0$ and $Q_2 > 0$. Suppose $Q_1 = Q_2 = 0$. Then, $Q_E > 0, Q_L > 0$, i.e. $n_E > 0, n_L > 0$, a contradiction. Suppose, $Q_1 = 0, Q_2 > 0$. Then, $n_E > 0$, i.e. $p_1 = p_m$. Now if some S-type firm produces $q_1 = 0$, it earns a loss of $C(0, 0)$. On the other hand if it produces $q_1 = q_m > 0$ (where $C(q_m, 0)/q_m = p_m$), then it has a lower cost function in period 2 while the current loss is zero. So producing $q_1 = 0$ cannot be profit maximizing. Thus, $q_1(i) > 0$ for almost all $i \in [0, n_S]$, that is, $Q_1 > 0$, a contradiction. Similarly, $Q_2 = 0, Q_1 > 0$ is ruled out.

From the first order conditions of profit maximization for S-type firms it is clear that $p_1 < C_q(q_1, 0)$ so that q_1 does not maximize period 1 profit at price p_1. Coupled with the fact that $p_1 < p_m$, this implies that for all $i \in [0, n_S]$, $[p_1 q_1(i) - C(q_1(i), 0)] < 0$, so that (iv) implies $[p_2 q_2(i) - C(q_2(i), q_1(i))] > 0$.

Conditions (iii) and (iv) imply that if $n_E > 0$, then for $i \in [0, n_E], q_E(i) > 0$ and $[C(q_E(i), 0)/q_E(i)] = p_m$. Similarly, if $n_L > 0$ then for $i \in [0, n_L], q_L(i) > 0$ and $[C(q_L(i), 0)/q_L(i)] = p_m$. //

In equilibrium, initially identical firms may behave very differently, some staying, some exiting, and some entering late. A socially optimal allocation would solve the following problem:

The Social Planner's Problem (SPP*):

Choose

(a) (n_S, n_E, n_L): the measures of type S, E, and L firms who enter;

(b) Functions $q_1(i)$ and $q_2(i)$, where $q_t : [0, n_S] \to \mathbf{R}_+, t = 1, 2,$ and $q_t(.)$ is integrable with respect to Lebesque measure, $q_t(i)$ being the output produced by staying firm i in period t;

(c) Integrable functions $q_E : [0, n_E] \to \mathbf{R}_+$ and $q_L : [0, n_L] \to \mathbf{R}_+$, where $q_E(i)$ and $q_L(j)$ are the output produced by the i-th E and j-th L type firms, respectively, in their periods of operation;

so as to maximize

$$\int_0^{Y_1} P(q)dq + \int_0^{Y_2} \delta P(q)dq - \int_0^{n_S}[C(q_1(i), 0) + \delta C(q_2(i), q_1(i))]di$$
$$- \int_0^{n_E}[C(q_E(i), 0)]di - \delta \int_0^{n_L}[C(q_L(i), 0)]di$$

where $Y_1 = Q_1 + Q_E$ and $Y_2 = Q_2 + Q_L$, and

$$Q_1 = \int_0^{n_S} q_1(i)di, \quad Q_2 = \int_0^{n_S} q_2(i)di, \quad Q_E = \int_0^{n_E} q_E(i)di, \quad Q_L = \int_0^{n_L} q_L(i)di.$$

The social planner's problem includes choosing the entry and exit of firms as well as their output. The marginal social benefit of output in each period is exactly the demand price for that output. Maximization of social surplus implies that the net marginal cost of each firm is equated to the demand price for total output produced. Therefore, if the market price is equal to the demand price corresponding to the socially optimal total output in each period, then each active firm maximizes profit and the market clears in every period. The complementary slackness condition for the social planner's problem with respect to choice of entry of firms implies that every entering firm gets zero total profit and that no firm can make a strictly positive profit by entering the market. Thus, the solution to the social planner's problem corresponds to a competitive market outcome.

Under assumptions (A1) to (A6), not only does a competitive equilibrium exist, but it is unique in prices and it is socially optimal.

PROPOSITION 2. *Under assumptions (A1) to (A6), an equilibrium exists. It is unique in prices and aggregate output, and it is socially optimal.*

One way to prove Proposition 2 would be by following the arguments in Hopenhayn (1991), which analyzes a more general model of dynamic competitive equilibrium with stochastic shocks. We used a direct proof, an outline of which is contained in the Appendix, in line with the approach of Jovanovic (1982).

Before closing this section of the paper, one further explanation may be useful. Our assumption that the fixed cost of production is strictly positive is required to ensure the existence of a finite competitive equilibrium. If the fixed cost of production is zero, (i.e. $C(0,x) = 0$ for all x), and if costs are convex, then a firm accumulates experience only in order to reduce its marginal cost.[7] A well known result from standard price theory is that a competitive industry with increasing marginal costs, free entry, and no learning possibilities has no equilibrium if the fixed cost of production is zero. Loosely speaking, an infinite number of firms operate in the market, each producing an infinitesimal amount of output. This holds true even if firms are able to reduce their costs by accumulating experience.[8]

IV. Further Results: The Case of Convex Costs

Let us now introduce assumption (A7), convexity of the cost function, noting that (A7) does not necessarily imply (A5), which must still be retained, and that (A7) also requires that the marginal cost of production be

[7] A referee has pointed out that the limiting behavior of the competitive equilibrium in our model as the fixed cost converges to zero does not necessarily coincide with the market outcome when fixed cost is zero and learning is absent. Learning persists, as does its effect on prices and output, even as the fixed cost is reduced to zero.

[8] Another way to understand this is through the social optimality of competitive equilibrium, an implication of which is that the competitive market structure and allocation of output minimizes the social cost of production. If the firm-specific cost function is convex and the fixed cost is zero, social cost is minimized by having an indefinitely large number of firms, each producing infinitesimal output each period.

weakly decreasing with learning.

Earlier we saw that the equilibrium is unique in prices. When costs are convex, it is also unique in output and the number of firms.

PROPOSITION 3. *Under assumptions (A1)-(A7), the equilibrium is unique in prices, individual firms' outputs in each period, and the number of firms.*

Proof: See the Appendix.

Convexity also allows us to be more specific about the properties of the equilibrium, as shown in the next propositions. In an industry without the possibility of learning, identical firms produce the same output in equilibrium if the marginal cost curve is upward sloping. When the opportunity for learning is added, identical firms could behave differently in the same equilibrium.

PROPOSITION 4. *Under assumptions (A1)-(A7), the following is true in equilibrium:*

(a) Each of the staying firms behaves identically.

(b) If there is a positive measure of exiting firms, they produce at the zero-experience minimum efficient scale, which is less than the output produced by staying firms in period 1.

(c) There exist no late-entering firms.

Proof: See the Appendix.

Proposition 4 allows the unique equilibrium to take one of two distinct forms, depending on the cost and demand parameters: (i) with exit at the end of the first period, or (ii) without exit.

In an equilibrium *with* exit, some firms, after producing in the first period, decide to leave the industry. Thus, two types of firms coexist, those staying for both periods and those exiting at the end of the first period. Furthermore, firms that are identical ex ante nonetheless produce different outputs even in the first period. For a given price in period 1, exiting firms will produce less than staying firms, because overproducing to reduce future costs has no value for a firm that plans to exit at the end of the first period.

In an equilibrium *without* exit, all firms entering in the first period stay in the industry both periods (i.e. all are staying firms). Firms make losses today in order to accumulate experience, while they earn profits tomorrow on their maturity. To break even, the present value of the future profits must equal the losses today.

It is perhaps surprising that assumption (A7) is needed to ensure that there exist no late-entering firms in equilibrium. After all, a late-entering firm must compete with staying firms that have lower costs, and Proposition 1 showed that if late-entering firms do exist, it must be the case that the price is p_m in the second period, so $p_2 = p_m$ and the experienced firms are charging no more than than inexperienced firms. Example 1, in which costs are nonconvex, shows what can happen.

In the preceding section, Proposition 2 stated that the competitive equilibrium is efficient, and convex costs is only a special case of this. This implies that when costs are convex, late entry is inefficient, even though the staying firms earn positive profit in the second period. The reason is that late entrants, lacking experience, would have higher costs even if their profits were zero.

Example 1: Nonconvex Costs and Late Entry

$D(p) = 40 - 3p$

$\delta = 1$

$$C(q,x) = \begin{cases} q^2 + (4 - \frac{x}{100}) & for\ x < 3 \\ \\ q^2 + \frac{1}{(8/3)x - 7} & for\ x \geq 3 \end{cases}$$

In Example 1, the learning is entirely in the fixed cost. The technology is nonconvex because the rate of learning increases at $x = 3$, but it does satisfy assumption (A5), because decreasing returns set in at a large enough scale of operation.[9]

In equilibrium, $n_S = 10, n_E = 0, n_L = 4, q_1 = 3, q_2 = 2, q_L = 2$, $p_1 = 10/3$, and $p_2 = 4$. These prices clear the market, because

$$D(p_1) = 40 - 3(10/3) = 30 = n_S q_1 + n_E q_E = 10(3) + 0$$

and

$$D(p_2) = 40 - 3(4) = 28 = n_S q_2 + n_L q_L = 10(2) + 4(2).$$

The prices yield zero profits for the late-entering firms because $q_m = 2$ and $p_m = 4$. They yield zero profits overall for the staying firms because their profits are

$$\begin{aligned} \pi_1 + \pi_2 &= [p_1 q_1 - (q_1^2 + 4 - \tfrac{x_1}{100})] + [p_2 q_2 - (q_2^2 + \tfrac{1}{(8/3)x_2 - 7})] \\ &= [(10/3)(3) - (3^2 + 4 - 0)] + [(4)(2) - (2^2 + \tfrac{1}{(8/3)(3) - 7})] = -3 + 3. \end{aligned}$$

Think of this from the point of view of a social planner. In the first period, he decides to introduce just a few firms, so that all of them can produce high output and acquire sufficient experience to cross the threshold for effective learning. In the second period, those firms cut back their output because further experience is not so valuable, but this means that for the social planner to satisfy demand he must introduce new firms.

[9] The technology violates assumption (A1) because it is not continuous and differentiable, but it should be clear that the cost function could be smoothed without doing more than making the numbers less tidy.

Example 1 incidentally illustrates a point that will be generalized in Proposition 6: learning can make prices *increase* over time, even though costs are falling. This is because firms overproduce in the first period, incidentally driving down the price, in order to learn and save on their fixed costs later.

The discussion so far has shown that exit may occur in equilibrium, which makes the question of market efficiency especially interesting. A firm that exits seems to waste its learning. Can it be socially efficient that some firms never make any use of their first-period learning? Surprisingly enough, Propositions 2 and 3 tell us that the answer is yes. The unique equilibrium may involve some firms entering in the first period, producing a positive output and thereby reducing their costs, but then exiting before the second period. Their learning is wasted. Propositions 2 and 3 say that this is socially optimal— a social planner would also require that some firms exit, rather than direct that there be fewer firms in period 1. Social optimality therefore does not imply the kind of "rationalization of industrial production" that governments favor when they try to consolidate firms in an industry.

With a little thought, it becomes clear why this is so. Suppose the marginal cost curve initially slopes steeply upwards at some production level q', but that after a firm acquires experience, its marginal cost curve is closer to being flat. In the first period, it would be very expensive to serve market demand with firms producing much more than q'. Therefore, the optimal plan is to have some firms produce only in the first period, to keep output per firm low, but to have those firms exit in the second period, because the diseconomies of scale then become less severe.

A variable that will be important to the issue of exit is $\theta(x)$, the ratio of the minimum efficient scale to the quantity demanded when the price equals minimum average cost. Let us call this the *natural concentration,* defined as

$$\theta(x) = \frac{q_m(x)}{D(p_m(x))}, \tag{8}$$

where

$$q_m(x) = argmin_q\{C(q,x)/q\}$$

and

$$p_m(x) = C(q_m(x),x)/q_m(x).$$

When the minimum efficient scale decreases with learning, the natural concentration θ is falling in x: loosely speaking, the market is then able to sustain more firms when firms are experienced than when they are not.

Proposition 5 lays out sufficient conditions under which exit does or does not occur in a competitive equilibrium.

PROPOSITION 5. *Let assumptions (A1)-(A7) be true. If $\theta(q_m) > \theta(0)$, where q_m is the minimum efficient scale for a firm with no learning, then there exists $\delta_0 > 0$ such that exit occurs in equilibrium for all $\delta \in (0, \delta_0)$. On the other hand, if $\theta(q_m) < \theta(0)$, there exists $\delta_0 > 0$ such that no exit occurs in equilibrium for all $\delta \in (0, \delta_0)$.*

Proof. See the Appendix.

Firms that overproduce initially in order to learn suffer losses in period 1 which they are able to recover later as they become inframarginal, with lower costs than potential entrants. Suppose that $\delta = 0$, so firms care only about first-period profits. Then the equilibrium price would be $p_m(0) = p_m$ in the first period, and each firm would produce $q_m(0) = q_m$. These firms would find in period 2 that their experience level was $x = q_m$. As we are doing calculations for discount factors sufficiently near $\delta = 0$, it is only this level of experience that we need consider. If $\theta(q_m) < \theta(0)$, then the market can sustain more firms with experience than without. If $\theta(q_m) > \theta(0)$, however, then once firms acquire experience, the market cannot sustain as many of them, and some are forced out in the second period.

Proposition 5 has implications for the important special case in which the marginal cost of production shifts down uniformly with experience:

$$C(q,x) = C_v(q) + q\phi(x) + F(x),$$

with $C_v(q)$ strictly convex in $q, \phi(x) < 0$ and $F'(x) < 0$. (Note that this specification also allows the fixed cost to fall with learning.) The function $q_m(x)$ is nonincreasing in x in this case. As a result, $\theta(x) < \theta(0)$ for all x, and Proposition 5 can be applied. Exit will not occur in equilibrium, if discounting is sufficiently heavy.[10]

Suppose, on the other hand, that learning reduces only the fixed cost. Then the minimum efficient scale decreases with experience, and so $\theta(x) < \theta(0)$ for all x, yielding Proposition 6.

PROPOSITION 6. *If learning reduces only the fixed cost, then in equilibrium there is no exit, the price rises and each firm's output falls over time:* $n_E = 0, p_1 < p_2$, *and* $q_1 > q_2$.

This is the price path illustrated in Example 1. Net marginal cost is always lower than the marginal cost of any experienced firm if learning reduces fixed cost alone. Thus, if $p_1 \geq p_2$, then $q_1 > q_2$, which contradicts the market equilibrium condition if later entry is impossible. Exit then does not occur because $p_1 < p_2 \leq p_m$. Thus, $n_E = 0$. As we have seen, we cannot

[10] The proof of the fact that $q_m(x)$ is nonincreasing in x is as follows. $q_m(x)$ is defined by equating marginal to average cost, i.e. $C_v'(q_m(x)) + \phi(x) = \frac{C_v(q_m(x))}{q_m(x)} + \phi(x) + \frac{F(x)}{q_m(x)}$. This yields $C_v'(q_m(x))q_m(x) - C_v(q_m(x)) = F(x)$. Since C_v is strictly convex, $C_v'(q)q - C_v(q)$ is strictly increasing in q. If F is nonincreasing in x, then $x_1 > x_2$ implies $q_m(x_1) \leq q_m(x_2)$ and then $\theta(x) < \theta(0)$.

draw general conclusions about the properties of the price path, because it depends on the initial costs, the type and intensity of learning, the market demand, and the discount rate. The same is true for the quantity path of staying firms.

Environments in which the equilibrium has exit are fully if less intuitively characterized in Proposition 7.

PROPOSITION 7. *The following are necessary and sufficient conditions for an equilibrium to have exit. Let (q_1^*, q_2^*) be the solution to the following minimization problem:*

$$z = \underset{q_1, q_2 \geq 0}{Minimum} \quad \frac{C(q_1, 0) + \delta C(q_2, q_1) - p_m q_1}{\delta q_2}.$$

Under assumptions (A1)-(A7), an equilibrium with $n_E > 0$ exists if and only if

$$[D(p_m)/D(z)] > [q_1^*/q_2^*].$$

Furthermore, if there exists an equilibrium with exit then $p_2 = z, q_1 = q_1^*, q_2 = q_2^*, n_E = [D(p_m) - n_S^* q_1^*]/q_m$.

Proof. We know that if exit occurs in equilibrium then $p_1 = p_m$. Consider the following minimization problem:

$$\underset{q_1, q_2 \geq 0}{Minimize} \ (1/\delta q_2)[C(q_1, 0) + \delta C(q_2, q_1) - p_m q_1]$$

It can be checked that there is a unique interior solution, say, (q_1^*, q_2^*). Let z be the value of the minimization problem. Then, one can easily check that:

$$\begin{aligned} &p_m q_1 + z\delta q_2 - C(q_1, 0) - \delta C(q_2, q_1) \leq 0 \ \ for\ all \ \ (q_1, q_2) \\ &p_m q_1^* + z\delta q_2^* - C(q_1^*, 0) - \delta C(q_2^*, q_1^*) = 0. \end{aligned}$$

Thus, the maximum profit earned by S-type firms is exactly zero if $p_1 = p_m$ and $p_2 = z$. So, in equilibrium with exit, $p_2 = z$ and each firm produces (q_1^*, q_2^*). Let $n_S^* = D(z)/q_2^*$. If there is an equilibrium with exit, then $n_S^* q_1^* < D(p_m)$ and $n_S^* q_2^* = D(z)$, so that

$$D(p_m)/D(z) > q_1^*/q_2^*. \tag{9}$$

Thus, (9) is a necessary condition for an equilibrium with exit. Now, suppose (9) holds. Let $n_E^* = [D(p_m) - n_S^* q_1^*]/q_m > 0$. It is easy to check that $(p_1 = p_m, p_2 = z, n_S = n_S^*, n_E = n_E^* > 0, q_1 = q_1^*, q_2 = q_2^*, q_E = q_m)$ is an equilibrium. //

Consider any cost function C and the minimization problem indicated in Proposition 7. By definition of the minimum, it must be true that at prices $p_1 = p_m$ and $p_2 = z$, the firm can earn at most zero profit by producing in both periods. Obviously, $z < p_m$. Furthermore , check that the solution (q_1^*, q_2^*) to this minimization problem is also a solution to the profit maximization problem of a staying firm facing prices $(p_1 = p_m, p_2 = z)$. The numbers z, q_1^*, and q_2^* depend only on the cost function and have nothing to do with market demand. The proposition indicates that if $q_1^* \geq q_2^*$, there does not exist any downward sloping market demand function for which exit occurs in equilibrium. On the other hand, if $q_1^* < q_2^*$, exit occurs in equilibrium for any demand function D which satisfies

$$\frac{D(p_m)}{D(z)} > \frac{q_1^*}{q_2^*}.$$

This is a restriction on the behaviour of the demand function at only two specific prices. Thus, for such cost functions the class of demand functions for which exit occurs in equilibrium is "large." The lower q_1^* is relative to q_2^*, the larger the class of demand functions for which exit occurs.

V. Examples and Implications

Earlier we found two types of equilibria under convex costs: with and without exit. Under what cost and demand parameters will an equilibrium with exit arise? Example 2 helps develop some intuition for what may happen. In it, if the demand function is somewhat inelastic, then after the active firms reduce their costs in the first period by learning, their potential second-period output is so great that the market is saturated and some of them must exit.

Example 2: Industry Dynamics Under Different Demand Parameters

$D(p) = 20 - bp$
$\delta = 0.9.$
$C(q, x) = q^2(1 + e^{-x}) + 10$

Table 1 shows the equilibrium in Example 2 for two different values of the demand parameter, b.

If $b = 1.3$, demand is weaker, and more elastic for prices with positive demand. In this case, there is no exit. All firms behave identically, producing higher output in the second period than in the first because costs fall enough with learning. Prices fall for the same reason. Overall profits are zero, but they are negative in the first period and positive in the second. The losses in the first period can be seen as the cost of learning, and the profits in the second period are quasi-rents on the acquired learning. Even though second-period profits are positive, no entry occurs, because an entrant would face higher costs, having never learned how to produce cheaply.

If $b = 1$, demand is stronger, and less elastic for prices below $20/1.3$. In this case, there is exit. The qualitative features of the staying firms are the same as when $b = 1.3$: output rises, prices fall, and profits go from negative to positive over time. When $b = 1$, however, there are also exiting firms in the market. These firms operate only in the first period, during which they have zero profits, instead of the negative profits of the staying firms. Their higher profits arise because their outputs are smaller, but that means they acquire less learning than the staying firms, and cannot compete profitably in the second period. "Shakeout" has occurred.

TABLE 1:
THE EQUILIBRIUM IN EXAMPLE 2

Demand parameter b:		1 (strong, inelastic)	1.3 (weak, elastic)
Type of equilibrium		*Exit*	*No Exit*
Prices	(p_1, p_2)	8.94. 6.63	8.76, 6.78
Industry output	(Q_1, Q_2)	(11.05, 13.38)	(8.60, 11.20)
Number of staying firms	(n_S)	4.40	3.60
Number of exiting firms	(n_E)	0.18	0
Staying-firm outputs	(q_1, q_2)	(2.42, 3.04)	(2.39, 3.11)
Exiting-firm outputs	$(q_E, 0)$	(2.24, 0)	—
Staying-firm profits	(π_1, π_2, π_S)	(-0.068, 0.076, 0)	(-0.478, 0.531, 0)
Exiting-firm profits	(π_E)	0	—

Learning and Concentration

Does learning-by-doing increase concentration? In an imperfectly competitive context, and without any learning in fixed costs, Dasgupta & Stiglitz (1988, p. 247) say that, "... firm-specific learning encourages the growth of industrial concentration... Strong learning possibilities, coupled with vigorous competition among rivals, ensures that history matters..." Yet Lieberman (1982, p. 886) could find no systematic relation between learning by doing and industry concentration.

We have already found that history matters, even without the Dasgupta-Stiglitz assumption of initial asymmetries, but Example 3 will show that the possibility of learning by doing can either increase or reduce concentration, depending on the particular industry. Empirical predictions must take into account the type of learning, not just its presence.

Example 3: Industry Concentration

$D(p) = 1/p$
$\delta = 0.9.$
$C(q, x) = q^2(x+1)^{-\kappa\lambda_{variable}} + (x+1)^{-\kappa\lambda_{fixed}},$

where κ represents the speed of learning, for $0 < \kappa < 1$, and $\lambda_{variable}$ and λ_{fixed} each take the value 0 or 1 to represent whether learning occurs in variable costs or fixed costs.

Let us denote as case (a) the case of learning in variable costs alone, where $\lambda_{variable} = 1$ and $\lambda_{fixed} = 0$. Let us denote as case (b) the case of learning in fixed costs alone, where $\lambda_{variable} = 0$ and $\lambda_{fixed} = 1$.

In both cases (a) and (b), if $\kappa = 0$ there is no learning. The cost function reduces to $C(q, x) = q^2+1$ and the equilibrium number of firms is $n = 0.5$. As κ increases, learning-by-doing becomes stronger. The difference between the two cost functions is that in (a) learning affects only marginal cost, whereas in (b) it only affects the fixed cost.

Although $n = 0.5$ when $\kappa = 0$, when $\kappa = 0.5$, the number of firms is 0.469 in case (a) and 0.553 in case (b). Since firms behave identically in this equilibrium, if learning influences mainly the marginal cost, it results in fewer and bigger firms, but if it reduces mainly the fixed cost it results in more and smaller firms. These are the results one would expect from basic price theory.

Further, as the speed of learning increases, industry concentration increases in case (a) and decreases in case (b). Finally, in both cases, consumers are enjoying lower prices as the speed of learning increases. The number of firms is greater in an industry with learning on fixed cost alone, however, than in an industry with no learning possibilities. Given that $p_2 < p_m$, and that p_2 equals marginal cost (the same for all experience levels), we have that $q_2 < q_m$. Hence, because no firm exits, the number of firms in the industry is $n = D(p_2)/q_2 > D(p_m)/q_m$, which is the number of firms in an industry with no learning possibilities.

Antitrust authorities may learn an additional lesson from this model. Consider the following scenario, which is possible for a wide range of parameters. In the first period, big firms and small firms operate and charge high prices. In the second period, the big firms all reduce their prices, the small firms go out of business, unable to compete, and the big firms start earning strictly positive profits. An antitrust authority might look at this and infer predatory pricing. That is wrong; the big firms earn zero profits viewed ex ante, and the price drop is not strategic, but a consequence of falling costs, and the exit of the small firms is socially optimal.

VI. Concluding Remarks

When so much of the teaching that microeconomic theorists do involves perfectly competitive partial equilibrium, it is curious that so much of our research has focussed on imperfect competition. Perfectly competitive partial equilibrium is by no means a closed subject, and there is more to be learned even about the models we teach our beginning students and use in everyday analysis. In particular, we still need a theory of endogenous market structure. Why do firms in an industry behave differently at different points in its history, and why at any one time is heterogenous behavior observed?

One line of research, exemplified by Hopenhayn (1992, 1993), looks at the evolution of an industry in which firms encounter heterogeneous productivity shocks. Such shocks can explain why industries evolve over time, and why so much heterogeneity is observed even when firms are price takers and entry is free. We have come to the same general result that industries evolve and that firms behave heterogeneously, but for different reasons: in a fully deterministic setting, but one with learning.

Our central purpose has been to show that learning and perfect competition are compatible, and that learning has curious implications for the evolution of a competitive industry. We have shown that in the presence of convex learning, firms must enter at the beginning of an industry or never, and that the number of firms may decline predictably over time. Firms may behave differently even though they all begin with the same production opportunities. Some firms may enter at a small scale knowing full well that they will be forced to exit later; and these firms, in fact, will initially be the most profitable. Whether the equilibrium contains such firms or not, it will contain other firms which make losses in the first period and profits in the second. Viewing the situation from the second period, it may appear oligopolistic, because these firms will then be earning positive profits, yet no entry will occur. Viewed from the start of the industry, however, these firms are merely reaping the returns to their early investments in learning, investments which potential entrants have not made.

This model has been quite general in some ways, but it is limited in others, and opportunities abound for extending the model. The main limitation of this model has been its restriction to two periods. By this simplification, we have been able to employ general cost and demand functions. Allowing such general functions is important in this context, because industry evolution can be different depending on the curvature of these functions. To specify linear demand, marginal costs, and learning would be to run the danger of missing important phenomena, something we conjecture is not true of limiting the model to two periods. The other limitation of the model is the assumption of convex costs used for the later propositions; in particular, the assumption that diminishing returns in learning and static production are greater than the effect of learning on marginal cost. This is certainly a reasonable case to consider, but it is not the only case. Convexity was not needed, however, to prove the existence, uniqueness, and optimality of equilibrium prices. Moreover, it is remarkable that the industry dynamics of entry and output are so rich even under convex costs. If the cost functions were less constrained, we would of course expect even more surprising results to be possible.

We have also shown that the competitive equilibrium is socially optimal. Learning does not necessarily destroy this conclusion of basic price theory. Even if the equilibrium involves some firms exiting early and not making use of the learning they acquired in the first period, this is socially optimal. This contrasts sharply with learning models which assume that marginal cost is constant in current output, because in those models the social planner would specify that the industry be a monopoly. Here, using standard U-shaped cost curves, monopoly is not optimal and no intervention is needed.

Appendix: Proofs (The full proofs are available in the technical version of this paper, Petrakis, Rasmusen & Roy (1994), from Erasmuse@indiana.edu, or from the World Wide Web site,
http://www.indiana.edu/~busecon/lrning.prf.)

Proof Outline for Propositions 2 and 3. Consider the social planner's problem (SPP) defined in the main text. The problem can be decomposed into two stages:

(i) For any vector of total output to be produced by different S,E and L type firms, the social planner decides on the minimum total cost of producing this vector by choosing the measure of active firms and their output.

(ii) The social surplus from any total output vector can be written as the area under the inverse demand curve and the social cost corresponding to that output, where the social cost function is defined in stage (i).

One can use a result from Aumann and Perles (1965) to show existence and characterize the social cost minimization problem in stage (i). The minimand in this problem is not necessarily convex (unless we assume [A7]) and there need not be a unique solution. Using the Lyapunov-Richter theorem, however, one can convexify the social cost possibility set generated by using a continuum of firms even though the individual firm's cost function is not necessarily convex. The social cost function (the value of the minimization problem) is therefore convex and differentiable. This makes the problem in stage (ii) a strictly concave maximization problem with a differentiable maximand.

Using a set of arguments based on the fact that $P(Q) \to 0$ as $Q \to +\infty$ and that the social marginal cost of output is bounded above zero, we can show that there exists a solution to the problem in stage (ii). As the maximand is strictly concave, the solution is unique (in terms of total output produced by different types of firms). The way the production of this output vector is organized depends on the cost minimization problem of stage (i). The inverse demand function generates a price in each period such that demand is equal to total output. The first order conditions for the social planner's maximization problem show that the price in each period is equal to the social marginal cost of production if a positive quantity is produced and the price is no greater than social marginal cost otherwise. The social marginal cost (for each of the types E,S and L) is the Lagrangean multiplier for the appropriate social cost minimization problem in stage (i). One can show that in any solution to the social cost minimization problem, each firm produces output that maximizes its profit if the Lagrangean multipliers are interpreted as prices. Furthermore, such profit is zero if a positive quantity is produced and never exceeds zero. One can then establish that every solution to the SPP is sustainable as a competitive equilibrium. Also, the way the total output vector is produced in equilibrium can be shown to minimize social cost. Using the concavity of the social surplus in problem (ii) and the first order conditions of profit maximization, one can directly check that the competitive allocation indeed satisfies all the conditions of social optimality. Hence, a production plan is socially optimal if and only if it is sustainable as a competitive equilibrium. As there exists a solution to the SPP, there exists a competitive equilibrium. Furthermore, since the solution to the SPP is unique in total output produced, the competitive equilibrium is unique in prices.

If, in addition, we assume (A7), the social cost minimization problem in stage (i) becomes a convex problem, so it has a unique solution in the measure of active firms of different types and their output. So the competitive equilibrium allocation is unique in output and measure of active firms under (A7), which is what Proposition 3 says.

Proof of Proposition 4. The first part of (a) and (b) follow immediately from strict concavity of the profit function for each type of firm. (Note that since the total amount (Q_1, Q_2) produced by all S-type firms is always strictly positive, $(q_1^*, q_2^*) >> 0$.) The second part of (b) results from Proposition 1, because the negative first-period profits of the staying firms result from their high production for the sake of learning. It remains to show that $n_L = 0$ for part (c).

Suppose that $n_L > 0$. Then from Proposition 1, $n_E = 0$ and $p_2 = p_m$. Under (A7), there exists a unique q_m which minimizes $[C(q, 0)/q]$ over $q \geq 0$. So, $q_L(i) = q_m$ and

$$p_2 = p_m = C(q_m, 0)/q_m = C_q(q_m, 0). \tag{10}$$

Furthermore,

$$D(p_2) = D(p_m) = n_S q_2^* + n_L q_m > n_S q_2^*. \tag{11}$$

From first order condition of profit maximization for firms which produce in both periods we have that $C_q(q_2^*, q_1^*) = p_2 = p_m$ and, therefore (using (A7), (10) and $q_1^* > 0$)

$$q_2^* \geq q_m. \tag{12}$$

Next we claim that the following inequality is true:

$$C_w(q_2^*, q_1^*)q_1^* + C_q(q_2^*, q_1^*)q_2^* - C(q_2^*, q_1^*) \geq 0. \tag{13}$$

By convexity of C on $\mathbf{R}_+^2$,

$$C(q_m, 0) - C(q_2^*, q_1^*) \geq C_q(q_2^*, q_1^*)(q_m - q_2^*) + C_w(q_2^*, q_1^*)(0 - q_1^*)$$

which implies that $C_w(q_2^*, q_1^*)q_1^* + C_q(q_2^*, q_1^*)q_2^* - C(q_2^*, q_1^*) \geq C_q(q_2^*, q_1^*)q_m - C(q_m, 0) = p_2 q_m - C(q_m, 0) = [p_2 - (C(q_m, 0)/q_m)] = 0$ (using (10)).

From the first order conditions of profit maximization for S-type firms and the fact that in equilibrium, the discounted sum of profits is zero, we have:

$$C_q(q_1^*, 0)q_1^* + \delta C_w(q_2^*, q_1^*)q_1^* + \delta C_q(q_2^*, q_1^*)q_2^* - C(q_1^*, 0) - \delta C(q_2^*, q_1^*) = 0.$$

Using (13) in the above equation we have:

$$C_q(q_1^*, 0)q_1^* - C(q_1^*, 0) \leq 0$$

which implies that

$$q_1^* \leq q_m \tag{14}$$

so that, from (12) , we have $q_1^* \leq q_2^*$. Thus,

$$n_S q_1^* \leq n_S q_2^*. \tag{15}$$

From (11) and (15) we have

$$D(p_2) > n_S q_2^* \geq n_S q_1^* = D(p_1),$$

and so, $p_1 > p_2 = p_m$, which violates Proposition 1 of the definition of equilibrium. //

Proof of Proposition 5. We will first consider the case where $\theta(q_m) > \theta(0)$ and show that Proposition 5 holds.

Suppose not. Then there exists sequence $\{\delta_t\} \to 0$ such that for all t, if the discount factor $\delta = \delta_t$, then no exit occurs in equilibrium. Let $(n_t, p_{1t}, p_{2t}, q_{1t}, q_{2t})$ be the equilibrium (with no exit) corresponding to each δ_t. Now, the sequences $\{p_{it}\}, \{q_{it}\}, i = 1, 2$ are all bounded sequences (the prices lie in $[0, p_m]$ and the quantities in $[0, K]$). There exists a subsequence $\{t'\}$ of $\{t\}$ such that the sequences of prices and quantities described above, converge to (say) $(p_i*, q_i*), i = 1, 2$. From first order and zero profit conditions, we have that

$$p_{1t'} = C_q(q_{1t'}, 0) + \delta_t C_x(q_{2t'}, q_{1t'}).$$

$$p_{2t'} = C_q(q_{2t'}, q_{1t'}).$$

$$[p_{1t'}q_{1t'} - C(q_{1t'}, 0)] + \delta_t[p_{2t'}q_{2t'} - C(q_{2t'}, q_{1t'})] = 0.$$

Taking limits as $t \to \infty$ yields

$$p_1^* = C_q(q_1^*, 0), \tag{16}$$

$$p_2^* = C_q(q_2^*, q_1^*), \tag{17}$$

$$p_1^* q_1^* - C(q_1^*, 0) = 0. \tag{18}$$

From (16) and (18) we have that

$$p_1^* = p_m, q_1^* = q_m. \tag{19}$$

By the definition of equilibrium, it must be true that firms earn non-negative profit in period 2 so that for all $t', [p_{2t'}q_{2t'} - C(q_{2t'}, q_{1t'})] \geq 0$. Taking limits, we have that

$$p_2^* q_2^* - C(q_2^*, q_1^*) \geq 0. \tag{20}$$

Combining (17) and (20) , we can see that

$$p_2^* \geq p_m(q_1^*), q_2^* \geq q_m(q_1^*). \tag{21}$$

Since $D(p_{1t'})/q_{1t'} = D(p_{2t'})/q_{2t'}$, we have after taking the limit as $t' \to \infty$

$$D(p_1^*)/q_1^* = D(p_2^*)/q_2^*. \tag{22}$$

From (19) ,

$$\frac{q_1^*}{D(p_1^*)} = \frac{q_m}{D(p_m)} = \theta(0) \tag{23}$$

From (21)

$$\frac{q_2^*}{D(p_2^*)} \geq \frac{q_m(q_1^*)}{D(p_m(q_1^*))} = \theta(q_1^*) \tag{24}$$

But $\theta(0) < \theta(q_1^*)$, as $q_1^* = q_m$. Thus, (23) and (24) contradict (22).

Let us now turn to the case where $\theta(q_m) < \theta(0)$. Suppose Proposition 5 is false in this case. Then there exists a sequence $\{\delta_i\} \to 0$ such that exit occurs in equilibrium for all i. Let $(p_{1i}, p_{2i}, q_{1i}, q_{2i}, n_i)$ be the associated equilibrium prices, outputs and numbers of staying firms. Then $p_{1i} = p_m$. Note that $\{(p_{1i}, p_{2i}, q_{1i}, q_{2i})\}$ is a bounded sequence, converging to, say $\{(p_1, p_2, q_1, q_2)\}$. Abusing notation, let this be the convergent subsequence itself. Observe that

$$p_{1i} = p_m = C_q(q_{1i}, 0) + \delta_i C_x(q_{2i}, q_{1i}).$$

Since $C_x(q_{2i}, q_{1i})$ stays bounded as $i \to \infty$, we have $p_2 = p_m(q_1) = p_m(q_m)$. (Note that $p_m(x)$ is continuous in x.) Observe that $p_{2i} = C_q(q_{2i}, q_{1i})$ and so, taking the limit, we have $p_2 = C_q(q_2, q_1) = C_q(q_2, q_m)$. Since $p_2 = p_m(q_m)$, we have $q_2 = q_m(q_m)$. Lastly, note that for each i,

$$\frac{D(p_{1i})}{q_{1i}} \geq n_i = \frac{D(p_{2i})}{q_{2i}},$$

so that taking the limit we have

$$\frac{D(p_m)}{q_m} \geq \frac{D(p_m(q_m))}{q_m(q_m)},$$

which is to say, $\theta(0) \leq \theta(q_m)$, a contradiction. //

References

Arrow, Kenneth. "The Economic Implications of Learning by Doing." *Review of Economic Studies.* 29 (June 1962): 152-173.

R.J. Aumann and M. Perles. "A Variational Problem Arising in Economics." *Journal of Mathematical Analysis and Applications.* 11 (1965): 488 - 503.

Bhattacharya, G. "Learning and the Behavior of Potential Entrants." *Rand Journal of Economics.* 15 (Summer 1984): 281-289.

Boldrin, M. and Scheinkman, J.A. "Learning by Doing, International Trade and Growth: A Note" in *The Economy as a Complex Evolving System*, edited by P. Anderson and K.J. Arrow, Santa Fe Institute Studies in the Sciences of Complexity, Addison Wesley, 1988.

Clarida, R.H. "Entry, Dumping and Shakeout." *American Economic Review.* 83 (March 1993): 180-202.

Cabral, L. & M. Riordan. (1991), "Learning to Compete and Vice-Versa," Working paper number 167, University of Lisbon.

Clarke, F., M. Darrough and J. Heineke. "Optimal Pricing Policy in the Presence of Experience Effects." *Journal of Business.* 55: (October 1982): 517-530.

Dasgupta P. and J. Stiglitz. "Learning-by-Doing, Market Structure and Industrial and Trade Policies." *Oxford Economic Papers.* 40 (June 1988): 246-268.

Davis, Steven and John Haltiwanger. "Gross Job Creation, Gross Job Destruction, and Employment Reallocation." *Quarterly Journal of Economics* 107 (1992): 819-863.

Dunne, Timothy, Mark Roberts, and Larry Samuelson. "The Growth and Failure of U.S. Manufacturing Plants." *Quarterly Journal of Economics* xxx (1989): 671-698.

Fudenberg, D. and J. Tirole. "Learning-by-Doing and Market Performance." *Bell Journal of Economics.* 14 (Autumn 1983): 522-530.

Ghemawat, P. and Spence, M. "Learning Curve Spillovers and Market Performance." *Quarterly Journal of Economics* 100 (1985 Supp.): 839-52.

Gort, Michael and Steven Klepper. "Time Paths in the Diffusion of Product Innovations." *Economic Journal* 92 (September 1982): 630-653.

Hopenhayn, Hugo. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica.* 60 (September 1992a): 1127-1150.

Hopenhayn, Hugo. "Exit, Selection, and the Value of Firms." *Journal of Economic Dynamics and Control.* 16 (1992b): 621-653.

Hopenhayn, Hugo. "Entry, Exit, and Firm Dynamics in the Long Run Equilibrium." Stanford University, Graduate School of Business Research Paper 1180, December 1991.

Hopenhayn, Hugo. "The Shakeout." Working paper, Economics Working Paper no. 33, University of Pompeu-Fabra, Barcelona, March 8, 1993.

Jovanovic, Boyan. "Selection and the Evolution of Industry." *Econometrica.* 50 (1982): 649-670

Jovanovic, Boyan and Lach, S. "Entry, Exit and Diffusion with Learning by Doing." *American Economic Review.* 79 (September 1989): 690-699.

Jovanovic, Boyan and Glenn MacDonald. "Competitive Diffusion." *Journal of Political Economy.* 102 (1994): 24-52.

Jovanovic, Boyan and Glenn MacDonald. "The Life Cycle of a Competitive Industry." *Journal of Political Economy.* 102 (1994): 322-347.

Lippman, S. & R. Rumelt. "Uncertain Imitability: An Analysis of Interfirm Differences in Efficiency Under Competition." *Bell Journal of Economics.* Autumn 1982, 13: 418-438.

Lucas, Robert. "On the Mechanics of Economic Development." *J. Monetary Econ.* 22 (July 1988): 3-42.

Majd, S. and Pindyck, R.S. "The Learning Curve and Optimal Production under Uncertainty." *Rand Journal of Economics.* 20 (Autumn 1989): 331-343.

Mookherjee, D. and D. Ray "Collusive Market Structure under Learning-by-Doing and Increasing Returns." *Review of Economic Studies.* 58 (October 1991): 993-1009.

Mookherjee, D. and D. Ray (1992) "Learning-By-Doing and Industrial Stucture: An Overview." in *Theoretical Issues in Development Economics*, ed. by B. Dutta, S. G., D. Mookherjee & D. Ray, New Delhi: Oxford University Press .

Panzar, J. C. (1989) "Technological Determinants of Firm and Industry Structure" in *Handbook of Industrial Organization, Volume I*, edited by R. Schmalensee and R.D. Willig, Elseviers Science Publishers B.V.

Petrakis, Emmanuel, Eric Rasmusen, and Santanu Roy (1994) "The Learning Curve in a Competitive Industry: Technical Version" Indiana University Working Paper in Economics no.94-004, May 1994.

Romer, Paul. "Increasing Returns and Long-Run Growth." *Journal of Political Economy* 94 (October 1986): 1002-37.

Schumpeter, Joseph (1950) *Capitalism, Socialism, and Democracy*, 3rd edition, New York: Harper.

Smiley, R.H., and S.A Ravid. "The Importance of Being First: Learning Price and Strategy." *The Quarterly Journal of Economics.* 98 (May 1983): 353-362.

Spence M. "The Learning Curve and Competition." *Bell Journal of Economics.* 12 (Spring 1981): 49-70.

Spence M. (1986) "Cost Reduction, Competition, and Industry Performance." in Joseph Stiglitz and G. Frank Mathewson, *New Developments in the Analysis of Market Structure*, Cambridge, Mass: The MIT Press, pp. 475-515.

Stiglitz, J. "Price Rigidities and Market Structure." *American Economic Review.* 74 (May 1984): 350-355.

Stokey, N. (1986) "The Dynamics of Industry-Wide Learning," in W.P. Heller, R. M. Starr and D.A Starrett, Eds,*Equilibrium Analysis: Essays in Honour of Kenneth J. Arrow*, Vol. II, Cambridge: Cambridge University Press.

Stokey, N. (1988) "Learning by Doing and the Introduction of New Goods." *Journal of Political Economy.* 96 (August 1988): 701-718.

Tirole, J. (1988) *The Theory of Industrial Organization*, Cambridge, Mass: The MIT Press.