Biased Experts in a Sender-Receiver Model

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Abstract

If someone biased is using an intermediary to collect and convey information, he does best by not being able to secretly convey his bias to the intermediary. He may do just as badly if there is a chance he might be caught imposing the bias, and he does best if he is caught with probability one.

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1. Introduction

In a sender-receiver game, the sender knows some item of information, a number x, and sends a message, possibly false, about it to the receiver, who takes an action. The sender would like the receiver to believe that the value of the information is $x + k$ and the receiver knows that the sender has that motivation. The receiver therefore does not trust the sender's message, and ignores it. This is worse for both receiver and sende than if the sender were forced to send only true messages.

One solution to the problem is for the pair of players to hire a third person, whom we might call an expert, to discover the intormation and send the message. (Equivalently, the sender could at some cost show the agent the information directly.) It is important that the expert be unbiased and truthful, however. If the sender chooses the expert and chooses an expert with bias or bribes a neutral expert, the receiver will ignore what the expert says , just as he ignored the sender himself. Then, the sender incurs the cost of the expert and the cost of the bribe uselessly. The sender would prefer not to be able to bribe an expert.

In this note, I explore a version of this game. The difference is that the sender may be able to secretly bribe the expert, but there is some probability that the receiver discovers whether the agent has been bribed or not.

More generally, this would apply whenever the sender tries to make his message more credible at some cost, so long as his method might be duplicitous.

2. The Model

The two players are the sender and the receiver. In addition, Nature and a hired expert make deterministic choices. The order of play is as follows, where x, b, m, k, and a are scalars.

(0) Nature chooses the value of x. The sender and receiver do not know $f(x)$, and place a diffuse prior with mean 0 on the distribution of x.

(1)The sender pays a bribe of b to the expert for him to bias his message upwards by amount k, or does not pay it.

(2) If the sender pays a bribe, the agent observes the bribery with probability θ. If the sender does not offer a bribe, the agent observes the "restraint" with probability γ (for discussion of this odd statement, see below) .

(3) If the sender does not bribe him, the expert sends the message $m = x$. If the sender does bribe him, he sends the message $m = x + u$.

(4) The receiver picks an action a after observing the message.

The sender's payoff is

$$\pi_p = -(x + k - a)^2 - I(b)b, \tag{1}$$

where $I(b)$ an indicator variable that equals 1 if a bribe is paid and 0 otherwise.

The receiver's payoff is

$$\pi_a = -(x - a)^2 \tag{2}$$

Discussion

The payoff functions have quadratic loss from the action not being at the player's ideal point, but quadratic loss is unimportant to the qualitative results.

We do not need to specify a payoff function for the expert, since he reacts very simply to the sender's action.

In this model the sender does not know the information before he decides whether to bribe the expert. If he does, we are in the world of Lanzi & Mathis (2004).

The assumption that "If the sender does not offer a bribe, the agent observes this with probability γ" allows for the possibility that the agent might or might not become sure that the sender did not pay a bribe. We might imagine that the agent posts a guard on the expert to see if a bribe is paid, but the guard falls asleep (and is honest enough to admit it) with probability $1 - \gamma$. If this scenario seems uninteresting, set $\gamma = 0$ for the rest of the paper.

I do not have eq. 3 well in mind. If the sender soetimes pays a bribe, if he can make t he message as big as he likes, he will make it bigger than x+k, knowing the receiver will scale it back. Ah– that's not hard to work in. It means that the agent WILL choose $a = x + k$ when the expert is bribed and the bribery is not detected. Or will it?

Equilibrium

In the table below the action is shown conditional on whether the receiver observes a bribe being paid, is sure no bribe was paid, or is uncertain. There are three kinds of equilibria. In a Restraint Equilibrium, the sender does not attempt to bribe the expert. In a Bribery Equilibrium he bribes him with probability one. In a Mixed Equilibrium, he bribes him with some probability between zero and one.

Equilibrium	a—bribe	a—restraint	a—nothing	Payoff
(1) Restraint	$m - k$	m	m	$-k^2$
(2)Bribery	0	x	0	$-k^2 - b$
(3) Mixed	$m - k$	x	$m - z$	$-(1-\theta)z^2 - \theta k^2 - b$

The mixed-strategy equilibrium payoff is between the other two. That makes sense.

What is interesting is that there can be multiple equilibria, maybe (?) Pareto ranked. Suppose receivers think the sender will bribe. Then he will– the Bribery equilibrium. Suppose receivers think the sender will maybe bribe. Then we can find ourselves in the mixed eq. This needs to be shown.

There usually exists a babbling equilibrium in sender-receiver games. Here there does not, because it is not the sender who sends the message directly, but the expert, who tells the truth even if he knows the receiver will ignore the message.

We will analyze each equilibrium in turn and see when multiple equilibria exist.

The Restraint Equilibrium

The sender's expected payoff in equilibrium is

$$\pi_p = -(x + k - x)^2 = -k^2 \tag{3}$$

If the sender deviated and paid the bribe his expected payoff would become

$$\pi_p = -(1-\theta)(x + k - (x + k))^2 - \theta(x + k - x)^2 - b = -\theta k^2 - b. \tag{4}$$

Deviation is not profitable if

$$-k^2 \geq -\theta k^2 - b. \tag{5}$$

i.e.

$$b \geq (1-\theta)k^2 \tag{6}$$

The Bribery Equilibrium

The sender's expected payoff in equilibrium is

$$\pi_p = -(x + u_1 - (x + k - k))^2 - b = -k^2 - b \tag{7}$$

If the sender deviated and did not pay the bribe his expected payoff would become

$$\pi_p = -(1-\gamma)(x + k - (x - k))^2 - \gamma(x + k - x)^2 = -4(1-\gamma)k^2 - \gamma k^2. \tag{8}$$

Deviation is not profitable if

$$-k^2 - b \geq -4(1-\gamma)k^2 - \gamma k^2, \tag{9}$$

i.e. if

$$[4 - 4\gamma) - 1 + \gamma]k^2 \geq b \tag{10}$$

i.e. if

$$b \leq 3(1-\gamma)k^2 \tag{11}$$

The Mixed Equilibrium

The sender pays the bribe with probability p. If the agent observes neither bribery nor restraint, he chooses action $m - z$.

The sender's expected payoff in equilibrium must equate his two pure-strategy payoffs, so

$$\begin{aligned} \pi_p(bribery) &= -(1-\theta)(x+k-(x+k-z))^2 - \theta(x+k-(x+k-k)^2 - b \\ &= \pi_p(restraint) \end{aligned} \qquad = -(1-\gamma)(x+k- \tag{12}$$

This means that

$$(1-\theta)z^2 + \theta k^2 - b = (1-\gamma)(k+z)^2 + \gamma k^2 \tag{13}$$

so

$$0 = (1-\gamma)k^2 - 2(1-\gamma)kz + (1-\gamma)z^2 - (1-\theta)z^2 + \gamma k^2 - \theta k^2 - b \tag{14}$$

By the quadratic formula,

$$z = \tag{15}$$

The receiver's utility from choosing $a = m - z$ must be maximal for the z we just found, which means the expected value of x must be $m - z$ when the receiver observes nothing.

$$E(x) = p(x+k) + (1-p)x = z. \tag{16}$$

Solving this yields

$$px + pk + x - px = z \tag{17}$$

so

$$pk = z + x \tag{18}$$

so so

$$p = \frac{z + x}{k} \tag{19}$$

It is interesting that p does not depend on θ or γ directly, only through z.

3. Extension: Making the Message Distortion Endogenous

Now let us think what happens when the message distortion is not fixed at k, but can take any value u. We will first assume that if the bribery is observed, so is u, and then ask what happens when u is never observed.

xxx

Equilibrium	u(bias)	a—bribe	a—restraint	a—nothing
(1) Restraint	u_1	$m - u_1$	m	m
(2)Bribery	—	0	x	0
(3) Mixed	u_3	$m - u_3$	x	m

The Restraint Equilibrium

The sender's expected payoff in equilibrium is

$$\pi_p = -(x + k - x)^2 = -k^2 \tag{20}$$

If the sender deviated and paid the bribe his expected payoff would become

$$\pi_p = -(1 - \theta)(x + k - (x + k))^2 - \theta(x + k - x)^2 - b = -\theta k^2 - b. \tag{21}$$

Deviation is not profitable if xxx

Out of equilibrium, what happens if the sender does pay a bribe? He will follow this deviation by choice of u_1 as a bias.

The Bribery Equilibrium

How much is the bias? If it were always k, then the sender would deviate to 2k. But it always has to be the same. Or does it? Does he have ot mix ovef the bias?

The receiver has to ignore the message in making his choice.

I'm very confused now.

The sender's expected payoff in equilibrium is

$$\pi_p = -(x + u_1 - (x + k - k))^2 - b = -k^2 - b \tag{22}$$

If the sender deviated and did not pay the bribe his expected payoff would become

$$\pi_p = -(1-\gamma)(x + k - (x - k))^2 - \gamma(x + k - x)^2 = -4(1-\gamma)k^2 - \gamma k^2. \tag{23}$$

Deviation is not profitable if xxx

The Mixed Equilibrium

The sender pays the bribe with probability p. If the agent observes neither, he chooses action $m - z$, where

The sender's expected payoff in equilibrium is

$$\begin{aligned} \pi_p(bribe) &= -(x + k - a)^2 - I(b)b \\ &= \pi_p(no\ bribe) \qquad = -(x + k - a)^2 - I(b)b \end{aligned} \tag{24}$$

If the sender deviated and paid the bribe his expected payoff would become

$$xs \tag{25}$$

$$sdf \tag{26}$$

$$sdf \tag{27}$$

$$sdf \tag{28}$$

$$sdf \tag{29}$$

9. Concluding Remarks

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