PERFECTLY CONTESTABLE MONOPOLY AND ADVERSE SELECTION

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Abstract

In a contestable market the possibility of "hit-and-run" entry prevents the price from rising above average cost. A contestable natural monopoly earns zero profits despite economies of scale. We show that informational imperfections can also result in a single firm serving the entire market with zero profits. This is possible even under constant returns to scale, and when barriers to exit preclude "hit-and-run" attacks and force potential entrants to consider the post-entry response of the incumbent firm. Furthermore, the equilibrium involves cross-subsidization, which is not possible in conventional contestable markets.

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April 20, 1993

1. Introduction

It has been widely accepted that the degree of market power possessed by firms is inversely related to the number of firms in the industry (Scherer (1980)). This relationship has been thrown into doubt by the concept of contestable markets (Baumol, Panzar, and Willig (1982)). In a perfectly contestable market, there is no relationship between the number of firms and the level of profits. Profits are always zero in equilibrium, even where cost functions lead the market to be served by a single firm. The reason is the existence of unobservable *potential* firms.

The theory of contestable markets was developed under the assumption of perfect information and has concentrated on the form of the production function of a multiproduct firm. We will use an example to show that informational imperfections can also lead to zero-profit monopoly equilibria. Furthermore, this can happen even though the technology exhibits constant returns to scale and exit barriers preclude "hit and run" attacks by potential entrants, forcing them, instead, to consider the incumbent's post-entry response. Our example is based on Spence's educational screening model (Spence (1973)). Workers have heterogeneous marginal products that cannot be observed before hiring takes place, and firms attempt to *screen* workers by conditioning each wage offer on some observable worker action called a *signal*. In our example, despite the possibility of entry, exactly one firm will engage in production, yet that firm will earn zero economic profits.

The organization of the paper is as follows: Section 2 presents Spence's model along with our assumptions about the entry and exit of firms from the market. Section 3 presents conditions under which all workers are employed by a single firm earning zero profits in equilibrium. These conditions are presented in three steps: a maximization problem that the equilibrium must satisfy, the equilibrium outcome, and the equilibrium strategies. Section 4 contrasts our model of firm competition with a game of frictionless entry and exit. Section 5 summarizes the results.

2. A Model of Educational Screening

Adverse selection models have been used to analyze such markets as business loans (Bester (1985)), insurance (Rothschild and Stiglitz (1976)), corporate bonds (Leland and Pyle (1977)), and used cars (Akerlof (1970)). Our example is based on Spence's seminal model (Spence (1973)) of education and the labor market.

An infinite number of identical, risk-neutral firms may freely enter as competitive buyers into the labor market. Firms try to screen workers by conditioning the wage on an observable activity of the worker called a *signal*, whose level is denoted by the real number y. A firm may tender one or more *offers*, each consisting of a wage-signal pair (y, p), where $y \geq 0$ and $p \geq 0$. Such an offer means the firm will pay a wage p to any worker who signals at the level y. The level of the signal has no effect on the worker's productivity.

There are two types of workers, who differ in their productivity and their costs of signalling. Proportion $1 - \pi$ of the workers have a "high" productivity of 2, while proportion π have a "low" productivity of 1. A worker's productivity cannot be observed before he is hired. In order to simplify the exposition, we will assume the preferences of the two types are represented by the following two utility functions:

$$\begin{aligned} U_L(y,p) &= log(p) - y \\ U_H(y,p) &= log(p) - y/2, \end{aligned}$$

The choice of these particular functional forms is unimportant. The important properties of these utility functions are: (1) they are increasing in p, (2) decreasing in y, (3) quasi-concave in both p and y, and the indifference curves of low ability workers are steeper than those of high ability workers. The latter means the marginal cost of signalling is higher for the low-ability workers than for high-ability workers.

Finally, we make two technical assumptions to deal with tie-breaking when workers are indifferent between offers. The first kind of tie-breaking arises when a worker faces two offers between which he is indifferent. In such a case, we assume that the worker chooses the offer that requires less signalling.[1] The second kind of tie-breaking arises when the most attractive offer to some group of workers is tendered by two different firms. We will assume that each of these two firms has a positive probability of attracting

[1] This assumption, which solves an open-set problem, would not be needed if there were a continuum of worker types. We will show later that it has no substantive importance.

a worker.[2]

3. The Order of Play

The way that the market is organized is very important in situations of asymmetric information. We will assume this labor market is a *screening market*: the firms (the uninformed players) move first and announce sets of offers, and the workers move last and choose among the available offers.[3] As a result, the workers are essentially passive players. The interesting game is played among the firms before the workers make their selections.

Firm i begins the game endowed with a finite set of *old offers*, denoted O_i. These old offers are givens, not moves of the game. They should be interpreted as offers that can persist in a steady-state equilibrium, given the

[2] This assumption rules out the following bizarre story. Suppose, contrary to the assumption above, that workers who are otherwise indifferent between firms follow a policy of going to the firm that offers the most contracts. Two firms each start by offering a pair of contracts, a profitable one that attracts high-ability workers and an unprofitable one that attracts low-ability workers. Neither firm drops the unprofitable contract, because then the high-ability workers would all depart for the other firm, which still offers two contracts. But if even a single worker remains, dropping the unprofitable contracts is a profitable deviation.

[3] In simpler adverse selection models (as opposed to screening) only a price is announced (see, e.g., Akerlof [1970]). A screening market is also different from a *signalling* market where the workers move first and choose signals, and the firms move second and tender offers after observing which signals were chosen. Stiglitz and Weiss (1989) discuss this crucial distinction.

rules for entry and exit. The game $W(O_1, O_2, \ldots)$ is then played as follows:[4]

(1) Each firm i may simultaneously tender a set of new offers, denoted N_i.

(2) Each firm i may simultaneously withdraw all or a subset W_i of O_i.

(3) Workers of each type simultaneously choose signal levels and employers.

(4) Wages are paid and profits are earned.[5]

(5) At every decision node in the game tree every player knows all previous moves made by all other players.

This specification is not the only way that offers and counteroffers could be made in a market. There are several distinct ways to specify the order of play. None of them can be called "correct," because each is appropriate to the institutional structure of a particular kind of market. The order of play used here implies that the market has the following three features:

(A) When a firm introduces new offers, it cannot then withdraw this offer before workers are hired; it cannot "back out" from its move.

(B) When a firm introduces new offers, its competitors have sufficient advance notice to withdraw some or all of their old offers before the workers make their choices.

[4]This process by which offers and counteroffers are made, follows the specification of Wilson (1980) as elaborated by Miyazaki (1977).

[5]The sets O_i, N_i and/or W_i may be empty, meaning that firm i does not tender any old offers, does not introduce any new offers, and/or does not withdraw any old offers.

(C) When a firm introduces new offers, its competitors do not have sufficient advance notice to introduce any new offers of their own.

Feature (A) is that legality, good industrial relations, or administrative inertia require firms to give workers the opportunity to accept new offers, rather than being able to withdraw them before the workers have time to react. Under frictionless exit, no such opportunity need be given. Feature (B) is that a firm cannot add new offers without its competitors becoming informed and being able to react before workers choose employers. Under frictionless exit, such advance warning is not given. Feature (C) is that considerations of technology or timing do not allow offers to be made instantly. This is the only one of the three features shared by our model and the frictionless-exit model.

Features (A) and (B) are not the only extra structure that can be added to a situation characterized by (C). Riley (1979) has proposed the "reactive game" in which (A) still holds, but (B) and (C) do not: firms cannot withdraw old offers, but they can make reactive new offers before workers choose. This kind of friction also adds enough structure to the game for a pure-strategy equilibrium to exist under weak conditions (Engers & Fernandez (1987)).

What is remarkable about friction in this model is that the result we will find, zero-profit monopoly, would not be possible if new entrants could then immediately exit at no cost. In the usual contestable monopoly market, by contrast, *frictionless* exit by potential entrants is needed to obtain the same result.

4. Equilibrium

By an "equilibrium outcome" we will mean a 4-tuple, $(y_L^*, p_L^*, y_H^*, p_H^*)$, for which there exists a sequence of old offers, $\{O_1^*, O_2^*, \ldots\}$, and a pure-strategy subgame-perfect equilibrium of the game $W(O_1^*, O_2^*, \ldots)$ in which in equilibrium, (1) $N_i = W_i = \emptyset, \forall i$, and (2) the Lows choose (y_L^*, p_L^*) and the Highs choose (y_H^*, p_H^*). The first condition simply means that no firm i wants to unilaterally add or subtract from its set of old offers, O_i^*. If (y_L^*, p_L^*) $= (y_H^*, p_H^*)$, the equilibrium is said to be "pooling"; otherwise, it is said to be "separating."

The equilibrium of our game is related to the solution of the following "Optimization Problem" (OP). The OP maximizes the welfare of the high-ability worker among all pairs of offers (not necessarily distinct) that are both incentive compatible and profitable. This optimal pair of offers constitute the offers chosen by the high and low ability workers in any sub-game perfect equilibrium of our game. The OP is

$$\underset{y_L, p_L, y_H, p_H}{Maximize} \quad U_H(y_H, p_H)$$

subject to:

$$
\begin{array}{lll}
(1) & \pi(1-p_L) + (1-\pi)(2-p_H) \geq 0 & \text{(Non-negative profits)} \\
(2) & U_L(y_L, p_L) \geq U_L(y_H, p_H) & \text{(Lows do not prefer the High offer)} \\
(3) & U_H(y_H, p_H) \geq U_H(y_L, p_L) & \text{(Highs do not prefer the Low offer)} \\
(4) & U_L(y_L, p_L) \geq U_L(0, 1) & \text{(Lows get their reservation wage)} \\
(5) & y_L \geq 0 & \text{(The Low signal is feasible).}
\end{array}
$$

For any particular value of π this optimization problem has a unique solution, but there are three qualitatively distinct solutions over three different ranges of π. These three solutions are given in Lemma 1.

LEMMA 1: *The optimal arguments $y_L^*, p_L^*, y_H^*, p_H^*$ for the Optimization Problem take the values shown in Table 1.*

TABLE 1:

SOLUTIONS TO THE OPTIMIZATION PROBLEM

Fraction of Lows	y_L^*	p_L^*	y_H^*	p_H^*
$\pi \leq 1/2$	0	$2-\pi$	0	$2-\pi$
$1/2 < \pi < 2/3$	0	$\frac{2-\pi}{2\pi}$	$log(\frac{\pi}{1-\pi})$	$\frac{2-\pi}{2(1-\pi)}$
$\pi \geq 2/3$	0	1	$log2$	2

Lemma 1 tells us how workers react to offers. Suppose that the workers have the two (not necessarily distinct) offers $(0, p_L^*)$ and (y_H^*, p_H^*) from which to choose. When $\pi \leq 1/2$, both types of workers choose the single offer $(0, p_L^*) = (y_H^*, p_H^*)$ and any firm that tenders this offer earns exactly zero profits. When $1/2 < \pi < 2/3$, the Lows choose the offer $(0, p_L^*)$ and the Highs choose the offer (y_H^*, p_H^*). Since in this case $1 < p_L^* < p_H^* < 2$, the Highs are paid less than their marginal product and the Lows are paid more than their marginal product; but the two differentials exactly offset each other, so any firm that tenders both offers exactly breaks even. When $\pi \geq 2/3$, the Lows and Highs again choose different offers, but now every worker is exactly paid his marginal product. Any firm that tenders either offer exactly breaks even.

It can now be shown why our assumption that indifferent workers choose the offer with the least signalling is non-substantive. Suppose we did not have it, and $\pi \geq 2/3$. The Lows would then be indifferent between the two offers $(0, p_L^*)$ and (y_H^*, p_H^*), from which we have assumed they all pick the offer with the lower signal, $(0, p_L^*)$. Suppose instead that some Lows choose (y_H^*, p_H^*). Then no firm will want to tender (y_H^*, p_H^*) because it is unprofitable if even one Low chooses it. But a firm would be willing to tender $(y_H^* + \epsilon, p_H^*)$ for small ϵ, because the Highs prefer it to $(0, p_L^*)$ but the Lows do not. The only problem is that some other firm could now tender the slightly more attractive offer of $(y_H^* + \epsilon/2, p_H^*)$. As a result, no equilibrium exists, in either pure or mixed strategies. But the problem is a modeling artifact. If the set of possible signal levels were discrete rather than continuous, so signals could only rise by increments of ϵ, the problem would disappear. Our tie-breaking assumption achieves the same result more simply than a model with a large number of discrete signal levels.

The OP problem is static; entry and exit play no role. We now come to the most important part of this paper: the demonstration that for a range of moderate parameters the equilibrium of our screening market has the characteristics of a contestable-market equilibrium. In what follows, an offer is "active" if some worker chooses it in equilibrium; otherwise it is "inactive." A firm is "active" if it tenders at least one active offer in equilibrium, and therefore hires at least one worker; otherwise, the firm is "inactive."

PROPOSITION 1. *Any equilibrium outcome solves the Optimization Problem, and all firms earn zero profits. In addition: (i) When* $\pi \geq 2/3$*, at least two firms are active; (ii) When* $1/2 < \pi < 2/3$*, exactly one firm is active; (iii) When* $\pi \leq 1/2$*, the number of active firms is indeterminate.*

PROOF: Suppose $(\tilde{p}_L, \tilde{y}_L, \tilde{p}_H, \tilde{y}_H)$ is a equilibrium outcome of the game. Then we claim this vector satisfies the five constraints of the OP. It satisfies constraint (5) trivially. Since the two offers $(\tilde{p}_L, \tilde{y}_L)$ and $(\tilde{p}_H, \tilde{y}_H)$ must be best choices of the Lows and Highs, constraints (2) and (3) must be satisfied as well. The profitability constraint (1) must be satisfied, for otherwise some firm could improve its profits by unilaterally withdrawing all its offers. Furthermore, (1) must be binding as well. Otherwise, the firms tendering offers would be making strictly positive profits overall, and some entrant could tender the two new offers $(\hat{y}_L, \hat{p}_L)$ and $(\hat{y}_2, \hat{p}_2)$ that attract both types of workers away from $(\tilde{p}_L, \tilde{y}_L)$ and $(\tilde{p}_H, \tilde{y}_H)$ and yet earn positive profits. Finally, suppose constraint (4) is violated. Then $0 = U_L(0,1) > U_L(\tilde{p}_L, \tilde{y}_L) = \tilde{u}$. Define $p' = e^{\tilde{u}/2}$. Since $p' < 1$, any firm which in equilibrium was tendering nothing, but which adds the offer $(0, p')$ will make positive profits, regardless of which offers are subsequently withdrawn.

Suppose $(\tilde{p}_L, \tilde{y}_L, \tilde{p}_H, \tilde{y}_H)$ is not equal to the OP solution $(y_L^*, p_L^*, y_H^*, p_H^*)$. Then there exists an ϵ such that $U_H(y_H^*, p_H^* - \epsilon) > U_H(\tilde{p}_H, \tilde{y}_H)$ and $U_H(y_H^*, p_H^* - \epsilon) > U_H(y_L^*, p_L^*)$. Suppose a firm adds the two offers (y_L^*, p_L^*) and $(y_H^*, p_H^* - \epsilon)$. Regardless of what old offers the other firms withdraw, this firm will attract only the Highs to $(y_H^*, p_H^* - \epsilon)$, which earns strictly positive profits. If the other firms all withdraw their old offers and the Lows choose (y_L^*, p_L^*), then our firm still earns positive profits. The contradiction shows $(\tilde{p}_L, \tilde{y}_L, \tilde{p}_H, \tilde{y}_H)$ must solve the OP, as claimed.

The proposition's claims about the number of firms remain to be proven. If $1/2 < \pi < 2/3$, then Table 1 tells us that a different offer is chosen by each type and the offer accepted by the Lows incurs losses for the offering firm (the wage of $\pi/(1-\pi)$ exceeds the marginal product of 1). A firm that tenders only (y_H^*, p_H^*) cannot be part of the equilibrium because then any firm offering (y_L^*, p_L^*) would not attract enough Highs to break even. Multiple firms offering both offers cannot be part of equilibrium because each firm would want to unilaterally drop (y_L^*, p_L^*) in the withdrawal stage. The only other possibility is for one firm to offer both offers, in which case no other firm is active.

If $\pi \geq 2/3$, Proposition 1 claims there cannot be just one active firm in equilibrium. If there were just one active firm tendering both separating offers, that firm would drop the High offer in the withdrawal stage, the Highs would accept the Low offer, and the firm would earn positive profits. Since there cannot be any incentive to make new offers or withdraw old offers in a equilibrium, there must be at least two active firms in equilibrium, with both firms offering the High contract and at least one offering the Low contract.

If $\pi \leq 1/2$, Proposition 1 claims that the number of active firms is indeterminate. Clearly there could be two or more active firms, each tendering the same pooling offer. Each would make zero profits, and none could benefit by adding new offers, because any offer that made profits by attracting away the Highs would make the old pooling offer unprofitable. The old pooling offer would be withdrawn, and the new offer would no longer be profitable. But there could also be a single active firm, tendering the pooling offer $(0, p_L^*)$. In this case, some other firm would have to tender two inactive offers: (0,1) and $(y_3^*, 2)$, where y_3^* is chosen so that the Highs are just indifferent between $(0, p_L^*)$ and $(y_3^*, 2)$. This would be an equilibrium because the active firm could not benefit by adding to or withdrawing from its offer: the alternative of (0,1) prevents it from profiting from the Lows by paying them less than $p = 1$ and the alternative of $(y_3^*, 2)$ prevents them from adding a more profitable pooling offer and withdrawing $(0, p_L^*)$. Thus, there can be either one or more firms active in equilibrium.
Q.E.D.

When $\pi > 2/3$, there are at least two firms active in equilibrium, an ordinary result. But whenever $\pi \leq 2/3$, there can be monopoly in equilibrium, even though the production function shows constant returns to scale. This range of π can be further divided into two smaller ranges with qualitatively different kinds of monopoly.

When $\pi \leq 1/2$, the situation is similar to a perfect-information model with constant returns to scale. The number of firms does not matter, but the possibility of entry does. Profits are zero whether one firm or many firms make the pooling offer. That is why we cannot determine the number of active firms for this parameter range.

When $1/2 < \pi < 2/3$, the situation might be termed a "natural monopoly," since the unique equilibrium outcome is for only a single firm to be active despite the absence of entry barriers of any kind. The firm earns zero profits, however, even without government regulation. The results, if not the assumptions, recall the paradigmatic contestable market: air service to a small town. Only one airline will provide the service because of technological economies of scale, but that airline cannot raise price above cost without provoking entry. In our model, a single firm hires all the workers, but the reason is not economies of scale. It is, rather, that by being the only active firm, the firm can internalize the benefits of cross-subsidization. At the same time, the firm cannot lower wages, or it will provoke entry.

Cross-subsidization, in fact, is where contestable monopoly arising from adverse selection differs most from contestable monopoly arising from scale economies. Baumol (1982) says that one of the three chief welfare characteristics of a contestable market is the absence of cross-subsidies: each product is sold at marginal cost.[6] Otherwise (in the markets he is considering), an outsider would enter and undercut the price of the product whose profits were cross-subsidizing the other product. If the incumbent then lowered his price on that product, the entrant would end up with no worse than zero profits.

In the screening equilibrium described above, the High workers subsidize the Low workers whenever $\pi < 2/3$, whether the market contains one firm or several. Should an entrant introduce an offer that would attract just the Highs, the incumbent would withdraw all active offers. Both High and Low workers would choose the entrant's offer, and the entrant's profits would be negative, not zero. The firm, because it is the only active firm, can internalize the benefits from cross-subsidization. Thus, the difference in cross-subsidization from scale-economies contestable monopoly is not just accidental, but is at the heart of adverse-selection contestable monopoly. The type of cross-subsidization in the screening model differs in the two parameter ranges. In the range $\pi \leq 1/2$, a single pooling offer is made. That offer is profitable when it is accepted by a High and unprofitable when accepted by a Low, but the firm does not know whether a particular transaction is profitable or not. An entrant might threaten to introduce a offer that would lure away the Highs, but the incumbent's optimal response would be to withdraw (y_H^*, p_H^*), which would result in the entrant ultimately hiring the Lows

[6]Baumol's other two welfare characteristics are efficient production and zero profits.

also, at a loss. In the range $1/2 < \pi < 2/3$, the contrast with the perfect-information market is even more striking. In that range, the single, active firm makes two offers, one of which attracts only Lows and is known to be unprofitable. The firm tenders that offer only to deter the Lows from accepting the profitable offer, which would be unprofitable if it were accepted by Lows as well as by Highs. Should an entrant enter with the offer (y_H^*, p_H^*), the incumbent's optimal response is to withdraw both of its old offers, which leaves the entrant hiring both Highs and Lows and earning negative profits.

Proposition 1 describes the offers that are active in any equilibrium, and thus characterizes the equilibrium outcome, but it does not prove that a pure-strategy sub-game perfect equilibrium exists not does it describe the equilibrium strategies. Given the nonexistence result of Rothschild & Stiglitz (1976) for insurance markets similar to this labor market, the existence of a pure-strategy equilibrium cannot be taken for granted. Proposition 2 in the Appendix provides the missing proof of the existence of a pure-strategy equilibrium for our game. One interesting aspect of the equilibrium strategies is that firms must tender inactive offers in order to prevent deviations from equilibrium.

Our model was designed to have the special feature of cross-subsidization in a monopolized separating equilibrium, but some of its other features can be found in simpler adverse selection models with monopoly pooling equilibria. An example is the following bid-ask spread model of the market for a security, which we will merely sketch out here, since the results parallel those described above.

Bagehot (1971) argued the bid-ask spread on a stock exchange exists to guarantee zero profits to the marketmaker, who trades with whoever appears at the market. Some of those who appear are informed traders, and the marketmaker always loses in trades with them. The rest of those who appear are uninformed traders, and the bid-ask spread allows the marketmaker to profit in those trades. The equilibrium is pooling because there is no signal, and the marketmaker must offer the same spread to both types. The uninformed effectively subsidize the informed, but if the marketmaker tries to charge too high a spread, he can be undercut by a competing marketmaker.

The reason this market would be monopolized is the marketmaker can make use of the volume of trades to learn the true value of the security. If, for example, he finds many more traders are buying than selling, he can conclude the uninformed traders are randomly distributed on each side, but the informed traders realize the price is too low. In response, he can raise the price. The marketmaker with the greatest volume of trade can amass more information in this way, set the price more accurately, and lower his bid-ask spread.[7]

This securities market, like our labor market over most of its parameter range, consists of one active firm earning zero profits and cross-subsidizing

[7] While we have not seen the conclusion that such a market is a natural monopoly published, it is unlikely the argument is new. We discuss it here to contrast pooling monopoly with the more complicated separating monopoly.

across its transactions. The difference is that the firm in the securities market does not know which transactions are profitable and which are unprofitable. There is no possibility of an entrant trying to skim off the trades with the uninformed traders, and hence the details of entry and exit are not so important to the model.

5. Concluding Remarks

The most important insight of the contestable-markets literature has been that when we observe one firm monopolizing a market we cannot immediately conclude there exists inefficiency or that the firm is earning positive economic profits. Instead, the conditions of production may be such that it is most efficient for one firm to serve the market, and other firms would enter if that one firm ever tried to raise price above average cost. The implication is that policymakers ought to check the conditions of production—is entry and exit costless, and are there economies of scale?

We have presented another reason why one firm might dominate a market without earning positive profits or restricting its output. In our example, it is not the conditions of production so much as the conditions of *distribution* that are important. The analyst need not be concerned with production economies of scale—we have assumed constant returns to scale— but he must worry about whether information problems make it important that only one firm operate.

Our example has two features which are very different from a standard contestable market. First, we assume a certain friction in the way the market operates: offers cannot be introduced and withdrawn instantly. This ensures that equilibrium exists in an adverse selection model like ours, but the idea of contestable markets, like that of perfect competition, has usually been associated with the absence of frictions. Second, cross-subsidization occurs in equilibrium in our model. The single firm offers two offers, one of which is profitable and the other, unprofitable. This cannot happen in a conventional contestable market; indeed, Baumol (1982) says a chief conclusion of the theory is that no cross-subsidization can occur in a perfectly contestable market. Although economists normally associate cross-subsidization with regulation, our model is one of *laissez faire* in which the subsidy is paid out of pure self-interest. Our model implies that if one of a monopoly firm's products is observed to be profitable, that does not mean the firm is making profits overall, for those profits may be balanced by losses on another of its products.

Appendix: Proofs of propositions

PROOF OF PROPOSITION 1: Because both utility functions are continuous, the constraint set is closed. We claim the constraint set is also bounded, which implies that an optimal solution $(y_L^*, p_L^*, y_H^*, p_H^*)$ exists. The incentive-compatibility constraints (2) and (3) of the OP imply $y_L \leq y_H$ and $p_L \leq p_H$, and constraints (1) and (4) imply $p_L \geq 1$ and $p_H \leq 2$. Finally, constraints (2) - (4) together imply $U_H(y_H, p_H) \geq U_H(0, 1)$, which in turn implies $y_H \leq log4$. We have shown $1 \leq p_L \leq p_H \leq 2$ and $0 \leq y_L \leq y_H \leq log4$, which proves that the constraint set is bounded, as we claimed. Note that these bounds imply all four parameters, not just y_L, take non-negative values.

Second, we claim that constraints (1), (2), and (5) are binding at any optimum. Suppose (y_L, p_L, y_H, p_H) satisfies the constraints of the OP, and $y_L > 0$. Direct calculation reveals that the vector $(0, p_L, y_H - y_L, p_H)$ also satisfies these constraints, but $U_H(y_H - y_L, p_H) > U_H(y_H, p_H)$. This shows that at any optimum, constraint (5) is binding. Next suppose $y_L = 0$, but $U_L(y_L, p_L) > U_L(y_H, p_H)$. It follows $y_H > log\frac{p_H}{p_L} = y_H'$. The vector (y_L, p_L, y_1', p_H) satisfies all the constraints of the OP, but $U_H(y_H', p_H) > U_H(y_H, p_H)$. This proves that at any optimum, constraint (2) is also binding. Finally suppose $y_L = 0$ and $U_L(y_L, p_L) = U_L(y_H, p_H)$, but $\pi(1 - p_L) + (1 - \pi)(2 - p_H) > 0$. Then $\beta = (2 - \pi)/(\pi p_L + (1 - \pi)p_H) > 1$. The vector $(y_L, \beta p_L, y_H, \beta p_H)$ satisfies all the constraints of the OP, but $U_H(y_H, \beta p_H) > U_H(y_H, p_H)$. This proves that at any optimum, constraints (1), (2), and (5) are binding.

It follows that $(y_L^*, p_L^*, y_H^*, p_H^*)$ is the solution of the modified optimization problem

$$\underset{y_L, p_L, y_H, p_H}{Maximize} \quad U_H(y_H, p_H)$$

subject to:

$$\begin{array}{ll}
(1') & \pi(1-p_L) + (1-\pi)(2-p_H) = 0 \\
(2') & U_L(y_L, p_L) = U_L(y_H, p_H) \\
(3') & U_H(y_H, p_H) \geq U_H(y_L, p_L) \\
(4') & U_L(y_L, p_L) \geq U_L(0, 1) \\
(5') & y_L = 0
\end{array}$$

Constraints $(1')$, $(2')$, and $(5')$ implicitly define y_H and p_H as functions of p_L. Specifically, $y_H(p_L) = log\frac{2-\pi-\pi p_L}{(1-\pi)p_L}$ and $p_H(p_L) = \frac{2-\pi-\pi p_L}{(1-\pi)p_L}$. Substitution of these expressions into constraint $(3')$ yields the inequality $p_L \leq 2-\pi$; and their substitution into constraint $(4')$ results in the inequality $p_L \geq 1$. Finally, $U_H(y_H(p_L), p_H(p_L)) = \frac{1}{2}log\frac{(2-\pi-\pi p_L)p_L}{1-\pi}$, which is an increasing function of the strictly concave function $V(p_L) = (2-\pi)p_L - \pi {p_L}^2$. It follows that any solution of the OP is of the form: $(0, p_L^*, y_H(p_L^*), p_H(p_L^*))$, where p_L^* is a solution to the one-variable constrained optimization problem

$$\underset{p_L \in [1, 2-\pi]}{Maximize} \quad (2-\pi)p_L - \pi {p_L}^2$$

It is simple to verify that this optimization problem has the following

solution: $p_L^* = 2 - \pi$ when $\pi \leq 1/2$; $p_L^* = \frac{2-\pi}{2\pi}$ when $1/2 < \pi < 2/3$; and $p_L^* = 1$ when $\pi \geq 2/3$. The conclusions of Lemma 1 follow directly.
Q.E.D.

PROPOSITION 2: *Let p_L^*, p_H^*, y_L^*, and y_H^* take the values in Table 1 and let $y_3^* = 2log(2/p_H^*) + y_H^*$. Suppose Firm 1 tenders the set of old offers O_1^* $= \{(0,1),(0,p_L^*),(y_H^*,p_H^*)\}$, Firm 2 tenders $O_2^* = \{(0,1),(y_3^*,2)\}$, and all remaining firms tender nothing. That is, $O_i^* = \emptyset$, for $i > 2$. The Wilson-Miyazaki game $W(O_1^*, O_2^*, \ldots)$ has a pure-strategy equilibrium in which no firm has an incentive to add new offers or withdraw old ones.*

Before proving Proposition 2, it may be useful to describe the equilibrium offers. When $\pi \leq 1/2$, Table 1 tells us that $(0, p_L^*) = (y_H^*, p_H^*)$, so the workers face three distinct offers— $(0,1)$, $(0,p_L^*)$, and $(y_3^*,2)$— from which both types of workers choose $(0,p_L^*)$. Only Firm 1 is active, and it makes zero profits overall: losses on the Lows are offset by profits on the Highs. Firm 2's offers are inactive, but they are important in constraining Firm 1's behavior. Figure 1 illustrates the offers. The indifference curve of the Highs labeled U_H passes through both $(0,p_L^*)$ and $(y_3^*,2)$, while the indifference curve of the Lows labeled U_L passes through the offer $(0,p_L^*)$ but lies above the offer $(y_3^*,2)$.

[SEE FIGURE 1]

When $1/2 < \pi < 2/3$, the workers face four distinct offers– $(0,1)$, $(0,p_L^*,)$, (y_H^*,p_H^*) and $(y_3^*,2)$– from which the Lows choose $(0,p_L^*)$ and the Highs choose

(y_H^*, p_H^*). Firm 1 is again the only active firm, and it hires all of the workers, both Highs and Lows. It makes losses on the Lows which are offset by the profits earned on the Highs. Figure 2 illustrates this. The indifference curve of the Highs labeled U_H passes through both (y_H^*, p_H^*) and $(y_3^*, 2)$. The indifference curve of the Lows labeled U_L passes through both $(0, p_L^*)$ and (y_H^*, p_H^*). The indifference curve of the Lows labeled U_L' passes through $(0, 1)$ and lies below $(y_3^*, 2)$.

[SEE FIGURE 2]

Finally, when $\pi > 2/3$, Table 1 tells us that $(0, 1) = (0, p_L^*)$ and $(y_H^*, p_H^*) = (y_3^*, 2)$. The workers face only two distinct offers— $(0, 1)$ and (y_H^*, p_H^*)— from which the Lows choose $(0, 1)$ and the Highs choose (y_H^*, p_H^*). Now both Firm 1 and Firm 2 are active, and they each hire Lows with one of their offers and Highs with the other, breaking even on each offer. Figure 3 illustrates this. The indifference curve of the Lows, labeled U_L, goes through both $(0, 1)$ and (y_H^*, p_H^*), while the indifference curve of the Highs, labeled U_H, goes through (y_H^*, p_H^*) and lies above the offer $(0, 2 - \pi)$ that is the pooling equilibrium offer chosen when $\pi \leq 1/2$.

[SEE FIGURE 3]

PROOF OF PROPOSITION 2: We need to show that when Firm 1 tenders O_1^* as an old offer, Firm 2 tenders O_2^* as an old offer, and the other firms tender no old offers, no firm can make positive profits by unilaterally deviating at any stage of the game— whether by tendering new offers and/or by withdrawing any of its old offers. Moreover, since we are interested in a perfect equilibrium, any deviant action is taken with the knowledge that the other firms will react in later stages of the game. Fortunately, we can sidestep the complex maze of possible deviations by analyzing the possible outcomes of any deviation.

Suppose that some firm makes positive profits by deviating at some stage of the game. In this case, at the end of the game the Low workers choose some offer $(\tilde{p}_L, \tilde{y}_L)$ and the Highs choose $(\tilde{p}_H, \tilde{y}_H)$, where possibly $(\tilde{p}_L, \tilde{y}_L) = (\tilde{p}_H, \tilde{y}_H)$. We will show the following: (i) $(\tilde{p}_L, \tilde{y}_H, \tilde{p}_H, \tilde{y}_H)$ satisfies the constraints of the OP and (ii) $U_H(\tilde{p}_H, \tilde{y}_H) < U_H(y_H^*, p_H^*)$, that is the deviation hurts the Highs. For this to happen, Firm 1 must withdraw the offer (y_H, p_H) and Firm 2 must withdraw the offer $(y_3, 2)$, since both offers generate identical levels of utility for the Highs. But we will then show: (iii) it is impossible to induce both firms to withdrawn both of these offers unilaterally.

(i) If constraint (1) of OP is violated, then total profits across all firms are negative and some firm would do better by withdrawing an active offer at stage 2. Constraints (2) and (3) are the self-selection constraints and are satisfied by the definition of $(\tilde{p}_L, \tilde{y}_L)$ and $(\tilde{p}_H, \tilde{y}_H)$. Constraint (5) is the feasibility constraint and, therefore, must be satisfied. Finally, constraint (4) must be satisfied if the offer $(0, 1)$ is still being tendered after stage 1.

But notice that after stage 1 at least one firm must still be tendering $(0, 1)$. Since $(0, 1)$ can never be unprofitable, neither of these two firms benefits from withdrawing this offer at stage 2.

(ii) It follows from Proposition 1 that $U_H(\tilde{p}_H, \tilde{y}_H) \leq U_H(y_H^*, p_H^*)$. Suppose $U_H(\tilde{p}_H, \tilde{y}_H) = U_H(y_H^*, p_H^*)$. Since the solution to the OP is unique, this implies $(\tilde{p}_L, \tilde{y}_H, \tilde{p}_H, \tilde{y}_H) = (y_L^*, p_L^*, y_H^*, p_H^*)$. It now follows from the proof of Proposition 1 that constraint (1) is binding— that is, total profits across firms equal zero. Since the deviating firm is making strictly positive profits by assumption, it follows some firm must be making strictly negative profits. But this in inconsistent with optimal behavior at stage 2, since every firm can guarantee itself zero profits by withdrawing all of its offers at that stage. It follows $U_H(\tilde{p}_H, \tilde{y}_H) < U_H(y_H^*, p_H^*)$.

(iii) Suppose no deviations occured at stage 1. There is no incentive for Firm 1 to unilaterally withdraw (y_H^*, p_H^*) at stage 2, since the Highs will just go to the offer $(y_3^*, 2)$ and the firm will be left hiring the Lows at a loss; and if Firm 1 withdraws both offers, then it makes no sales and therefore no profits. Likewise, there is no incentive for Firm 2 to unilaterally withdraw the inactive offer $(y_3^*, 2)$. So any profitable deviation from the equilibrium must begin at stage 1. Suppose the new offers were introduced by a firm other than Firm 1. Since we have deduced that the Highs must strictly prefer (y_H^*, p_H^*) to any of these new offers, Firm 1 is guaranteed to earn positive profits on (y_H^*, p_H^*) as long as it continues to tender $(0, p_L^*)$– in which case it is guaranteed to earn at least zero profits on both offers. So Firm 1 cannot be induced to withdraw (y_H^*, p_H^*). So the new offers must have been added at stage 1 by Firm 1. Furthermore, the Highs must consider any of these offers strictly inferior to $(y_3^*, 2)$.

We claim in this case, it is an equilibrium strategy for both firms to continue to tender their old offers, in which case Firm 1 earns zero profits, a contradiction. For if Firm 1 continues to tender (y_H^*, p_H^*), then Firm 2 cannot do better by withdrawing $(y_3^*, 2)$. And if Firm 2 continues to tender $(y_3^*, 2)$ then Firm 1 cannot do better by withdrawing (y_H^*, p_L^*) and (y_H^*, p_H^*). For suppose Firm 1 withdraws (y_H^*, p_H^*). It then must lose the Highs to Firm 2. As for the Lows, either it also loses the Lows to Firm 2; or it retains the Lows at (y_L^*, p_L^*), which results in a loss; or it hires the Lows at one of its new offers. But this new offer can attract the Lows away from Firm 2's offer of $(0, 1)$ only by paying the Lows more than 1, resulting in a loss. If Firm 1 continues to tender (y_H^*, p_H^*), then it gains nothing from withdrawing (y_L^*, p_L^*). For it can prevent the Lows from choosing (y_H^*, p_H^*)— which results in a loss— only by tendering a new offer that pays them more than p_L^*— which also results in a loss.

Q.E.D.

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APPENDIX: **Generalization**

(In this section we suggest several generalizations that will not be made in the submitted version of this paper, but which might be the basis for future work. We include them in the working-paper version in the hope of generating useful comment.)

The Spence screening model makes overly restrictive assumptions about the preferences of the sellers and the buyers. It is possible to greatly relax these assumptions and still be able to construct examples of equilibria in which there is a single firm earning zero profits. The essential assumptions are these:

Assumption 1: *There are two types of sellers, high quality sellers (Highs) and low quality sellers (Lows). The proportion of Highs is* π *and the proportion of Lows is* $1-\pi$.

Assumption 2: *The seller utility functions* $U_L(y,p)$ *and* $U_H(y,p)$ *are differentiable, strictly increasing in* p*, and quasi-concave in both* p *and* y.

Assumption 3: *Suppose* $y_L < y_H$*, and* $U_L(y_L,p_L) < U_L(y_H,p_H)$*. Then* $U_H(y_L,p_L) < U_H(y_H,p_H)$.

Assumption 3 is equivalent to assuming at any offer the indifference curves of the high quality sellers are flatter than those of the low quality sellers.

Assumption 4: *The buyer utility per unit of consumption purchased from a Low seller at the offer* (y,p) *is* $V_L(y)-p$*, and the buyer utility per unit* *of consumption purchased from a High seller at the offer* (y, p) *is* $V_H(y) - p$, *where* V_L *and* V_H *are differentiable and non-decreasing in* y, *and for every* y, $V_L(y) \leq V_H(y)$.

We conjecture examples can also be constructed where there are more than two discrete types or where there is a continuous distribution of seller types.

DIAGRAMS.