Strategic Implications of Uncertainty Over One's Own Private Value in Auctions

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Abstract

Suppose a bidder must decide whether and when to incur the cost of estimating his own private value in an auction. This can explain why a bidder might increase his bid ceiling in the course of an auction, and why a bidder would like to know the private values of other bidders. It also can explain sniping– flurries of bids at the end of auctions with deadlines– as the result of other bidders trying to avoid stimulating the uninformed bidder to examine his value.

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A STORY OF MISTAKEN BIDDING

Jeff happily awaited the end of the Ebay auction. He'd submitted a bid ceiling of $2,100 for a custom-made analog stereo amplifier, and the highest anybody else had submitted was $1,400, so he was sure to win. Since he'd followed the advice of Ebay and academic auction theory, submitting his true maximum price, he looked forward to a cool $700 in consumer surplus.

It was five minutes before the auction deadline. And then disaster struck. The winning bid rose to $1,800, and then $1, 900, and $2,000. And then it rose to $2,150, and Jeff was losing! Worse yet, as he feverishly thought hard about how much the amplifier was worth to him, he realized he actually would have been willing to pay $2,500.

But by then it was too late. The auction was over.

VALUE DISCOVERY AND ITS IMPLICATIONS

What if a bidder in a private value auction does now know his own value precisely, but can discover his value at some cost?

Value discovery can explain:

1. Why bidders would like to know how much other bidders are going to bid.
2. How other bidders can benefit when an uninformed bidder learns his value more precisely.
3. How improved buyer information on the value of the object hurts the seller.
4. Why bidders update the bid ceilings they submit during the course of an auction such as those on Ebay and Amazon that uses proxy bidding.
5. Why bidders use “pre-emptive bids”, bidding early in auctions rather than later.
6. Why bidders use “sniping”— the practice of submitting bids at the last minute.

THE MODEL

The two players in an auction are both risk-neutral and have private values which are statistically independent and distributed over the same support $[0, z]$, on which the densities are strictly positive. Bidder 1 has value u distributed according to the atomless density $f(u)$. Bidder 2 has value v distributed according to the atomless density $g(v)$.

Bidder 1 knows neither u nor v. At any time he may, unobserved by Bidder 2, pay c and learn u after additional time δ has passed. Bidder 2 knows v, but not u.

We will look at three sets of auction rules: a sealed-bid second-price auction, an Amazon auction, and an Ebay auction. In each, a player submits his "bid ceiling", but the winner pays the second-highest bid ceiling submitted.

The Amazon auction has a "soft deadline," ten minutes after the last bid ceiling update.

The Ebay auction has a "hard deadline": at a preannounced time.

THE SECOND-PRICE SEALED BID AUCTION

Equilibrium. Bidder 1 pays to discover his value and submits a bid ceiling of u if c is sufficiently low. Otherwise, he submits a bid ceiling of Eu. Bidder 2 submits a bid ceiling of v.

Bidder 1's Bidding Strategy. First, suppose Bidder 1 has paid c and discovered his value, u. Once Bidder 1 knows u, if Bidder 2 submits a bid ceiling of p with probability $m(p)$, Bidder 1's expected payoff is

$$\pi_1(u) = \int_0^b (u-p)m(p)dp. \tag{1}$$

Maximizing by choice of b yields $(u-b)m(b) = 0$, so $b^* = u$. Bidder 1 should bid his value, u.

Second, suppose Bidder 1 has not discovered his value. His payoff if he bids b and Bidder 2 bids p with probability $m(p)$ is

$$\pi_1 = \int_0^z \left(\int_0^b (u-p)m(p)dp\right) f(u)du. \tag{2}$$

Maximizing by choice of b yields

$$\int_0^z (u-b)m(b)f(u)du = 0, \tag{3}$$

so

$$\int_0^z bm(b)f(u)du = \int_0^z um(b)f(u)du, \tag{4}$$

and $b^* = \int_0^z uf(u)du$, the expected value of u. Bidder 1 should bid his expected value.

Bidder 2's Bidding Strategy. Bidder 2's payoff if he submits a bid ceiling of b and Bidder 1 submits a bid ceiling of p with probability $m(p)$ is

$$\pi_{sniper} = \int_0^b (v - p)m(p)dp, \tag{5}$$

because Bidder 2 wins the value v and pays the price p if his bid of b exceeds Bidder 1's bid of p, and otherwise his payoff is zero. Maximizing his payoff by choice of b yields

$$(v - b)m(b) = 0, \tag{6}$$

so $b^* = v$. Bidder 2 should bid his value.

THE DECISION OF WHETHER TO PAY TO DISCOVER ONE'S VALUE

For given Bidder 2 value v, Bidder 1's expected payoff before he actually learns u is

$$\pi_1^d(v) = -c + \int_0^v (0) f(u)du + \int_v^z (u - v) f(u)du. \tag{7}$$

Proposition 1: If the discovery cost c is low enough, Bidder 1's expected payoff is higher if he learns Bidder 2's value, v, before he must decide whether to pay to learn his own value, u.

If c is low enough, there are two reasons why Bidder 1 might benefit from knowing v.

First, if $\pi_1^d > \pi_1^{nd}$, Bidder 1 would switch from always paying to discover his value to paying only if $\pi_1^d(v) > \pi_1^{nd}$.

Second, if $\pi_1^d < \pi_1^{nd}$, Bidder 1 would switch from never discovering his value to discovering it if $\pi_1^d(v) > \pi_1^{nd}$.

THE PAYOFF FROM DISCOVERING ONE'S VALUE

$$\pi_1^d(v) = -c + \int_0^v (0) f(u)du + \int_v^z (u-v) f(u)du. \tag{8}$$

$$\pi_1^d = -c + \int_0^z \left(\int_v^z (u-v) f(u)du \right) g(v)dv. \tag{9}$$

$$\begin{aligned} \pi_1^d &= -c + \int_0^{Eu} \left(\int_v^z (u-v) f(u)du \right) g(v) + \int_{Eu}^z \left(\int_v^z (u-v) f(u)du \right) g(v \\ &= -c + A_1 + A_2. \end{aligned} \tag{10}$$

Now let us find Bidder 1's payoff if he does not learn u. He will bid Eu. If $Eu < v$, then $\pi_1^{nd}(v) = 0$. If $Eu > v$, then

$$\pi_1^{nd}(v) = \int_0^v (u-v) f(u)du + \int_v^z (u-v) f(u)du. \tag{11}$$

Integrating $\pi_1^{nd}(v)$ over v yields the expected payoff

$$\begin{aligned} \pi_1^{nd} &= \int_0^{Eu} \left(\int_0^v (u-v) f(u)du + \int_v^z (u-v) f(u)du \right) g(v)dv + \int_{Eu}^z (0) g(v \\ &= \int_0^{Eu} \left(\int_0^v (u-v) f(u)du \right) g(v)dv + \int_0^{Eu} \left(\int_v^z (u-v) f(u)du \right) g(v)d \\ &= -A_3 + A_1. \end{aligned} \tag{12}$$

SUMMARY OF THE VALUE OF INFORMATION

We can summarize the results for small discovery cost c as follows:

1. Learning v is useful to Bidder 1 if he does not yet know u (Proposition 1).

2. Learning v is useless to Bidder 1 if he already knows u.

3. Learning u is useful to Bidder 1 either if he does not know v or if he knows v and $v > Eu$.

THE EFFECT OF BIDDER 1'S VALUE DISCOVERY ON BIDDER 2

Proposition 2: Bidder 2's expected payoff is higher if Bidder 1 knows u at the start of the auction than if Bidder 1 submits a bid ceiling of Eu.

Proof: First, suppose $v < Eu$. Bidder 2's payoff is zero without value discovery, because Bidder 1 will bid Eu and win. Bidder 2's payoff is positive with value discovery, because there is probability $F(v)$ that u will be less than v and Bidder 1 will win. Thus, if $v < Eu$, Bidder 2 benefits from Bidder 1 knowing u.

Second, suppose $v > Eu$. Without value discovery, Bidder 2's payoff is

$$v - Eu, \tag{13}$$

which can be rewritten as

$$\int_0^z vf(u)du - \int_0^z uf(u)du. \tag{14}$$

With value discovery, Bidder 2's payoff is

$$\int_0^v (v-u)f(u)du + \int_v^z (0)f(u)du = v\int_0^v f(u)du - \int_0^v uf(u)du. \tag{15}$$

We need to show that

$$\int_v^z vf(u)du < \int_v^z uf(u)du, \tag{16}$$

which is true because in the right-hand-side integral u is taking values that are v or greater.

THE EFFECT OF BIDDER 1'S VALUE DISCOVERY ON THE SELLER

Proposition 3: The seller prefers that Buyer 1 not know u at the start of the auction; if Bidder 1 submits a bid ceiling of Eu instead of u, the expected price is higher.

Proof: Take a given v. First, suppose $v < Eu$. The winning price would be v. Value discovery will either keep the winning price at v (if $u \geq v$), or reduce it to below v (if $u < v$). Thus, the expected winning price is higher if Bidder 1 does not know u.

Second, suppose $v \geq Eu$. The winning price would be Eu if Bidder 1 does not know u. Value discovery will change the winning price to $Min\,(u, v)$. The winning price is u if $u < v$ and v if $u > v$, so its expected value is

$$\int_0^v uf(u) + \int_v^z vf(u). \tag{17}$$

This is less than Eu if

$$\int_0^v uf(u) + \int_v^z vf(u) < \int_0^z uf(u), \tag{18}$$

which is true if

$$\int_v^z vf(u) < \int_v^z uf(u), \tag{19}$$

which is true since in the second integral u ranges from v up to z whereas in the first integral v is a constant.

EXAMPLE 1: SECOND-PRICE SEALED BID

Bidder 1 has private value u uniformly distributed on [0,100]. He can take 5 minutes and pay amount 5 to discover u if he wishes; otherwise, his estimate is 50. Bidder 2 has value v of either 10 or 60, with equal probability.

Bidder 2 will submit his value, either 10 or 60. If Bidder 1 submits a bid ceiling of 50, his expected payoff is $\pi_1^{nd} = .5(Eu - 10) + .5(0) = .5(50 - 10) = 20$. Bidder 2's payoff is $\pi_2 = .5(60 - 50) + .5(0) = 5$. The expected price would be $.5(10) + .5(50) = 30$.

Suppose Bidder 1 decides to pay to discover his value before he submits his bid ceiling. He will then submit a bid ceiling of u. With probability .5, $v = 10$. If that is the case, then $u \geq 10$ and Bidder 1 wins the auction with probability .9, and the expectation of his payoff if he wins is $Eu|(u \geq 10) - 10$. With probability .5, $v = 60$. If that is the case, then $u \geq 60$ and Bidder 1 wins the auction with probability .4, and the expectation of his payoff if he wins is $Eu|(u \geq 60) - 60$. Bidder 1's overall expected payoff is therefore $\pi_1^d = -5 + .5(.9)[Eu|(u \geq 10) - 10] + .5(.4)[Eu|(u \geq 60) - 60] = -5 + .45[55 - 10] + .20[80 - 60] = -5 + 20.25 + 4 = 19.25$. Thus, Bidder 2 will choose not to pay to discover his value.

EXAMPLE 1: SECOND-PRICE SEALED BID, continued. PROPOSITIONS 1,2,3

Bidder 1 has private value u uniformly distributed on [0,100]. He can take 5 minutes and pay amount 5 to discover u if he wishes; otherwise, his estimate is 50. Bidder 2 has value v of either 10 or 60, with equal probability.

If, however, $c = 0$ instead of $c = 5$, Bidder 2 would pay to discover his value; for small enough c, value discovery would occur. And if $c = 5$ and Bidder 1 knew that $v = 60$, he would pay to discover his value, because his payoff from not discovering it would be 0, whereas his payoff from discovering it would be $-5 + .4(Eu|(u \geq 60) - 60] = -5 + .4(80 - 60) = 3$. This illustrates Proposition 1, that a bidder could benefit from knowing the other bidder's value.

This example can also illustrate Propositions 2 and 3. If Bidder 1 knew his value and bid u, Bidder 2's expected payoff would be $.5(.1)(10 - Eu|u \leq 10) + .5(.6)(60 - Eu|u \leq 60) = .05(10-5) + .30(60 - 30) = .25 + 9 = 9.25$. As Proposition 2 predicts, Bidder 2's payoff of 9.25 when Bidder 1 knows u is greater than his payoff of 5 when Bidder 1 does not know u.

The expected price if Bidder 1 bids u is $.5[(.1)(Eu|u \leq 10) + .9(10)] + .5[(.6)(Eu|u \leq 60) + .4(60)] = .5[.5 + 9] + .5[18 + 24] = 25.75$. This is less than the expected price of 30 if Bidder 1 does not know u, as Proposition 3 predicts.

Proposition 4: The phenomenon of bidders increasing their bid ceilings during the auction can be a necessary part of equilibrium.

Equilibrium. Bidder 1 submits a bid ceiling of either Eu, if c is high enough, or $\overline{p} > 0$ otherwise. If he has submitted a bid ceiling of $\overline{p}$ and the current winning bid rises to $\overline{p}$, he pays c to discover u and then increases his bid ceiling to u if $u > \overline{p}$. Bidder 2 submits a bid ceiling of v.

Explanation. Bidder 1's expected payoff conditional on v is

$$\pi_1^d(v) = \int_0^z (u - v) f(u) du. \tag{20}$$

If $v \geq \overline{p}$, Bidder 1's expected payoff is

$$\pi_1^d(v) = -c + \int_0^v (0) f(u) du + \int_v^z (u - v) f(u) du. \tag{21}$$

$$\pi_1^d = \int_0^{\overline{p}} \left(\int_0^z (u - v) f(u) du \right) g(v) dv + \int_{\overline{p}}^z \left(-c + \int_v^z (u - v) f(u) du \right) g(v) dv \tag{22}$$

Our results from the second-price sealed-bid auction tell us immediately that $\pi_1^d > \pi_1^{nd}$ if c is low enough but not otherwise.

It can also be shown that $\overline{p} > 0$.

WHY VALUE DISCOVERY WOULD NEVER OCCUR BEFORE THE START OF THE AUCTION

Lemma: *Bidder 1 will set his initial bid ceiling to be strictly positive, delaying value discovery:* $\overline{p} > 0$.

Proof: Differentiating Bidder 1's payoff with respect to $\overline{p}$ (which we can do if the densities are atomless) yields

$$\frac{d\pi_1^d}{d\overline{p}} = \left(\int_0^z (u-\overline{p})f(u)du\right)g(\overline{p}) - -cg(\overline{p}) - \left(\int_{\overline{p}}^z (u-\overline{p})f(u)du\right)g(\overline{p}) \tag{23}$$

If $\overline{p} = 0$, the first and third terms of this derivative cancel out. The second term is positive, however, given our assumption that the value density is everywhere positive. Thus the payoff derivative is positive at $\overline{p} = 0$, in which case the optimal value of $\overline{p} = 0$ is positive, which was to be proved.

The advantage of increasing $\overline{p}$ is that possibly the bidder will win at a price $p < \overline{p}$ and not have to pay the discovery cost c. The disadvantage is that possibly the bidder will win at $p < \overline{p}$ such that p exceeds his value: $p = v > u$. The size of this disadvantage depends on the likelihood that Bidder 1's value is below $\overline{p}$, which is $\int_0^{\overline{p}} f(u)du$. If $\overline{p} = 0$, this disadvantage vanishes; there is no risk that Bidder 1's value will be below $\overline{p}$. Bidder 1 should increase his initial bid ceiling until the marginal gain from avoiding the discovery cost equals the marginal loss from winning when his value is below the price he pays.

Proposition 5:A bidder may purposely bid early, so as to stimulate value discovery.

Equilibrium. If c is low enough, Bidder 1 submits a bid ceiling of $\bar{b}$ before time $T - \delta$, and then pays to discover his value if the current winning bid reaches $\bar{b}$. If the current winning bid does not reach $\bar{b}$, he raises his bid ceiling to Eu. If c is higher, Bidder 1 submits a bid ceiling of Eu and never discovers his value. Bidder 2 submits a bid ceiling of $\bar{b}$ before time $T - \delta$, and raises his bid ceiling to v after time $T - \delta$.

Explanation. Now Bidder 2 can wait to bid until after $T - \delta$ and prevent value discovery. Whether Bidder 2 wants to do this depends on whether he wants to provoke Bidder 1 to discover u.

Does he?

IN THE EBAY AUCTION DOES BIDDER 2 WANT TO PROVOKE VALUE DISCOVERY?

If Bidder 2 has value v and provokes value discovery by submitting a bid ceiling of $\bar{b}$ or more, his expected payoff is made up of three parts, depending on whether he wins at a price of $\bar{b}$, wins at a price of u, or loses the auction.

$$\pi_2 = \int_0^{\bar{b}} (v - \bar{b}) f(u) du + \int_{\bar{b}}^{v} (v - u) f(u) du + \int_v^z (0) f(u) du. \quad (24)$$

$$\frac{d\pi_2}{d\bar{b}} = (v - \bar{b}) f(\bar{b}) - \int_0^{\bar{b}} f(u) du + (v - \bar{b}) f(\bar{b}). \quad (25)$$

If $\bar{b}$ is greater, then Bidder 2 benefits less from Bidder 1 learning u because Bidder 1 cannot reduce his bid ceiling below $\bar{b}$ after learning that $u < \bar{b}$.

We can deduce from Proposition 2 that there is some positive value k of $\bar{b}$ low enough that Bidder 2 will benefit from value discovery, since if $\bar{b} = 0$. In choosing a discovery threshold, $\bar{b}$, Bidder 1 is constrained to set $\bar{b} \leq k$, because otherwise Bidder 1 would want to avoid provoking value discovery and would delay submitting v as his bid ceiling until after $T - \delta$. Thus, $\bar{b}$ will be less than or equal to the $\bar{p}$ of the Amazon auction. If c is too high, then Bidder 1 will prefer not to discover his value and to simply bid Eu at the start of the auction.

EXAMPLE 2: EBAY AUCTION

Bidder 1 has private value u uniformly distributed on [0,100]. He can take 5 minutes and pay amount 5 to discover u if he wishes; otherwise, his estimate is 50. Bidder 2 has value v of either 10 or 60, with equal probability.

If Bidder 1 submits a bid ceiling of 50, $\pi_1^{nd} = 20$ and $\pi_2 = 5$.

Suppose Bidder 1 instead follows the strategy of submitting a bid ceiling of $\bar{b} = 11$ and then either paying to discover his value if the current winning bid rises to 11 or raising his bid ceiling to 50. Bidder 2 will respond by bidding v.

With probability .5, Bidder 1 will win at $p = 10$ because $v = 10$, and with probability .5, Bidder 2 will bid 60 and stimulate value discovery. If that happens, Bidder 1 incurs the cost 5, and with probability .4 finds that he wants to overbid Bidder 2 and win at the $p = 60$. With probability .6, he finds that he does not want to increase his bid past 11, and Bidder 2 will win. $\pi_1^d = 21.5$ and $\pi_2 = 14.7$. Thus, the payoffs of both bidders are higher with value discovery.

Example 2 illustrates Proposition 4, that a bidder may wish to increase his bid ceiling in the course of an auction, and Proposition 5, that a bidder may wish to stimulate value discovery by bidding early.

NAIVE BIDDERS

Suppose Bidder 1 is "naive": he does not realize that Bidder 2 is present (i.e., he assigns probability 0 to Bidder 2 being present). After all, the Ebay instructions say:

> "For example, if the current bid on an item is $5 and you are willing to pay up to $10, you would enter $10 as your maximum bid. Your bid would be shown on the item page as $5, but if another bidder places a bid for $6, then eBay will place a higher bid on your behalf. The bid would be just above the other member's bid. This would continue until either you win the auction at or below $10 or the bidding exceeds the $10 you were willing to pay. eBay will notify you via email if you are outbid and you can return to place another bid if you like. Your maximum bid is never disclosed to other bidders or to the seller."
>
> –EBay Tutorials, "Place Your Bid"
> http://pages.ebay.com/education/tutorial/course1/bidding_3.html
> (May 25, 2002)

THE RESULT OF NAIVETE

Bidder 1 would submit a bid ceiling of Eu regardless of how low c was, under all three auction rules.

In the Ebay and Amazon auctions, this hurts Bidders 1 and 2 and helps the seller.

In the Ebay auction, if $v > Eu$, then if parameters are such that Bidder 1 would pay to discover u if he were aware of Bidder 2 being present, Bidder 2 would submit a bid ceiling of v in the time interval $[T - \delta, T]$. If Bidder 1 would not pay, Bidder 2 could submit a bid ceiling of v at any time.

Proposition 6: "Sniping" can occur in equilibrium. A bidder may purposely delay submitting a bid ceiling higher than the current winning bid until near the auction deadline.

EXAMPLE 3: EBAY AUCTION WITH NAIVETE

Let Bidder 1 have a private value u uniformly distributed on [0,100]. He can take 5 minutes and pay amount 5 to discover his value precisely if he wishes; otherwise, his estimate is 50. Let Bidder 2 have a value v of either 10 or 60, with equal probability.

Bidder 1 will submit a bid ceiling of 50. Bidder 2 will submit a bid ceiling of 10 if $v = 10$, and timing will not matter. If $v = 60$, though, he should wait until within 5 minutes of the deadline and submit a bid ceiling of 60. He will win at a price of 50, for a payoff of 10.

What if Bidder 2 were to submit a bid ceiling of 60 earlier? Bidder 1's payoff from paying 5 to discover u would be $-5+.6(0)+.4[(Eu|u > 60)-60] = -5+.4(80-60) = -5+8 = 3$, compared to a payoff of 0 from not discovering his value and retaining his bid ceiling of 50. Thus, Bidder 1 would pay to discover his value and with probability .4 he would increase his bid ceiling to more than 60 and win the auction. $\pi_2 = .6(60 - 50) + .4(0) = 6$, less than the 10 Bidder 2 would get from delaying his bid submission and sniping.

Example 3 illustrates the value of sniping (Proposition 6). Bidder 2 does not want to bid early, because it would stimulate value discovery, and that would either increase the price Bidder 2 pays or cost him victory in the auction.

OTHER EXPLANATIONS FOR SNIPING

1. Alvin Roth and Axel Ockenfels (2000): players making bids in the last minute may find the computer has not been able to get their bids in time. In that case, players will submit low bids early in the auction and higher bids in the last minute. There is some chance that none of the high bids will be accepted, and so some bidder wins with his very low initial bid.

2. The auction is common value, and updating valuations requires time.

3. There is no point to submitting a bid early, and possibly you will discover something about your private value exogenously as time passes.