Permutations

Definition 1. A permutation is an ordered arrangement of K objects. For example, ABC and ACB are permutations of the objects A, C, and B.

Definition 2. For number N, we define N Factorial, also called "the factorial of N" as $N \cdot (N-1) \cdot (N-2) \cdot \ldots 3 \cdot 2 \cdot 1$. For example $4! = 4 \cdot 3 \cdot 3 \cdot 2 \cdot 1 = 24$. We define 0! as 1, because in advanced math that turns out to make things simpler sometimes

(see https://en.wikipedia.org/wiki/Empty_product), and it is hard to know whether to define an empty set, $\{\}$, as having 1 permutation or 0 permutations. Is there one way to arrange nothing, or no way?

Symbols. The Greek letter for the "A" sound, is Alpha: α as a small letter and A as a capital. See "I am Alpha and Omega, the beginning and the end, the first and the last," in Revelation 22:18. The Greek letter for the "O" sounds is Omega: ω as a small letter and Ω as a capital. Omega is the last letter of the Greek alphabet, the way they arrange it. They also had a letter Omicron: o, O for "O" that is the same as our "O".

Proposition 1. There are 6 permutations of the letters $\{A, B, C\}$.
Proof. There are three ways to start the permutation: A, B, and C. For each of these, there are two ways to choose the second letter (e.g., for A first, B or C can be second). Then there is one letter left that can be used for the third letter. Thus there are $3 \cdot 2 = 6$ ways to arrange the the three letters, which is what it means to permute them. Q.E.D.

A diagram is helpful for this.

Proposition 2. There are $N!$ permutations of N objects.
Proof. There are N ways to choose the first object in the permutation. For each of those, that leaves $N-1$ ways to choose the second object. That leaves $N-2$ ways to choose the third object. This pattern continues until there are 3 ways to choose the 3rd-from-last object, 2 ways to choose the second-from-last, and just 1 way to choose the last one. Thus, there are $N \cdot N - 1 \cdot N - 2 \cdot \ldots 3 \cdot 2 \cdot 1$ permutations, which can be written at $N!$. Q.E.D.

Proposition 3. There are $N(N-1)\dots(N-K)+1$ permutations of K objects taken from a set of N objects with no duplicates.

Proof. There are N ways to choose the first object in the permutation. For each of those, that leaves $N-1$ ways to choose the second object. That leaves $N-2$ ways to choose the third object. This continues until we have reached the Kth object. Thus, there are $N \cdot N-1 \cdot N-2 \cdot \dots N-K+1$ permutations. Q.E.D.

Proposition 4. There are N^K permutations of K objects taken from a set of N objects if duplicates are allowed.

Proof. There are N ways to choose the first object in the permutation. For each of those, there are still N ways to choose the second object, since duplicates are allowed. Then there are N ways to choose the third object. This continues until we have reached the Kth object. Thus, there are $N \cdot N \cdot N \dots N$ with N repeated K times permutations, which is N^K. Q.E.D.