How to Do a Proof

A proof is a rigorous explanation in math. In a proof, you first write a statement that you are going to prove. Then you go step by step to prove every part of it. For example, suppose we want to prove that $2 * [(2 + 10)/4 + 8 - 4] = 14$.

Proposition 1: 2[(2+10)/4 + 8 - 4] = 14.*
The left-hand-side of the equation in the proposition can be rewritten as 2*[12/4 + 8 - 4] by adding the 2 and the 10. In turn, this can be rewritten as 2*[3 + 4]by by dividing 12 by 4. Next, we can rewrite it as 2*(7) by adding 3 to 4. This equals 2*7, which in turn is equal to 14. Quod erat demonstrandum.

"Quod erat demonstrandum" means "Which was to be shown" in Latin, and is put at the end of proofs, sometimes abbreviated to "Q.E.D."

Proposition 1 is just a math problem. Usually proofs are for more general statements. Proposition 2 is an example.

Proposition 2: $x^2 * x^4 = x^6$ *for any number* x*.*
Proof: $x^2 = x * x$.
$x^4 = x * x * x * x$
Thus, $x^2 * x^4 = (x * x)(x * x * x * x) = x * x * x * x * x * x$ by the associative property, and equals $x * x * x * x * x * x = x^6$. Q.E.D.

We can make Proposition 2 more general:

Proposition 3: $x^a * x^b = x^{a+b}$ *for any number* x *and non-negative whole numbers* a *and* b *.*
Proof: $x^a = 1 * x * x...$ with x repeated a times on the right-hand-side (so $x^0 = 1$ and $x^1 = x$). The same is true for x^b except x is repeated b times on the right-hand-side.
Thus, $x^a * x^b$ multiplies $1 * x * x...$ with x repeated a times by $1 * x * x...$ with x repeated b times. That means, by the associative property, that $x^a * x^b = 1 * x * x...$ with x repeated $(a + b)$ times, which equals x^{a+b}. Q.E.D.

We could make Proposition 3 even more general by allowing a and b to be negative, but that would require a more complicated proof. Let's move on to different topic:what numbers can be divided evenly.

Proposition 4: If and only if the last two digits of a number can be divided by four without remainder, so can the entire number.
Proof: We can write the number as xyz, where y and z are the last two digits and x is all the other digits. Thus, for $53,678, x = 536, y = 7$, and $z = 8$.
We can break the number xyz up into a hundreds part and a number from 0 to 99: $xyz = x*100+yz$.
100 is evenly divisible by 4 because $100/4 = 25$.
Thus, $x * 100$ also evenly divisible— $x * 100/4$ equals $25x$, and x is a whole number.
Thus, xyz, which we found equalled $x*100+yz$ is evenly divisible by 4 **if** yz is divisible by 4. Also, this is true **only if** yz is divisible by 4, because if it is not divisible by 4 then $xyz/4 = 25x + yz/4$ and $yz/4$ is not a whole number. Q.E.D.

The last proof is harder, but it is the most surprising proposition here.

Proposition 5: If and only if the sum of the digits of a four-digit number can be divided by three without remainder, so can the number itself.
Proof: Call the digits of the number a, b, c, d, so the number is $abcd$ e.g., 3,523 has $a = 3, b = 5, c = 2, d = 3$. It will be true that
$abcd = a * (1,000) + b * (100) + c * (10) + d$
Thus,
$abcd = a * (999 + 1) + b * (99 + 1) + c * (9 + 1) + d$
and
$abcd = a * 999 + a + b * 99 + b + c * 9 + c + d$
and
$abcd = (a * 999 + b * 99 + c * 9) + (a + b + c + d)$
We know that 999 and 99 and 9 are divisible even by 3, so everything in the first set of parentheses is divisible by 3.
Thus, whether $abcd$ is divisible by 3 depends on whether $(a+b+c+d)$ is divisible by 3. Quod erat demonstrandum.

It is hard to know how many steps to write out in a proof. Proposition 1 had lots of steps, and it was easy enough that some of them could be combined. Proposition 5 is harder, so having lots of steps make it easier to understand.