February 7, 2023

Handout: The Vertical Angle Theorem

The Vertical Angle Theorem: The vertical angles of two intersecting lines are congruent.

When two lines cross, they form four pairs of adjacent angles (ones next to each other) and two pairs of vertical angles (pairs not adjacent). You can think of vertical angles as "opposite angles". If two angles are congruent, they have the same size.

Proof 1. Draw two intersecting lines and label the angles a, b, c, and d, so a and c are vertical angles.

We know that a and b are adjacent and add up to a straight angle, 180 degrees. Thus, $b + a = 180$.

We know that c and b are adjacent and add up to a straight angle, 180 degrees. Thus, $b + c = 180$.

But that means that $b + a = 180 = b + c$.

If we subtract b from both sides, the equation remains true, so $b + a - b = b + c - b$, so $a = c$.

In the same way, $a + d = 180$ and $a + b = 180$, so $a + d = a + b$, so $d = b$.

Thus, the vertical angles are equal in size. Quod erat demonstrandum.