**February 16, 2023**

**Handout: Inscribing a Right Triangle in a Circle**

**Theorem:** *A right triangle can be inscribed in a circle between any two opposite points on the circle A and B and any other point D on the circle.*

Proof. Let C be the center of the circle and A and B be opposite points on the circle. Let D be any other point. We will show that angle ADB is a right angle, 90 degrees in size.

Consider the triangle ACD. That is an isosceles triangle, because both AC and CD are radii of the circle so they have the same length. This means that angle A equals angle ADC. Since it is a triangle, A + ADC + ACD = 180. Since A = ADC, that means 2ADC + ACD = 180. Thus, ACD = 180 – 2ADC.

Consider the triangle BCD. It also is an isosceles triangle, because both CD and BC are radii of the circle so they have the same length. This means that angle B equals angle BDC. Since it is a triangle, B + BDC + BCD = 180. Since B = BDC, that means 2BDC + BCD = 180. Thus, BCD = 180 –2BDC.

Note that ADC + BCD = 180, because they are supplementary angles, adding up to a straight line.

Thus, ADC + BCD = 180 and also, from what we found earlier, ADC + BCD = (180 – 2ADC) + (180 – 2BDC).

Thus, 180 – 2ADC + 180 – 2BDC = 180.

We can add up the 180’s on the left-hand side to get 360 – 2ADC - 2BDC = 180, so if we subtract 180 from each side we get 180 - 2ADC – 2BDC = 0.

Then 180 = 2ADC + 2BDC, adding (2ADC + 2BDC) to both sides of the last equation.

Dividing both sides of the equation by 2, we get 90 = ADC + BDC.

This means that since angle ADB of the triangle ABD equals the sum of ADC and BDC, ABD = ADC + BDC = 90.

Thus, the triangle ABD has one right angle, and we have succeeded in inscribing a right triangle in the circle. Q.E.D.

