February 1, 2025

Inscribing a Right Triangle in a Circle

Theorem: *A right triangle can be inscribed in a circle between any two opposite points on the circle A and B and any other point D on the circle.*

Proof. Let C be the center of the circle and A and B be opposite points on the circle. Let D be any other point. We will show that angle $\gamma + \delta$ of triangle ABD is a right angle, 90 degrees in size.

Consider the triangle ACD. That is an isosceles triangle, because both AC and CD are radii of the circle so they have the same length. This means the two angles on edge AD both equal δ. Since it is a triangle, the three angles add up to 180: $\alpha + \delta + \delta = 180$, so $\alpha + 2\delta = 180$. Thus,

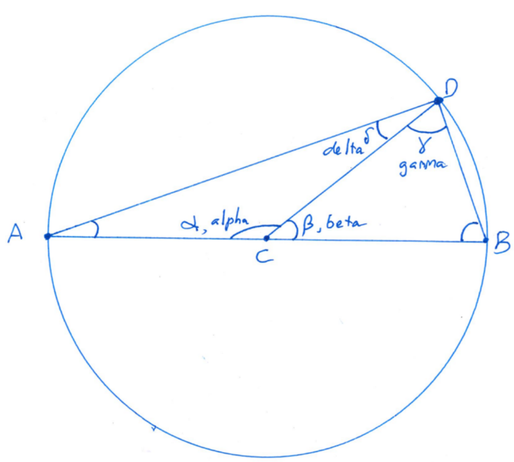
(1) $$\alpha = 180 - 2\delta.$$

Consider the triangle BCD. It also is an isosceles triangle, because both CD and BC are radii of the circle so they have the same length. This means that both of the angles on edge BD are equal to γ. Since it is a triangle, $\beta + \gamma + \gamma = 180$, so $\beta + 2\gamma = 180$. Thus,

(2) $$\beta = 180 - 2\gamma.$$

Combining equations (1) and (2), we have

(3) $$\alpha + \beta = (180 - 2\delta) + (180 - 2\gamma) = 360 - 2\delta - 2\gamma.$$



Note that

(4) $$\alpha + \beta = 180$$

because they are supplementary angles, adding to a straight line.
Combining (3) and (4), we get

$$\alpha + \beta = 360 - 2\delta - 2\gamma = \alpha + \beta = 180,$$

so

$$360 - 2\delta - 2\gamma = 180,$$

so

$$180 - 2\delta - 2\gamma = 0$$
$$180 = 2\delta + 2\gamma$$
$$90 = \delta + \gamma$$

So the angle $\delta + \gamma$ of triangle ABD is 90 degrees and we have succeeded in inscribing a right triangle in a circle.

Q.E.D.

α β γ δ
alpha beta gamma delta