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Formulas for π

This list is selected from the *Wikipedia* article, "List of Formulas involving π". Some of them I adapted to be $\pi = f(x)$ instead of $x = f(\pi)$ I should really write this all out as eqnarrays.

$$\pi = \frac{C}{d} \quad \text{(where C is the circumference and d is the diameter of a circle.)} \tag{1}$$

$$\pi = \frac{C}{2r} \quad \text{(where C is the circumference and r is the radius of a circle.)} \tag{2}$$

$$\pi = \frac{A}{r^2} \quad \text{(where A is the area and ris the radius of a circle.)} \tag{3}$$

$$\pi = \frac{A}{\frac{major}{2} \cdot \frac{minor}{2}}, \tag{4}$$

where A is the area of an ellipse, $major$ is its major axis, and $minor$ is its minor axis.

$$\pi = \frac{3}{4} \cdot \frac{V}{r^3} \quad \text{(where V is the area of a sphere and r is its radius.)} \tag{5}$$

$$\pi = \frac{A}{4r^2} \quad \text{(where A is a sphere's surface area and r is its radius.)} \tag{6}$$

$$\pi = \frac{\sqrt{2H}}{r^2}, \quad \text{(where H is the volume and r is the radius of a 3} - \text{hypersphere in 4} - \text{D space.)} \tag{7}$$

$$\pi = \frac{Sum}{N-2} \quad \text{(where Sum is the sum of an N} - \text{sided regular convex polygon's angles, in radians.)} \tag{8}$$

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\right) \quad \text{(Leibniz's formula for } \pi\text{)} \tag{9}$$

$$\pi = 2\sqrt{2}\left(1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \cdots\right) \qquad \text{(Newton's formula for } \pi) \tag{10}$$

$$\pi = \sum_{k=1}^{\infty} k\frac{2^k k!^2}{(2k)!} - 3 \qquad \text{(Euler's series)} \tag{11}$$

$$\pi = \sqrt[4]{90\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots\right)} \tag{12}$$

$$\pi = \sqrt{12\left(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\right)} \tag{13}$$

$$\pi = \sqrt{24\left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \frac{1}{8^2} + \cdots\right)} \tag{14}$$

$$\pi = \sqrt{8\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right)} \tag{15}$$

$$\pi = \sqrt[3]{32\left(\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots\right)} \tag{16}$$

$$\pi = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n + 1/2} \tag{17}$$

$$\pi = \sqrt{\sum_{n=-\infty}^{\infty} \frac{1}{(n + 1/2)^2}} \tag{18}$$

$$\pi = \sqrt{6\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)} \qquad \text{(from Euler)} \tag{19}$$

$$\pi = 4\left(\frac{3}{4} \cdot \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{9}{8} \cdot \frac{11}{12} \cdot \frac{13}{12} \cdots\right), \tag{20}$$

from Euler, where the numerators are the odd primes; each denominator is the multiple of four nearest to the numerator.

$$\pi = 2\int_{-1}^{1} \sqrt{1-x^2}\,dx \qquad \text{(which is to integrate two halves of a unit circle)} \tag{21}$$

and, similarly,

$$\pi = \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\,dx \qquad \text{(which is to integrate two halves of a unit circle)} \tag{22}$$

$$\pi = \int_{-\infty}^{\infty} \frac{1}{1+x^2}dx \qquad \text{(from the Cauchy density and the Witch of Agnesi.)} \tag{23}$$

$$\text{Since } f(x) = \tfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \text{(the normal density),} \tag{24}$$

$$1 = \int_{-\infty}^{\infty} \tfrac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \qquad \text{(integrating and setting } \sigma = 1, \mu = 0\text{), so} \tag{25}$$

$$\pi = \tfrac{1}{2}\left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)^2 \tag{26}$$

$$\pi \approx \left(\sum_{x=0}^{N} \tfrac{1}{N} e^{-x^2/2}\right)^2 \qquad \text{(doubling the sum from 0 to N)} \tag{27}$$

$$\tag{28}$$

$$\pi = \int_{-\infty}^{\infty} \frac{sin(x)}{x} dx, \tag{29}$$

Add the fluid dynamics equation.

Add the Euler identity.

Peter Borwein, The World of Pi website.

Peter Borwein, "The Amazing Number Π" (2000).