February 10, 2023
Math Class

Odd Squares Minus One Are Divisible by Eight

Theorem: Suppose a square's side is an odd number. Then the area of the square minus one is divisible by eight.

Proof: Let the side of the square have length X. Since X is odd, we can write it as X = 2N+1 for some number N. What we must show is that $(2N+1)^2 - 1$ is divisible by 8.

$$
\begin{aligned}
(2N+1)^2 - 1 &= (2N+1)(2N+1) - 1 && (1) \\
&= (2N+1)(2N) + (2N+1)(1) - 1 && \\
&= 2N \cdot 2N + 2N \cdot 1 + 2N \cdot 1 + 1 \cdot 1 - 1 && (2) \\
&= 4N^2 + 2N + 2N + 1 - 1 && (3) \\
&= 4N^2 + 4N && (4) \\
&= 4(N^2 + N) && (5) \\
&= 4N(N+1) && (6)
\end{aligned}
$$

Clearly this is divisible by 4. But we can show that $N(N+1)$ is even, so $N(N+1)$ is divisible by 2 and the whole thing is divisible by $4 \cdot 2 = 8$. The reason $N(N+1)$ is even is that either N is even or $(N+1)$ is even, so $N(N+1)$ is an even number multiplies by an odd number. But anything multiplied by an even number is even. So $N(N+1)$ is even, so it is divisible by 2 and $4N(N+1)$ is divisible by 8. QED.