April 2, 2023
Math Class

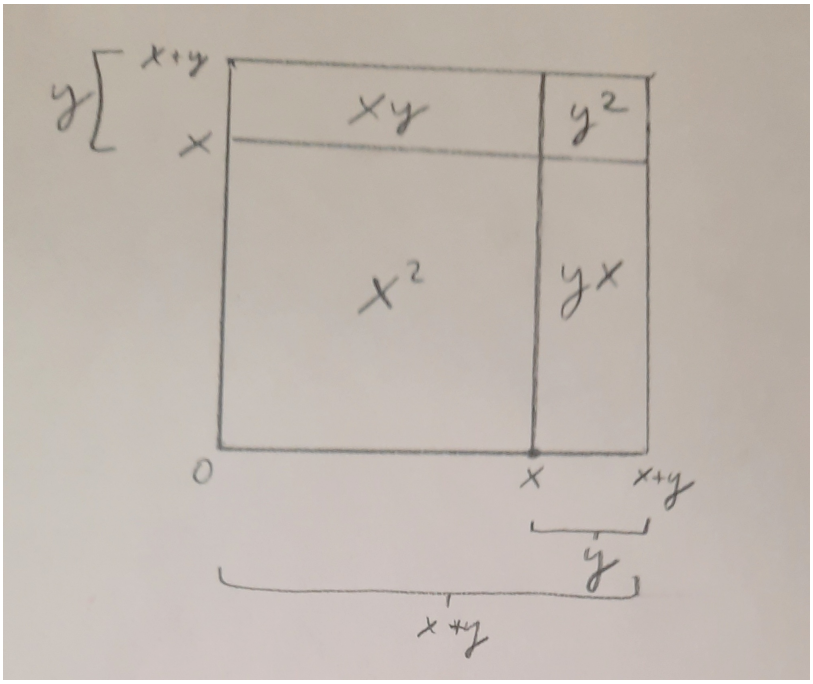
For Opera in the Arts Cycle, some of you are memorizing the Major-General's Song from *Pirates of Penzance*. One verse is:

I am the very model of a modern Major-General,
I've information vegetable, animal, and mineral,
I know the kings of England, and I quote the fights historical
From Marathon to Waterloo, in order categorical;
I'm very well acquainted, too, with matters mathematical,
I understand equations, both the simple and quadratical,
About binomial theorems I'm teeming with a lotta news,
With many cheerful facts about the square of the hypotenuse.

We covered the hypotenuse last month, and are doing pre-algebra this month, so I used this onto which to hang some review and some new material. "$-3 + x = 4$" is a *simple equation*, while "$x^2 = 9$" is *quadratical*. "$(x + y)^2$" is a *binomial quadratic equation* because it has two variables ("bi-" as in "bicycle"). We have come to the Distributive Property in the text, and it says that we can expand that binomial equation thus:

$$\begin{aligned}(x+y)^2 &= (x+y)\cdot(x+y) && (1)\\ &= x(x+y)+y(x+y) && (2)\\ &= xx+xy+yx+yy && (3)\\ &= x^2+2xy+y^2 && (4)\\ & && (5)\end{aligned}$$

A Quadratic Binomial



The figure shows this in a picture. The square is $(x + y)$ on each side, and we can divide it into four pieces, of area x^2, y^2 and two piecesof size xy.

Similarly, we could expand a trinomial equation $(x+y+3)^2$. Or, we could expand a *cubic* (power 3) or higher binomial equation. I won't show you the steps, just one result:

$$(x+y)^5 = (x+y)(x+y)(x+y)(x+y)(x+y) \quad (6)$$

$$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 \quad (7)$$

$$(8)$$

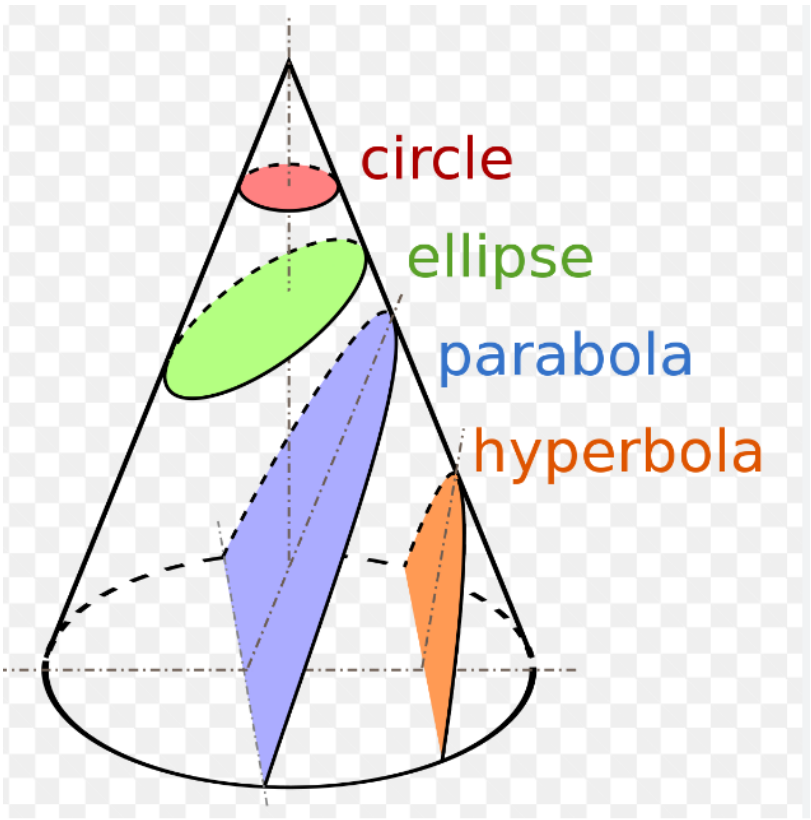
The Binomial Theorem gives a formula for $(x+y)^n$ for any value of n you might choose. It's a complicated formula which you'll probably learn in high school.

The $(x+y)^2$, I told them, relates to how we proved the Pythagorean Theorem in class, which says that the square of the hypotenuse (more Major General) equals the sum of the squares of the other two sides of any right triangle. I attach the handout on that proof. One student then pointed out that it also came up in another proof we had in class, two months ago, for Odd Numbers Minus One Are Divisible by Eight (also attached).

"I'm very good at integral and differential calculus;
I know the scientific names of beings animalculous:"

Integral calculus is about computing the area under curves. Differential calculus is about computing the slope of curves.

Conic Sections



"In conics I can floor peculiarities parabolous;"

The Greeks found that you could get the most important shapes of curves by slicing a cone different ways: circle, ellipse, parabola, and hyperbola. A parabola has an equation like $y = 6x - x^2$; it is a symmetric hill that rises gently to a highest point and then declines.