The Mathematical Writings of Diderot

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IT IS by no means surprising that DIDEROT, an encyclopedist not only by profession but also by the natural bent of his mind and the diversity of his talents, for many years turned his inquiring genius to the field of mathematics. It is apparent, however, that almost no significance has been attached to his mathematical studies. At least, those who have explored the intricacies of his Dedalus-like mind have shunned—perhaps for lack of technical knowledge—his writings on that subject. Nevertheless, these endeavors dominated DIDEROT's youthful activities and represent an important phase of his universal interests. Especially, we cannot lay claim to complete understanding of his scientific talents and accomplishments until we have explored a field in which his published contributions extend from 1748 to 1761, and in which his studies cover the fifteen years preceding this period.[1] Since much of DIDEROT's mathematics consists of applications to physical questions, it is impossible to dissociate the two; nor would it be desirable, since they show how mathematics led him to wider fields of thought.

This study is concerned less with the mathematics of DIDEROT than with DIDEROT the mathematician. Here the man is greater than his work. Yet it cannot be denied that even in the history of mathematics, DIDEROT's writings possess a certain interest, entirely apart from the larger interest associated with the author, for they show some originality even though they are admittedly minor contributions. Reference to them in several mathematical histories attests that they are not to be looked upon as a quaint or amateurish effort.

In view of this fact, it is necessary to examine immediately a story about DIDEROT that has been circulated for so many years that historians have finally accorded it credence. This tale first attracted considerable attention in Anglo-Saxon countries when it appeared in AUGUSTUS DE MORGAN's famous *Budget of Paradoxes*.[2] DE MORGAN relates the tale as follows:

> DIDEROT paid a visit to the Russian Court at the invitation of the Empress. He conversed very freely, and gave the younger members of the Court circle a good deal of lively atheism.

[1] J. ASSÉZAT, editor of DIDEROT's works, recalls that he was an active collaborator of DEPARCIEUX. Cf. *Œuvres*, édition ASSÉZAT et TOURNEUX, Paris, 1875, IX, 75. All further page references will be to this edition and this volume.

[2] London, 1872, pp. 250–51 (first edition).

The Empress was much amused, but some of her councillors suggested that it might be desirable to check these expositions of doctrine. The Empress did not like to put a direct muzzle on her guest's tongue, so the following plot was contrived. DIDEROT was informed that a learned mathematician was in possession of an algebraical demonstration of the existence of God, and would give it him before all the Court, if he desired to hear it. DIDEROT gladly consented: though the name of the mathematician is not given, it was EULER. He advanced toward DIDEROT, and said gravely, and in a tone of perfect conviction:

$$\textit{Monsieur, } \frac{a+b^n}{n} = x, \textit{ donc Dieu existe; répondez!}$$

DIDEROT, to whom algebra was Hebrew, was embarrassed and disconcerted; while peals of laughter rose on all sides. He asked permission to return to France at once, which was granted.

Since then, many historians of mathematics have taken up the story and repeated it, with slight variations, as a choice morsel of mathematical gossip.[3] One of them has even attempted to give it more spice by means of a few details born of his own imagination. This scholar, in his widely-read work,[4] changes "Hebrew" to "Chinese," and not content with applying that scornful term to DIDEROT's algebra, declares of CATHERINE's plot: "This was easy, because *all mathematics* was Chinese to DIDEROT."[5]

It is perhaps excusable that these chroniclers, not being students of DIDEROT, should accept unquestioningly the substance of the anecdote. That ANDRÉ BILLY should repeat it unhesitatingly, in his usually trustworthy biography of the *philosophe*, is more surprising.[6] Furthermore, it is scarcely sound historical method conveniently to omit DE MORGAN's explanation that the tale may well be apocryphal. He admits taking it from a volume generally received as trustworthy, but whose author confesses in turn "that he has no personal knowledge of the truth of the story, but that it was believed throughout the whole of the north of Europe."[7]

Analysis of the anecdote strengthens the suspicion that it is entirely fictitious. It is incredible that DIDEROT was nonplussed and silenced by EULER's supposed formula. The subject of theism was too close to his heart, his reflections on it too deep, to make it at all conceivable that he would bow before such a trick. And, indeed, he had no reason to. Algebra was *not* "Hebrew," nor "Chinese," to him; he had used it, both often and with ease, as well as calculus and geometry, in his writings.[8] An equation such as EULER's would be simple enough for him.

The dénouement is no more worthy of belief: that DIDEROT, because of

[3] Cf. A. CAJORI: *A History of Mathematics*, New York, 1919, p. 233; DAVID E. SMITH: *History of Mathematics*, Boston, 1923, pp. 522–23.

[4] E. T. BELL: *Men of Mathematics*, New York, 1937, pp. 146–47.

[5] Italics inserted.

[6] Paris, 1932, p. 567. Instead of EULER, BILLY also writes "a Russian philosopher."

[7] The volume was THIÉBAULT's *Souvenirs de vingt ans de séjour à Berlin* (1804).

[8] Curiously enough, the assertion that "algebra was Hebrew to DIDEROT" may be literally *true*, because it is possible that DIDEROT did know Hebrew! Cf. R. SALESSES: "Les Mystères de la jeunesse de DIDEROT," *Mercure de France*, 15 décembre 1937, pp. 511–12.

this humiliation, requested of CATHERINE permission to depart, and that she promptly accepted. It is well known that CATHERINE was loath to lose the conversationalist who charmed her leisure hours; that for three months he had longed to return home, because of illness and nostalgia for his family and friends.[9] Most certainly, he did not leave abruptly, nor because of any such humiliation.[10]

The origin of this anecdote apparently does not lie in any personal animosity between the two men. The only time DIDEROT speaks of EULER—and that during his Russian sojourn—is to call him "the good and respectable EULER," and no traces exist elsewhere of a quarrel.[11] When we consider how numerous were the anecdotes arising from DIDEROT's stay at St. Petersburg, it seems reasonable to assume that this particular one may be merely legendary.[11bis]

DIDEROT's mathematical work includes five *Mémoires* published in 1748, written to counteract the rather scandalous reputation he had acquired from his *risqué* novel, *Les bijoux indiscrets*;[12] and two later essays, composed

[9] Cf. M. TOURNEUX, *Diderot et Catherine II*, Paris, 1899, pp. 461, 465–66.

[10] DIDEROT's alleged atheistic conversations with courtiers may possibly have become annoying to CATHERINE; but Professor BELL's ironical assertion, that "DIDEROT earned his keep by trying to convert the courtiers to atheism," is certainly unjust (*op. cit.*, p. 146). Given DIDEROT's aversion to proselytism, and the characteristic enthusiasm of his conversation, it was more probably the courtiers who amused themselves, as did so many of DIDEROT's compatriots, by launching him on eloquent tirades. No one was more familiar with these outbursts than CATHERINE herself.

[11] TOURNEUX: *op. cit.*, p. 73 nb. 1. CONDORCET does not mention DIDEROT in his *Eloge d'Euler* (*Œuvres*, édition ARAGO-O'CONNOR, 1847, II, 1–42), nor does K. HAGENBUCH in his study, *Leonhard Euler als Apologet des Christenthums* (Basel, 1851).

[11bis] For further discussion of this story see note by DIRK T. STRUIK (*Isis* 31, 431–32).

[12] Paris, "chez Durand et Pissot," 1748, in-8°. There were no other separate editions. The titles follow:

1. "Principes généraux d'acoustique"
2. "De la développante du cercle"
3. "Examen d'un principe de mécanique sur la tension des cordes"
4. "Projet d'un nouvel orgue"
5. "Lettre sur la résistance de l'air au mouvement des pendules."

The last is a letter in reply to a question from an unknown person. In the dedication to Mme de P . . . (according to ASSÉZAT, Mme DE PRÉMONTVAL), DIDEROT declares his intention to avoid scandalous writing in the future and to remain serious.

The presence of six vignettes, in the 1748 mémoires, signed "N. BLAKEY," is at first glance somewhat mystifying and even suggests the possibility of English sources. In reality, it is not necessary to look so far for an explanation. Had ASSÉZAT known anything about the engraver, he could easily have cleared up the mystery himself. "Nous aurions été heureux," he writes regretfully, "de pouvoir donner quelques renseignements sur l'artiste auquel on doit ces élégantes compositions." (IX, 76 nb. 1). Various modern dictionaries and histories agree in affirming that "little is known" about him. Yet we do glean important bits of information: that he was Irish and not English, that he lived mostly in Paris (THIEME-BECKER: *Künstler-Lexikon*), that he had a French wife (*Nouvelles archives de l'art français*, V (1884), p. 271), and that he worked for publishers (PILKINGTON: *Dictionary of Painters*). REDGRAVE (*Century of English Painters*, London, 1866) informs us that he illustrated an edition of POPE (I, 438). These facts explain with sufficient plausibility the presence, at first disconcerting, of the "English vignettes"; doubtless the publishers alone were responsible for them.

in 1761.[13] We have, however, discovered the apparently unsuspected fact that the fourth mémoire ("Projet d'un nouvel orgue") had previously been published in the *Mercure de France*.[14] DIDEROT wrote at least one other work, but he claims to have destroyed it when forestalled by another publication; this was a commentary on NEWTON's *Principia*.[15] Finally, the article "Probabilité," in the *Encyclopédie* (later reprinted in the *Encyclopédie méthodique*), is generally attributed to him.[16] It is excluded from this study, because of its uncertain authenticity, and especially because it is not an original piece of work, but a summary of existing principles and problems, as treated by other mathematicians.[17]

The publication of the *Mémoires* earned comment in several leading journals. The notable exception is the *Journal des sçavans*, which has no mention of them, a fact that is not flattering to DIDEROT's fame. The *Nouvelles littéraires* of RAYNAL announces their publication;[18] while CLÉMENT's *Cinq Années littéraires*, the Jesuit *Journal de Trévoux* and the *Mercure de France*, in their regular scientific sections, honor DIDEROT with reviews of some length.[19]

The second mémoire—to begin with the purely mathematical works—deals with involutes and their properties. Its origin was the desire to add to the geometrical instruments of straight edge and compass a new device to permit the mechanical drawing of involutes. Why should we not have, inquires DIDEROT, an instrument to draw transcendental as well as algebraic curves, and why should such a new tool not be welcomed?[20] His inquiring

[13] "Mémoire sur la cohésion," published in the *Mémoires de Trévoux*, avril 1761, vol. II, p. 976; reprinted in the *Journal des sçavants combiné avec les Mémoires de Trévoux*, Amsterdam, LIX (mai 1761), I, 121.

"Mémoire sur le Calcul des probabilités" (not published in the eighteenth century).

[14] Octobre 1747, pp. 92–109.

[15] The other book was the commentary on NEWTON by Fathers FR. JACQUIER and TH. LESUEUR (*Philosophiae naturalis principia mathematica . . .*, Genève, 1739–42). There is an unpublished manuscript of DIDEROT's at Leningrad, titled *Premiers principes sur les mathématiques*, which may be the work in question. At any rate, the date of JACQUIER and LESUEUR's book confirms DIDEROT's early interest in serious mathematical work.

[16] Lack of signature, in the *Encyclopédie*, is supposed to identify DIDEROT's work; however, it is not a certain indication of his authorship. The article which appears in the *Encyclopédie*—general in nature, touching superficially on mathematics and philosophy—doubtless belongs to DIDEROT. The second part, however, which appears only in the *Encyclopédie méthodique*, is a later addition, and the complicated techniques involved designate it clearly as the work of another hand.

[17] This is confirmed by I. TODHUNTER, in his *History of the Mathematical Theory of Probability* (Cambridge and London, 1865, p. 260). "It gives the ordinary view of the subject. . . ."

[18] Cf. *Correspondance littéraire*, éd. TOURNEUX, 1877, I, 202.

[19] Cf. *Cinq Années littéraires*, 20 avril 1749, Lettre XXIX (quoted by ASSÉZAT, unavailable here); *J. de Trévoux*, avril 1749, pp. 602–20; *Mercure de France*, sept. 1748, pp. 133–35. The latter both praise DIDEROT as an "homme d'esprit." The *Mercure* adds: "En lisant ces Mémoires, on reconnaîtra qu'il joint à cet avantage celui d'être savant musicien, mécanicien ingénieux et profond géomètre" (p. 135).

[20] "Si l'on augmentait le nombre de ses instruments d'un nouveau compas, qui fût d'un usage aussi

mind is, as usual, seeking to enrich contemporary ideas with the ferment of a *nouveauté*.

DIDEROT's frank purpose is to interest mathematicians and mechanics in the applications of such a device. He has not himself constructed the instrument, but presents a fairly clear conception of how it should be built: he suggests a copper or steel ring to which would be attached a very thin chain. In order to demonstrate its value, he proposes to illustrate, by a series of problems and theorems, the use that could be made of involutes in the field of geometry.

DIDEROT gives us, then, a series of propositions concerning the properties of involutes, dealing with equivalence of areas. They are closely related, but do cover the problem quite thoroughly. Starting from the simplest forms, such as sectors and arcs, he works up to the more difficult and more general—to interior and exterior segments, to areas with two curved sides—and by means of involutes, reduces their areas to straight-line equivalents. He tries to present all possible sides of a problem, overlooking none of the elements it contains.

HUYGENS, long before DIDEROT, had studied the more elementary properties of involutes.[21] DE LA HIRE had made further reference to them in his paper on roulettes.[22] However, no earlier mathematician seems to have investigated the properties of involutes in relation to the particular applications in which DIDEROT uses them. It is not unreasonable to conclude that the problems studied and the theorems proposed are, as he claims, an original, as well as a sound piece of mathematical research, though of specialized nature and limited value.[23]

One of the more curious applications is to the age-old problem of the quadrature of the circle. This riddle, which has attracted geometricians from the earliest times, continued in popularity during the eighteenth century.[24] DIDEROT may well have been the first to use involutes in an attempt to solve it. Unfortunately (perhaps necessarily), he gets away from figures and equations at this point and resorts to verbal description, whereby much is taken for granted and the conclusions do not follow obviously.[25] It is more inter-

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sûr et aussi exact que celui dont on se sert pour tracer le cercle, et qui facilitât un grand nombre d'opérations; serait-elle (la géométrie) bien fondée à le rejeter?" (pp. 133–34).

[21] Cf. *Horologium oscillatorium* (1673), ch. III.

[22] "Traité des roulettes," *Mémoires de l'Académie des sciences*, 1706, pp. 369–79. DE LA HIRE touches briefly on the problem of the area and length of the involute.

[23] This judgment is corroborated by G. LORIA: "This curve—of which LA HIRE spoke in his *Traité des roulettes*—receives methodical treatment from DIDEROT in his *Examen de la développante du cercle*." (*Curve piane speciali*, Milano, 1930, II, 125, nb. 5.)

[24] In 1775, the Académie des sciences "found it necessary to pass a resolution that no more solutions on the quadrature of the circle should be examined by its officials." (CAJORI: *op. cit.*, II, 246.)

[25] Cf. pp. 139–40. G. SARTON: MONTUCLA (*Osiris*, vol. 1, 528–31, 1936).

esting to note that when DIDEROT divides an arc into a given ratio, he has given a solution of the equally ancient problem of trisecting an angle, although he does not himself refer to it.

DIDEROT lacks clarity and rigor in some of his work, but the same thing can be said of some of his contemporaries.[26] His tendency is to omit the trivial or elementary, and to insist upon the essential. Occasionally, too much condensation makes the proof clumsy, or somewhat difficult to follow. For the most part, however, his demonstrations are sound and precise. At the present day, of course, other methods would replace much of the geometrical proof here used.[27]

The *Mémoire sur le calcul des probabilités* is another purely mathematical work. It is a criticism of two mémoires of D'ALEMBERT,[28] themselves an attack on the work of another mathematician, DANIEL BERNOULLI.[29] Although DIDEROT announces that he is writing for the layman, he is occasionally obscure and repetitious, especially when he gets away from mathematical symbolism. Nevertheless, it is by the very use of figures and mathematical induction that DIDEROT is able to contradict and correct what D'ALEMBERT had attempted to prove by words and reasoning.[30]

The most important of the questions raised concerns betting odds. If Jacques bets that Pierre will not get a head in two flips of a coin, what odds should he get? The question is approached this way: there are four possibilities (*h-h*, *h-t*, *t-h*, *t-t*), three winning chances for Pierre, one losing. Now D'ALEMBERT had argued that if a head comes the first time, there is no second play, and therefore the first two combinations are reduced to one—hence there are only three possible combinations, two winning and one losing; the odds are therefore two to one.

DIDEROT finds D'ALEMBERT'S distinction between the first throw as "certain" and the second as "probable" to be metaphysical and rational, not

[26] MONTUCLA, for example, does the same thing in criticizing solutions to the quadrature problem: "toute tangente à la spirale détermine une ligne droite égale à un arc de cercle aisément assignable. A quoi tient il donc, dira quelqu'un, que l'on n'ait la quadrature du cercle? J'en ai déjà donné la raison; il faudrait pouvoir tirer cette tangente d'une manière qui ne dépendît pas de la rectification de cet arc, et c'est ce qui est impossible." (*Histoire de la quadrature du cercle.* (1754), Paris, 1831, pp. 54–55.) Even a very careful reading of this is apt to leave the meaning none too clear.

[27] These pages of ASSÉZAT'S edition have not been carefully copied or proof-read. On p. 136, fig. 5, "B" is omitted. On p. 148, prob. XII, demonstration "arc$\times AF$" should be "arc AF". P. 147, line 12 should have "3C" instead of "AC," etc. P. 171, line 5, should be "$=-a$," not "$-=a$"; p. 172, "$+fv^2/g^2$" instead of "fv/g^2".

[28] *Sur le calcul des probabilités, Sur l'inoculation*, in *Opuscules mathématiques*, 1761, v. II.

[29] BERNOULLI'S applications to gambling of the theories of probability are to be found in his *Specimen theoriae novae de mensura sortis* (St. Petersburg, 1738). Cf. also his correspondence with GOLDBACH. (P. L. F. VON FUSS: *Correspondance mathématique et physique de quelques célèbres géomètres du XVIIIe siècle*, St. Pétersbourg, 1843, pp. 198–305 *passim*). EULER had also discussed the same questions (*Opera omnia*, vol. VII).

[30] Cf. pp. 196–201.

mathematical. He declares that we must calculate the chances before Pierre makes the first throw. If we suppose P the sum of the bets, then Pierre's chances are $3P/4$, those of Jacques $P/4$, or odds of 3:1. For when Pierre is about to make his first throw, he has an equal chance for P or for another try that will assure him P or 0. The first play is worth $P/2$; the probability of getting heads at the second throw is $P/2 \times 1/2$ or $P/4$. Therefore the sum of favorable probabilities is $1/2+1/4$ compared to $1/4$ or 3:1. Modern algebra has proven DIDEROT correct.[31] Having established this principle, DIDEROT applies it to more advanced series, and again refutes D'ALEMBERT, constantly attacking as non-mathematical the latter's reasoning.

D'ALEMBERT had also argued that it is necessary to distinguish between metaphysical possibility and physical possibility. We know that many things (such as the throwing of six sevens in succession) never really occur, although they are theoretically possible. In other words, when the probability of an event is very small, we must treat it as zero. This is approximately so, but not mathematical, and DIDEROT justly inquires: "At what point will the probability cease to be nil and begin to be worth considering?"[32] Because in gambling it is frequently more practical to rely on common sense than on mathematical principles, D'ALEMBERT committed the error of denying the existence of these principles.

The next part, *Sur l'inoculation*, is a particular application of probability. D'ALEMBERT attempts to prove that however great the ultimate gain, the immediate stake of possible death is too great to make inoculation worth while. DIDEROT questions the figures and again refutes the principle of probability advanced by his former friend and collaborator.[33]

It is regrettable that DIDEROT did not publish this mémoire. A sound piece of work, it might have combatted D'ALEMBERT's prestige and dissipated certain current misapprehensions. Thus MONTUCLA, in discussing the same questions, considers D'ALEMBERT's arguments specious, but is not able to refute them.[34] This mémoire contains much of interest. It establishes DIDEROT's ability to analyze an abstract problem and to discuss it intelligently.

[31] This discussion is a paraphrase of DIDEROT's. The above problem is treated by simple probability theory at present. The chance of not throwing a head in one trial is $1/2$, in two trials, $(1/2)^2$. Therefore the chance of throwing at least one head in two trials is $1-(1/2)^2=3/4$. Cf. BARNARD and CHILD: *Higher Algebra*, London, 1936, p. 510. [32] Pp. 194–95.

[33] But he supports D'ALEMBERT's plea for the development of statistical methods and the compilation of statistics on life expectancy, mortality, etc. (Cf. pp. 211–12.)

[34] *Histoire des mathématiques*, 1799–1802, III, p. 405. CONDORCET accepts and praises D'ALEMBERT's theories on probability (*Eloge de d'Alembert, Œuvres*, II, 92–94). D'ALEMBERT himself never renounced his theory on the "heads or tails" problem; he maintained his position with varying firmness, from 1754 until the end of his life. For a summary of his various articles on the subject, cf. TODHUNTER, *op. cit.*, pp. 276–93.

We must now consider DIDEROT's mathematics in relation to various problems of physics which aroused his interest. The first mémoire of the 1748 group is entitled "Principes généraux d'acoustique."[35] The subject was at the time a common one,[36] and DIDEROT will again refer to it in 1754, in his *Pensées sur l'interprétation de la nature.*[37] This essay is, in the first place, a clear, elementary discussion of sound waves, their nature and propagation, including such things as whispering galleries and the reflection of sound. This part of the mémoire reads quite like a chapter in an out-of-date physics text.

Mathematics re-enters the discussion as soon as DIDEROT goes from sound to music. He uses logarithms to determine the intervals which will be in a fixed ratio of vibration and thus produce harmonious sounds. He uses geometry, algebra and calculus to solve various specialized questions concerning musical instruments, especially the determination of pitch and amplitude of sounds in a string and in a wind instrument, and the questions of sound relationships that arise from dividing a string into various sections by insertion of bridges. Some problems are proposed and solved mathematically.[38] All in all, it is a thorough study of sound in relation to various types of instruments: string, wind, bells. DIDEROT apparently has read much on the subject, studied it extensively and finds new questions to explore.[39]

The third mémoire, "Sur la tension des cordes," is only two pages long. DIDEROT proposes an experiment to prove that if a cord is fixed at one end and stretched at the other by a weight, second and equal weight may be substituted for the fixed point without changing the tension. The problem was at the time a disputable one. At present DIDEROT's conclusions are universally accepted, in accordance with NEWTON's third law of motion, as can be verified by reference to an elementary physics text; yet the reviewer of the *Mercure de France* politely expresses his disagreement with these conclusions: "Nous serait-il permis d'en douter, sans cesser d'avoir toute la déférence possible pour l'ingénieux auteur?"[40]

The fifth mémoire, "Lettre sur la résistance de l'air au mouvement des pendules," finds DIDEROT in disagreement with the great NEWTON. But so

[35] The *Mercure* comments: "Sa brièveté n'a point empêché M. DIDEROT d'y examiner plusieurs questions intéressantes et difficiles" (p. 133); while the *J. de Trévoux* remarks: "L'auteur appelle la geométrie et le calcul algébrique, infinitésimal même, au secours de ce que l'expérience nous en apprend."

[36] Cf. D'ALEMBERT: "Recherches sur les vibrations des cordes sonores" (1747); EULER: "Dissertatio physica de sono" (Basel, 1727). DIDEROT, using similar mathematical technique, tackles different problems, or reaches different conclusions.

[37] "Cinquièmes conjectures," II, 30–31.

[38] *E.g.* "Find the greatest speed of the string, or that which it has on finishing its first half-vibration" (p. 110). "The pulsating force being given, find the greatest displacement of the string" (p. 112).

[39] It must be noted that proposition III (pp. 94–95) does not coincide with its figure; the symbolism, too, is confusing, "A" being both a point and a force.

[40] P. 610. The *J. de Trévoux*, on the other hand, has considerable praise for DIDEROT's linking of physics and music: "L'auteur dans son premier mémoire a renvoyé les musiciens au thermomètre et au baromètre. Ici, il renvoie les physiciens au clavecin" (p. 134).

great was the respect for this genius in the eighteenth century, that DIDEROT, who was emphatic in his denial of D'ALEMBERT's theories, is extremely hesitant and diplomatic when he dares to contradict NEWTON. The latter had "proved" that the retardation of a pendulum, due to resistance of air, in falling through an arc, is proportional to that arc. DIDEROT "proves" that it is proportional to the square of that arc.[41]

DIDEROT tackles here a rather complicated problem in mathematical physics, and gives it considerable development. He composes and follows through a series of equations and integrations, and brings his problem to a solution. He not only shows ability to handle algebra and some calculus, but again gives proof of his power of mathematical analysis.[42]

The last work we have to consider is the essay, "Sur la cohésion des corps" (1761). Here DIDEROT comes to NEWTON's defense. Various physicists of the day were asserting that there must really be a double law of gravity, one for celestial bodies, and one to account for the great force apparent in cohesion. The latter might be in inverse ratio to the cube of the distance, it was suggested. DIDEROT rather cautiously casts doubt upon this hypothesis. He modestly admits that in matters so profound, one is likely to err, when not supported by experiment;[43] yet we can see that he still considers geometry as the principal means of solution of physical questions.[44]

DIDEROT is correct in his essential contention that the law of gravity is invariable, that other factors enter into cohesion, such as the nature of the surfaces, the "impulsion," etc.[45] Many of his ideas, however, show the lack of knowledge of the time; thus he believes that fermentation results from cohesion; that at celestial distances reciprocal attraction is so slight as to be considered nil.[46] As in the mémoire previously discussed, we can see the indeterminate state of physical theory, the groping, exploratory procedure. DIDEROT is even ready to concede variations in natural laws: the law of refraction is not the same, he admits, for large and for small bodies of light.[47]

As we glance back over his writings, the most obvious conclusion is that DIDEROT, as a mathematician, was essentially a geometrician. Probably as a result of teaching experience and of his manifest interest, he handles ele-

[41] According to modern physics, neither is correct: the resistance varies as the square of the velocity. However, both NEWTON and DIDEROT use approximations and arrive at a fairly close result, considering the velocity studied and the short arc traversed. NAIGEON (*Mémoires sur Diderot*, 1798, p. 129), justly criticized DIDEROT's conclusions: "mais peut-être, comme j'en fis un jour l'objection à DIDEROT, que les différences sont si peu considérables, qu'on peut prendre sans erreur les arcs et leurs carrés pour l'expression des retardations. . . ."

[42] "Une suite de calcul fort, c'est à dire analytique et intégral, qu'il manie fort bien. . . . " (*J. de Trévoux*, p. 618).

[43] DIDEROT's modesty manifests itself on several occasions, which is quite interesting in view of his complete lack of it in less scientific fields.

[44] P. 185. [45] P. 191. [46] Cf. p. 183. [47] P. 184.

mentary geometry with ease and understanding.[48] His figures, too, are neat and well constructed. Nevertheless, DIDEROT is by no means averse to using algebra. In this field, he prefers proportions and constantly employs them, as was the custom of his time. Yet he also uses quadratic and higher degree equations, and radicals. Logarithms appear occasionally, calculus rather frequently, especially simple integration; in his integration, he mixes both algebraic and geometric evaluation, which is a bit awkward. At one point, DIDEROT uses negative exponents, continued fractions and even a touch of the theory of numbers.[49] The use of continued fractions is significant, for it shows DIDEROT attempting to handle newly introduced mathematical techniques.[50] This is again illustrated by his apparent conception of π as an irrational number.[51] EULER had argued just recently in favor of the irrationality of π, a fact which was not to be proved definitely until the later work of LAMBERT and LEGENDRE.[52]

DIDEROT's notation is worthy of a brief comment. On the whole, it is fairly clear and easy to follow. We have already referred to his habit of omitting some of the steps which to him seemed obvious; he uses, too, a condensed notation where separate, expanded forms and equations would now be employed. In expression and choice of letters he is occasionally careless. He does not readily accept innovations. In proportions, for example, the new symbols were used, even by the Académie des sciences, but DIDEROT constantly adheres to the old ones.[53] He does not adopt the symbol π, but prefers to express that relationship by his own "1/*c*."[54] He uses indifferently "log." or "l."[55]

DIDEROT's tendency is conservative, but the obvious inference that he did not keep up with current trends is unjustified. The very problems that interest him, in his writings, were most current; his readings clearly include recent publications in England, France and Germany, especially the works of EULER and D'ALEMBERT which exercised the greatest influence on him. He was well grounded in the earlier mathematical literature, judging from his acquaintance with the ideas of PYTHAGORAS, ARISTOXENES, GASSENDI, HALLEY and FLAMSTEED, NEWTON and others referred to in his *Mémoires*. Although DIDEROT avoided the grand problems of mathematical theory

[48] DIDEROT, in his youth, had given mathematics lessons to support himself in Paris, after he had quarreled with his father. [49] IX, 100, 101.

[50] EULER was largely responsible for the development of continued fractions, in his *De Fractionibus continuis*, published the same year as DIDEROT's *Mémoires*. Cf. MONTUCLA: *Histoire des mathématiques*, III, 309. [51] IX, 94 ff.

[52] Cf. EULER's *Introductio in analysin infinitorum* (1748); E. W. HOBSON: *Squaring the Circle* (Cambridge, 1913, pp. 41–43); F. KLEIN: *Famous Problems of Elementary Geometry* (Boston, 1897, pp. 58–60).

[53] For an exhaustive treatment of symbolism, including references to DIDEROT, cf. CAJORI: *A History of Mathematical Notations*, esp. I, 291.

[54] *Ibid.*, II, 11; cf. DIDEROT: *Œuvres*, IX, 94 ff. [55] IX, 100

which preoccupied scholars of his period, it is none the less true that he was as an amateur keenly interested in such questions as probability, rectification, quadrature, and physical applications of mathematics—questions that also engaged his more famous contemporaries. He knew sufficient mathematics to understand and discuss abstract questions or principles and to invent new problems and situations to which he attempted to apply them. Quite definitely, his interest was less in theory than in practical application. Most of these applications were of principles he had taken from NEWTON, D'ALEMBERT and EULER.

DIDEROT created just a little ripple on the great pond. "Voilà bien des vues nouvelles," is the laudatory but unenthusiastic conclusion of the *Journal de Trévoux*. The greatest testimony in favor of DIDEROT's contemporary reputation is that twenty-five years later an Italian friend of D'ALEMBERT, writing his *Eloge*, speaks of DIDEROT's many talents, including that "di geometrizzare sulle proprietà delle curve, sulle vibrazioni delle corde sonore et sulle resistenze dei pendoli."[56]

DIDEROT's later disparagement of mathematical science assumes significance only in the light of his own interests and abilities. His censure falls, first, on theories of probability. He agrees with D'ALEMBERT in finding that branch of the science in a confused state, and in declaring that a satisfactory theory presupposes the solution of several questions which possibly are insoluble.[57] Then he continues with a general criticism of mathematics: "Toute la science mathématique est pleine de ces faussetés. . . . D'où naissent les *incommensurables?* l'impossibilité des *rectifications* et des *quadratures?* C'est la fable de Dédale. L'homme a fait le labyrinthe et s'y est perdu."[58] DIDEROT was impressed by the point that mathematics is a man-made science, and that starting with any given set of suppositions, consistency will be the only criterion of truth.[59] The more he studied mathematics the more convinced he became that their "reign was over and the reign of natural science about to begin."[60] DIDEROT thus mirrors accurately the general change in outlook since the beginning of the century, when FONTENELLE, for instance, prefaced his *Eloges des académiciens* with a discourse *Sur l'utilité des mathématiques*.

It is notable that DIDEROT the philosopher intrudes frequently on DIDEROT the mathematician. Even in this objective and technical work, he is unable to put aside entirely his ubiquitous moralizing. In the mémoire on acoustics, he digresses to write upon the obstacles to scientific progress,

[56] PAOLO FRISI: *Elogio del Signor d'Alembert*, Milano, 1786, p. 38.

[57] Such as determination of the point when probability is slight enough to be treated as nil, fixing of relations of probability in impossible cases or cases of unequal possibility, etc. Cf. p. 197.

[58] P. 203. [59] Cf. *Pensées sur l'interprétation de la nature*, II, 10.

[60] "Nous touchons au moment d'une grande révolution dans les sciences . . . j'oserais presque assurer qu'avant qu'il soit cent ans, on ne comptera pas trois géomètres en Europe" (*ibid.*, II, 11).

which, he declares, are twofold. There is, first, general laziness or discouragement of the mind: "plus la cause d'un phénomène est cachée, moins on fait d'efforts pour la découvrir."[61] The second obstacle is that particular manifestation of human vanity "qui aime mieux s'attacher à des mots, à des qualités occultes, ou à quelque hypothèse frivole, que d'avouer de l'ignorance." Needless to say, DIDEROT is right on both scores.

In the same work he discusses at some length the source of pleasure in musical appreciation. The principle there evolved, of the perception of relationships, he then applies to all esthetic pleasure. Again the discussion is of considerable interest, but is none the less a digression.[62]

In the essay on inoculation, he attacks D'ALEMBERT for assuming a selfish, individualistic attitude. DIDEROT writes on the contrary from a social point of view and he urges that individual risks must be disregarded in favor of the health of the nation. This, of course, is in accord with his general conception of virtue as an individual sacrifice which, by assuring the general welfare, eventually brings greater happiness to those who have sacrificed their immediate and personal instincts.[63] DIDEROT's conclusion concerns the reluctance with which a novelty is accepted, however good and useful it may be. He warns D'ALEMBERT that their grandchildren will shake their heads sadly on reading his inept remarks and exclaim: "Le bien a donc beaucoup de peine à s'introduire dans le monde!"[64] Once more, DIDEROT had foreseen a definite line of progress.

Even in the first part of the mémoire on probability, he could not refrain from philosophizing. If men refuse to wager large sums where the risk is great, though the possibility of gain is tremendous, that does not justify D'ALEMBERT's principle of the reduction of slight probability to nullity. It merely proves that there are games which are not meant for men, and men who are not made for gaming.[65] D'ALEMBERT's proposition is not only false, but contrary to the constant practice of gamblers and merchants. Those who make a fortune, moralizes DIDEROT, "n'ont autre supériorité sur les autres que de discerner une petite probabilité et que de l'ôter à leurs concurrents. A la longue, ceux qui négligent les petits avantages se ruinent."[66]

He holds that the analysis of probability may be considered either as an abstract science or as mathematical physics. In the first case, every problem is solved in the head of the mathematician; time and the smallest finite quantities are given infinite value, and all combinations become possible.[67] In the second case, phenomena and chances are considered from the view-

[61] P. 115. [62] Pp. 85, 104.

[63] To do D'ALEMBERT justice, we must read his entire mémoire. Essentially he is in favor of inoculation, and attempts to dissipate many prejudices opposing it. In view of the risks involved, however, he holds that we have no right to urge, command or even to advise others to undergo treatment. Cf. *Œuvres* (1805), v. IV, *passim*. [64] P. 212. [65] P. 205. [66] P. 206.

[67] Cf. p. 203: "Si l'éternité multiplie le moindre degré de vraisemblance, le produit égalera la plus énorme vraisemblance multipliée par l'instant qui fuit."

point of man, "a being who passes like lightning and who relates everything to his own duration." DIDEROT thus applies to mathematics his principle of relativity, and extends it to include the possibility of there being two truths, entirely contradictory.[68]

Everywhere in DIDEROT's mathematical writings, there is evidence of his typical inquiring attitude, of the fertilizing intent of his suggestions. He seizes opportunities of relating the sciences.[69] Practical technical applications, we have noted, seem to interest him at least as much as theory. He is constantly seeking ideas for improvements, and in each mémoire an experiment or invention of some kind is suggested. We have discussed the new compass he proposes. Elsewhere he strives to apply the laws of sound to the construction of musical instruments.[70] He suggests improvements, amounting to a new invention, on the currently used orchestral tuning pipe, in order to allow correction for temperature and barometric pressure.[71] The fourth mémoire consists entirely of a projected invention: a new type of hand organ which can be played equally well by musicians and those who are ignorant of that art.[72] This is a significant tendency of DIDEROT's mind: even before the *Encyclopédie* was launched, he reveals a keen interest in the technical problems of the crafts.

In DIDEROT's intellectual development, mathematics is a starting point which led him directly to physics, biology and philosophy. This evolution is apparent in his very criticisms of mathematics, which were inspired partly by his limited talents, partly by a growth to new fields of interest. It is evident in his first original work, the *Pensées philosophiques*, where the principal argument in support of an atheistic explanation of the universe is based on mathematics. Given an infinity of time, he argues, and an infinity of atoms, the self-creation of the universe is not only possible, but necessary (*Pensée* XXI). DIDEROT's synthetical mind did not stop at the equations before him; mathematics became related to philosophy, to physics, to experimentation and to life.

This was, indeed, a not uncommon result of the eighteenth century intellectual atmosphere. Knowledge was not yet divided into sharply specialized spheres. Thus EULER and DANIEL BERNOULLI, in their correspondence, discuss applications of mathematics to political science, morals and physiology.[73] Physical questions were closely bound to theological. When D'ALEM-

[68] The mathematicians' calculations of probable chances are based on a game lasting through all eternity (hence being an eternal solution). If you apply it to Pierre and Jacques' game, you are applying an eternal solution to a reconstructed game lasting but a moment—applying the indefinite to the determinate. [69] Pp. 128–30. [70] P. 110.

[71] Pp. 128–30. The *J. de Trévoux* thinks little of the proposed invention. "Il n'est pas douteux que cela ne menât à une plus grande perfection. Il reste à savoir si la peine . . . de s'y conformer n'est pas un plus grand inconvénient que le manque de cette exactitude ne l'est dans la pratique, où tout va par des à-peu-près suffisants" (p. 608).

[72] The same journal ridicules this project as useless and impossible (pp. 643–45).

[73] Cf. P. H. VON FUSS: *op. cit.*, II, 495–96, 541–42.

BERT approaches the problem of the contingency or necessity of mechanical laws, he reduces it to the following question: "Has God made uniform laws or has he reserved the right to change them?"[74]

Further proof, if needed, is supplied by NAIGEON, one of DIDEROT'S closest friends:

DIDEROT partagea son temps entre l'étude des langues anciennes et modernes, et celle des mathématiques, dont l'inexorable précision lui plut extrêmement et lui inspira du dégoût et même du mépris pour la théologie . . . Il entrevit que c'est la physique et la physiologie qui peuvent seules donner quelque base à l'analyse du métaphysicien.[75]

Thus the evolution of DIDEROT's mind led him from mathematics to physics, thence to general questions about matter and the universe—that is, to metaphysics—and finally to biology and physiology which were to be permanent interests. Mathematics, then, were not merely DIDEROT's first love; they were the springboard from which he rose to the realms of science and philosophy. By studying their rôle, we have tried in some measure to fill one of the many gaps in the history of his intellectual formation.

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[74] Cf. J. BERTRAND: *D'Alembert*, 1889, pp. 38–40. Significantly, BERTRAND sees in this a possible influence of DIDEROT. [75] *Op. cit.*, pp. 5, 9.

The Silphium of the Ancients: a Lesson in Crop Control*

By ALFRED C. ANDREWS

GREEK σίλφιον is apparently a loan word from some non-Indo-European source, perhaps Cyrenaica, the homeland of the plant.[1] Latin *sirpe*, similarly used as a term for the plant itself, probably came independently from the same source.[2] *Lasserpicium*, a term for the resinous juice of

* Of the numerous articles and monographs on silphium, the following are noteworthy: B. BONACELLI' Il silfio dell' antica Cirenaica, Roma, Libr. d. Stato, 1924; Il silfio cirenaico e l'asiatico in una nuova interpretazione di Teofrasto, Riv. Trip., 2 (1925–26), pp. 183–193. M. D. CAURET, Sur le silphion, thèse, Paris, 1884. E. KÜSTER, Noch einmal die Silphionfrage, Natur, 7 (1912), pp. 588–590. VICTOR HEHN, Kulturpflanzen und Haustiere, 8th ed., 1912, pp. 193 ff. STEIER, PAULY-WISSOWA-KROLL, Realencyclopädie der classischen Altertumswissenschaft, Second Series, III, 103–114; Supplement V, 972–974. ELSE STRANTZ, Zur Silphionfrage, kulturgeschichtliche und botanische Untersuchungen über die Silphionpflanze, Berlin, 1909. C. TEDESCHI, Il silfio, un enigma nella storia botanica della Cirenaica, Rivista delle Colonie Italiane, 3 (1929), pp. 1276–1292.

[1] The most recent detailed discussion of the origin of σίλφιον and *sirpe* is in GIOVANNI NENCIONI, Innovazioni africane nel lessico latino, Studi Italiani di Filologia Classica, 16 (1939), pp. 30–32. The *l*:*r* alternation is a not uncommon feature of languages in the eastern and western Mediterranean region, and the