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CAPITAL MARKET EQUILIBRIUM AND THE TERM STRUCTURE OF INTEREST RATES

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INTRODUCTION

Most of the literature on the term structure of interest rates has made use of models that are inconsistent with capital market equilibrium. Various market imperfections are assumed that make individual issuers of debt securities or holders of debt securities prefer one maturity to another. Hicks [1], for example, assumes that the supply of bonds is fixed, or constrained, and that investors' demands for bonds of different maturity cause rates of return to vary with maturity. Even given a fixed supply, he does not consider investors' holdings of bonds in a context with their holdings of other assets. Kessel [2] assumes that liquidity differences make bonds of different maturity attractive to different investors, and also fails to think of bonds as held in portfolios containing other assets. Modigliani and Sutch [3] assume that both borrowers and lenders have various institutional reasons for preferring one maturity to another, and that the interaction of supply and demand schedules as a function of rate and maturity determines the term structure. None of these writers explains adequately why intermediaries do not spring up who can convert the maturities that borrowers prefer into the maturities that lenders prefer at zero economic cost.

While market imperfections may be important in explaining certain aspects of the observed term structure, we would certainly like to have a theory of the term structure that holds when there are no market imperfections. The effects of market imperfections can then be explored in the context of this theory.

Roll [4] has begun the development of a theory of the term structure that is consistent with capital market equilibrium, by applying the capital asset pricing model developed by Treynor [5], Sharpe [6], Lintner [7], and Mossin [8] to bond prices. Unfortunately, he is not able to obtain either a complete theory of the term structure that is consistent with the capital asset pricing model, or an adequate empirical test of the theory.

The capital asset pricing model states that under certain assumptions, the expected return on any capital asset for a single period will satisfy:

$$E(\tilde{R}_i) = R_f + \beta_i[E(\tilde{R}_m) - R_f] \quad (1)$$

(One of the problems in Roll's analysis is that he states the capital asset pricing model incorrectly.) The symbols in equation (1) are defined as follows:

$\tilde{R}_i$ The return on asset i for the period. The return is the change in the price of the asset, plus any dividends, interest, or other distributions, divided by the price of the asset at the start of the period.

$\tilde{R}_m$ The return on the market portfolio of all assets taken together.

R_f The return on a riskless asset for the period.

β_i The "market sensitivity" of asset i. It is equal to the slope of the regression line relating $\tilde{R}_i$ and $\tilde{R}_m$.

The market portfolio used in defining $\tilde{R}_m$ should in theory include all assets: common stocks, bonds, real estate, privately held businesses, human capital, and so on. There is reason to believe, however, that the return on a market portfolio of common stocks alone, or of common stocks plus bonds, will be very highly correlated with the return on the theoretical market portfolio that contains all assets. Thus equation (1) will hold approximately if a market portfolio of common stocks, or of common stocks plus bonds, is used instead of the theoretical market portfolio.

A riskless asset is a pure discount bond (with no coupons), that has no default risk, that matures at the end of the period. When the period is very short, the rate of inflation will be known in advance, and the asset will be riskless in both real and nominal terms. The return on any asset, however, is meant to be a nominal return, not a real return.

The market sensitivity of an asset is a measure of that part of the asset's risk that cannot be diversified away by combining it with other assets in a portfolio.

In market equilibrium, it is only this part of the asset's risk that influences its expected return. This is very important in analyzing bonds, because all or almost all of the risk in a long term bond can be diversified away. Thus in market equilibrium, most or all of the risk in a bond has no effect on its expected return. The market sensitivity of an asset is defined algebraically as follows:

$$(2) \qquad \beta_i = \mathrm{cov}(\tilde{R}_i, \tilde{R}_m)/\mathrm{var}(\tilde{R}_m)$$

The assumptions that are generally used in deriving equation (1) are as follows:

(a) All investors have the same opinions about the possibilities of various end-of-period values for all assets. They have a common joint probability distribution for the returns on the available assets.

(b) The common probability distribution describing the possible returns on the available assets is joint normal.

(c) Investors choose portfolios that maximize their expected end-of-period utility of wealth, and all investors are risk averse. (Every investor's utility function on end-of-period wealth increases at a decreasing rate as his wealth increases.)

(d) An investor may take a short position of any size in any asset, including the riskless asset. An investor may borrow or lend any amount he wants at the riskless rate of interest.

(e) There are no taxes, transactions costs, or other market imperfections.

The length of the period for which the model applies is not specified. The assumptions of the model, however, make sense only if the period is taken to be infinitesimal. For any finite period, the distribution of possible returns on an asset is likely to be closer to lognormal than normal; in particular, if the distribution of returns is normal, then there will be a finite probability that the asset will have a negative value at the end of the period.

The model is derived as a single period model; but in principle it is easy to generalize to a multiperiod model. We can simply say that it must apply to each successive infinitesimal period in time.

The riskless asset, in this context, may be thought of as a savings account with an interest rate that fluctuates continually. If an investor has a short position in the riskless asset, then he has what amounts to a demand loan with a fluctuating interest rate.

Given the assumptions underlying the capital asset pricing model, it must apply to all assets, including bonds of different maturities. We will start our analysis, then, by assuming that equation (1) applies to all bonds, when the length of the period is taken to be infinitesimal.

HOLDING PERIOD RETURNS

The return $\tilde{R}_i$ on a bond in equation (1) is a "holding period return," and has little to do with the bond's yield to maturity. Two bonds that mature at different times, or have different coupons, or have different amounts of default risk may have different yields to maturity, but if they have the same β's, they will have the same expected holding period returns.

If we regress the return on an index of government bonds against the return on a market portfolio containing both stocks and bonds, we find that the β of the bond index is very near zero. Given the uncertainty in our estimates of β, we cannot reject the hypothesis that the β's on all government bonds are equal to zero. It will be convenient, and a good approximation to reality, to assume that the β's of all bonds that have no default risk are equal to zero.

This means that the expected return on any bond free of default risk is equal to the return R_f on the riskless asset. The expected return on such a bond is independent of its maturity.

RISK PREMIUMS

Assuming that the β's of all bonds are zero, there should be no risk premiums. Investors should be indifferent to the greater risk in long term bonds, because it is risk that can be diversified away.

MARKET SEGMENTATION

Under the assumptions of the capital asset pricing model, there is no reason for market segmentation. An investor cares only about the expected return and risk of his portfolio. Indeed, as shown by Sharpe [6] and others, every investor holds the same portfolio of risky assets, including long term bonds, and mixes it with a long or short position in the riskless asset to suit his risk preferences. Financial intermediaries and firms are completely indifferent to the term structure of their assets and liabilities, because the investors who hold their shares can offset any undesirable characteristics of these shares by taking appropriate long or short positions in bonds of different maturity.

TRANSACTIONS COSTS

Let us assume that the transactions costs are lower on short maturity bonds than on long maturity bonds, as observed by Malkiel [10]. Then Kessel [2] and others have argued that the returns on shorter maturity bonds will be lower than the returns on longer maturity bonds, because shorter maturity bonds have value as liquid assets. They can be sold at low cost to meet sudden needs for cash. Given a choice of holding a short maturity bond or a long maturity bond offering the same return, an investor will choose the short maturity bond because of its greater liquidity.

But an investor does not have to hold a short maturity bond to be able to sell a short maturity bond to meet a sudden need for cash. He can always sell short: that is, he can borrow at the short term rate. A lender will be happy to take his long term bond as collateral, and to lend him a very large percentage of its value. It is true that transactions will be made by buying and selling the shortest maturity bonds (the riskless asset), but this need have no effect on an investor's holdings of risky assets.

CONCLUSIONS

Under conditions of market equilibrium, the expected returns on bonds of all maturities will satisfy equation (1). If the risk in a bond is entirely independent of the risk in the market portfolio, then the expected returns on bonds of all maturities will all be equal to the return on an ideal savings account. There can be neither risk premiums, nor habitat premiums, nor liquidity premiums in long maturity bonds.

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