HOW TO USE THE OPTION FORMULA IN PRICING CORPORATE BONDS

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Myron Scholes and I, as most of you know, worked out a formula for valuing options, warrants, and similar securities. It is a relatively simple formula to those who are familiar with mathematical notation. I won't bother to write it out, but these are the things that the formula depends on: the value of an option depends on the price of the stock--that is "X;" "T" the time that it is now; "C" the exercise price of the option; "T*" the time that the option matures; "R" the interest rate (and in our development, we assumed that the short-term interest rate was known and was constant through time, so we don't have any term structure or anything in here): and "V" a measure of the volatility of the stock (this is the standard deviation of the return on the stock; it is the total variability of the stock price; it is not a measure of the beta or the relation between the stock price movements and movements in the market).

We have done a lot of tests of the option version of this formula on various kinds of data: on warrants, over-the-counter options, and now the CBOE options--and it works well.

The tests are of two kinds: first of all, you can compare the predicted values of the options with the actual prices. In many cases, you get a remarkable correspondence between values and prices. For example, on the CBOE, if we use the same volatility estimate for a given stock that the market seems to be using, say, on Polaroid, which may have twelve different options outstanding at a given point in time, then you can tick right down the options. Our values and the options' prices will frequently be right in line for almost all of the options. So you often get a good correspondence. When there isn't a good correspondence, when the prices deviate from the values that the formula gives, then the simulations that Dan Galai and others have done indicate that, if your transaction costs are not too large, you make a lot of profit by taking the indicated positions--buying the undervalued options and selling the overvalued options. So, with both those kinds of tests, the formula seems to be borne out by the data.

Now, the first thing I want to try to explain is how you can apply just this simple option formula to the simplest kind of corporate bond. It is just a matter of re-interpreting the variables. Suppose you have a bond that has just one final payment and no coupons--a pure discount bond, you might call it. Then what do you have? You have a firm that is worth something at any given point in time. The value of the firm changes over time, much like the value of the stock changes the option formulation. And nothing really happens until you get to the maturity of the discount bond because there are no coupons. There are no payments to be made in the middle; so there is no chance of default on the bond--at least default from not paying the coupons--and so you don't ultimately know what is going to happen until the end.

At the time the bond matures, there is, in principle, a rather simple formula for what the stockholders get and what the bondholders get. If the bond has a face value of $100 million, then, if the firm is worth more than $100 million at the time the bond matures, the bondholders get their $100 million and the stockholders get what is left.

If the firm is worth less than $100 million when the bond matures, then the stockholders get nothing in this formulation and the bondholders get whatever the firm is worth.

In actual practice it doesn't quite work that way and you can take these practical things into account. For example, in Equity Funding, where there are bonds outstanding and the company is in trouble and it turns out that the company is probably worth less than the face value of the debt liabilities, but the rules of the game have somehow been interpreted as saying that the stockholders ought to get something; so, even though the bondholders are not going to be paid off fully, the stockholders are going to get something. This simple priority rule that you think of as being the basic rule in dealing with creditors and stockholders isn't always followed to the letter. Nevertheless, in this simplest problem that we are looking at here, we will assume it is.

We are now going to interpret this formula differently. We are going to take "X" to be, instead of the stock price, the value of the firm--the total value of the firm--and you can think of that as either the value of the assets of the firm or as the sum of the values of the liabilities. If there are stock and bonds in the firm, then it is the value of the stock plus the value of the bonds and, just as the book value of the assets has got to equal the book value of the liabilities, the market value of the assets has got to equal the market value of the liabilities, too, or there is something wrong with your analysis of the market value of the assets. So "X" is the total market value of the firm. "T" is time, just as before. We will interpret "C" as the face value of the bond--this pure discount bond. "T*" is the time that the bond comes due. "R" is still the known, constant, short-term interest rate. And "V" now is a measure of the variability of the value of the firm, or the variability of the value of the firm's assets, rather than just the variability of the firm's common stock. The analogy here is, the stockholders have the equivalent of an option on the value of the firm. It is really just like an option and maybe this will help make it clear. So, in the case where, at the end, the value of the firm is greater than the face value of the bonds, the stockholders get X minus C , and the bondholders get C , the face value of the bond. In the case where the value of the firm is less than the face value of the bond, the stockholders get nothing; the bondholders get the value of the firm.

Well, looking at the stock part, this is like an option. When you have an option, at the time the option matures, if the stock price is greater than the exercise price, then the option is worth the difference between the two, or X minus C . If the stock price is less than the exercise price, the option is worth zero. So, the common stock of a firm which has a pure discount bond outstanding, assuming that the variability of the firm's assets is known and constant, has a value that comes right out of the same option formula.

Now, once we see what the value of the stock is, or the value of the bonds are, at time T* , at the end, then we can figure out the value of the stock as a function of the value of the firm at all earlier times, too. And the key to the way we do that is to note, first of all, that the only variables in the way we have set the problem up are X and

$\dot{T}$; these other things, C , T* , R , and V are all constants, they are all assumed to be known and constant. So the only thing that is affecting the value of the stock and therefore affecting the value of the bond is the value of the firm and time. At a given point in time, if you know the value of the firm, you know the value of the stock and the value of the bond. And so you can, in theory, set up a riskless hedge, say, by going long in the stock and short in the bond, or long in the bond and short in the stock--in the right proportions-- and continually changing those proportions as the value of the firm changes. If you have a riskless position, you have to earn the interest rate on that position if prices are in equilibrium; and that is how you get a formula for the value of the stock in terms of the value of the firm.

Now, you can recast this formula. Instead of thinking of the value of the stock as depending on the value of the firm, and the value of the bond as depending on the value of the firm, you can think of the value of the bonds as just depending on the value of the stock and time. Note what is left out of this: Since I am assuming that the interest rate is constant here, you do not have the bonds fluctuating in value because of the changes in interest rates. What we are trying to get at here is the effects of possible default on the value of the bonds, and we are ignoring for the moment the effects of changes in interest rates on the value of the bond. So, at this stage of the analysis, we are just trying to figure out the discount in the value of the bond due to the possibility of default, or the increase in the yield to maturity of the bond due to the possibility of default.

With those assumptions, you can use the price of the stock as a measure of the likelihood of default on the bond. Naturally, the lower the price of the stock, the more likely it is that the final payment on this bond will not be made and the larger the possible size of the default will be.

Again, note that for the bond as well as for the option, the price of the bond, or the discount due to the possibility of default, depends on the total variability of the value of the firm; not on the beta of the firm. Any source of risk will give you the possibility of default on the bond; it doesn't matter whether it is market risk or risk that is independent of the market. And note, also, that I haven't had to talk here about the cash flow of the firm, or its credit rating, in any formal sense. The only variable in here is the value of the firm, or the value of the stock, and this "V," the variability of the value of the firm. Those are the key ingredients here in figuring the quality of a bond. The basic idea is, if the firm is worth more than the face value of the bond at maturity, then there is a way to get the money to pay the bondholders. The firm may not have the money in its checking account or in its current assets or anything like that; but the firm has assets that, if necessary, can be sold to get the money to pay the bondholders.

When you get into the analysis of complicated, real world bonds, some of these other considerations do come in. This is by far the biggest consideration, however, in determining the quality of a bond--the relationship between the value of the firm and the amounts that the bondholders are due to be paid at various points in time.

Now we have a formula for this simple problem, which is easy to calculate on the computer. So this simple bond problem, we can say, is already solved, namely, the case of a single-payment bond, or a pure discount bond.

What we are now making progress on is the more complicated cases. Let's consider, for example, a bond with coupons. Let's take the simplest case of that, a one-coupon bond. And let's assume again that the risk of the assets of the firm is going to be constant, that is, let's say, the firm is worth $200 million and the change in value of the firm is such that the standard deviation of the percentage change in value is 20% per year. Now, when we say that we assume that the variability of the firm stays constant, we mean such things as, if the firm sells half of its assets, then the percentage variability in the value of the firm will stay the same. It will still have a standard deviation of 20% per year, even though the dollar variation in the value of the firm will be half of what it was because the firm is only half the size it was.

Now we have two dates. We have the date that the coupon is due and we have the date of the final payment. We will assume that, when the coupon comes due, the firm will get the money to pay the coupon by selling a proportional part of itself, by selling some of its assets. There are other ways that the firm might get the money to pay the coupon. For example, the firm might issue some kind of securities. It might issue subordinated bonds to get the money to pay the coupon on the oustanding bonds. It might issue common stock. The stockholders have an interest in that and, in general, the rule is that the stockholders are better off if the firm sells assets to get the money to pay the bondholders than if the firm issues more common stock, let's say. So, since the stockholders would generally prefer that, I will assume that that is what is done--that the firm sells assets to pay the coupon.

So, when you come up to the coupon date, what happens is, if the value of the assets of the firm is at least as large as the coupon that is due, then that payment can be made. If necessary, most of those assets can be sold to make the payment, but somehow, that payment can be made. And we will assume that it is made. Then the value of the firm's assets drop by the amount of that payment. And then the value of the assets fluctuates according to the usual pattern after that, until it comes time to make the final payment. Again, we have an analysis similar to the earlier one, where, if the value of the firm at the time of the final payment is worth as much as that payment, then the stockholders get what is left and the bondholders are paid off. And, if it is worth less, then there is a default by the difference between the value of the firm and the payment that is due.

How do we apply the option type of analysis to this situation? Well, first we look at the time just after the first coupon is due. Just after the first coupon is paid, if it can be paid, we have a pure discount problem--there is just one final payment left. So we know what the value of the stock is as a function of the value of the firm just after that coupon is paid and from that we can figure out what it is just before the coupon is paid. If the firm was worth more than the coupon, then it is the option formula applied to what is left after you pay the coupon. If the firm is worth less than the coupon, then it is zero.

So, again we have an option problem, where the only relevant variables are the value of the firm and time, but where the end point isn't just the value of the stock minus the coupon any more: it is an option value.

So, we can use the same basic mathematical techniques for solving the problem, working backward. We start with the last payment that has to be made; we use the simple option formula to figure the value just after the next to the last payment is made; then we work backward, using the same basic techniques to find the value of a bond that has two payments; then we work backward from that to find the value of a bond that has three payments, and so forth.

For some problems of this sort, we will be able to get, but do not now have, relatively simple formulas like the basic option formula--analytic solutions, that is. For other problems of this sort, we will have to use numerical methods to find the solutions. We will just have to grind away with the computer. One of the problems in this is that it can get expensive to grind away with the computer at problems of this sort because you are having to analyze the possible values of the stock and the bond for all possible values of the firm, as you work back. There get to be a lot of different numbers that the computer is handling and it can get to be expensive. So part of the challenge here is to figure out ways of doing it on the computer that are not too expensive. The same general technique can be used on such things as convertible bonds, adding the conversion feature to the bond, putting in dividends on the stock, and complications of that sort. Actual bonds have all sorts of complications and some of those we don't yet know much about how to handle. The biggest one, of course, is the fact that interest rates are not known and constant; that they are uncertain; that they may change over time; and the whole yield structure changes. In principle, I don't believe that is going to be hard to put into this formulation; but from the point of view of computational problems, it is going to magnify the problems immensely. If we have trouble solving problems like this on the computer, then, if we put in changes in the term structure over time, we are going to have ten times as much trouble solving these problems at reasonable cost. But obviously, that is something that has to be put in; it is a very important part of bond valuation.

Another thing we don't know how to handle very well is the effects of taxes on deep discount bonds. Maybe we will find a way, but we don't know now.

Some other things that have to be put in are the possibility of changes in the nature of the firm, or in the terms of the relationship between the stockholders and the bondholders that will affect the value of the bonds. For example, the variability of the firm's assets affects the value of the bonds; the riskier the firm's assets, in general, the lower the value of the bonds. So the stockholders have an incentive to get the firm into riskier businesses because that will tend to reduce the value of the firm's bonds. It turns out that, if you reduce the value of the firm's bonds and leave the value of the firm unchanged, you increase the value of the stock. It is similar to the problem you have when you have warrants outstanding, except there, it works the other way; when a firm has warrants outstanding and the warrants are due to expire in 1975 and the company decides that it will just simply extend the expiration date until 1980, that increases the value of the warrant and reduces the value of the stock. This is something that isn't always recognized. And so, we have to deal with problems like that in worrying about the valuation of bonds that extend over 25 years. There is a long time for stockholders and others to figure out things to do to hurt the bondholders, or possibly vice versa.

Well, I think I have covered, in general, most of the things that we know now. Clearly, there is a lot of work still to be done.