

Problems
15.440 Advanced Topics in Financial Markets and Institutions
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Capital Market Equilibrium

1. Assume that all the assumptions of the capital asset pricing model hold, except that there is a tax on interest income at a constant rate, and a subsidy on interest expense at the same rate. If there is a riskless real asset, the returns on that asset are not taxed. How will the equilibrium be changed? Now assume that there is a tax on the excess return on an investor's portfolio, whether realized or unrealized, with a subsidy at the same rate when the excess return is negative. Assume the revenues from the tax or the cost of the subsidy are distributed evenly across investors. How will the equilibrium be changed this time?

2. Assume that securities prices are always correct and are always equal to the present value, properly computed, of the joint distribution of possible future dividends. Assume that the return on the market portfolio is lognormal, with constant expected return and variance rate. Write a discrete time model for dividends on the market portfolio that will give a standard deviation for percentage changes in dividends strictly less than the standard deviation for percentage changes in the level of the market. What are the key features of your model? What might cause dividends to behave in this way?

3. Assume that individuals have identical time-separable constant relative risk aversion utility functions on total consumption (but not leisure). Assume that their utility functions are state-independent. There are two real investment opportunities: a riskless asset with constant returns to scale, and a risky asset with constant stochastic returns to scale. The return on the riskless asset is constant, but the expected excess return per unit standard deviation of the risky asset is state dependent. Can we estimate the degree of risk aversion in the utility function from the average excess return on the market? Given the time discount rate and degree of risk aversion in the utility function, is there any possible feature of the time series of total consumption that cannot be matched using the right state dependence in the opportunity set?

4. Assume that tastes and technology are the same as in problem 3, except that the utility function is allowed to be state-dependent. Is there any possible feature of the time series of total consumption that cannot be matched using the right state-dependence in both the utility function and the opportunity set? Can you think of any observable state variables for tastes? For technology?

5. Describe some possible tests of the simple capital asset pricing model (with constant parameters). What are the advantages and disadvantages of each? How accurate do you expect the best test to be? Now suppose that firms in the aerospace business have higher expected returns on average than predicted by the CAPM. Will the best test pick that up? Is there another test that will pick it up?

6. Assume a one-period world in which all the assumptions of the capital asset pricing model hold but one: the supplies of available assets are not fixed. Each person starts with an endowment of wheat, and can plant it using any of a number of technologies. There is a riskless technology that returns 1.03 bushels for each bushel planted. Asset 1 is the total investment in risky technology 1. The joint distribution of the returns on investments in the different technologies is independent of the number of bushels planted in each technology. After all investment decisions are made, the portfolio of all risky assets has an expected return of 12%. (The market portfolio may contain both risky and riskless assets.) Will a version of the capital asset pricing model hold in this world? If so, how much can you say about the relation between the expected return on asset 1 and its β ?

7. Assume a one-period world like the one in problem 6, except that the expected return on the portfolio of all risky assets is not known. A very wealthy person takes so much of a liking to a special risky asset that he is willing to hold a huge long position in it, even though it has a negative expected return and would not have been included in the portfolio of all risky assets. All other investors are willing to hold short positions in the special asset. If we omit the wealthy person's holdings, the market portfolio will contain a large negative position in the special asset. Will total holdings of the special asset be positive, so that it is part of the overall market portfolio? From the point of view of all other investors, will a version of the capital asset pricing model hold? What kind of β will the special asset have? Are the other investors made better off or worse off by the existence of the very wealthy person?

8. Assume that all the assumptions of the capital asset pricing model hold except that only one person is allowed to borrow. How will the expected returns on securities differ from what is predicted by the capital asset pricing model? How will the interest rate be affected? What kinds of portfolios will most individuals who cannot borrow hold? What kind of portfolio will the individual who can borrow hold if he is very tolerant of risk? What kind will he hold if he is very averse to risk? How will fluctuations in the degree to which borrowing is restricted affect the economy?

9. Suppose that the government wants to prevent people from taking too much risk, because it believes that they don't fully understand the losses they may incur. Assume that real investments are fluid, so society can shift between safe and risky investments easily. The government sets a limit on the total risk, measured in dollars or percentages, that each individual can take. Will the capital asset pricing model hold in this world (assuming that it would hold without these restrictions)? What kinds of portfolios will people hold? Will the restrictions affect the interest rate or the expected return on the market portfolio of all real investments?

10. Assume that securities markets are efficient, and that changes in the money stock are associated with both past and future changes in the general level of economic activity. Assume that the current level of the money stock is widely known, without error. What relationships might we find between changes in the money stock and past, present, or future returns on the market portfolio?

11. Assume that markets are efficient in the sense that all investors have the same information, including information about the process the managers of a firm are going through in making investment decisions. The managers of one firm decide to save themselves the effort of making investment decisions in the following way. Each time an investment is being considered, they will announce a decision to make that investment. If the firm's stock price goes up, they will go ahead with the investment. If the firm's stock price does not go up, they will reverse the decision. Will the firm be able to make optimal investment decisions this way? Will they be optimally timed?

Corporate Finance

12. As a second approximation, what factors might influence a firm's optimal and actual asset and liability structure?
13. Assume no inflation-indexing is allowed. Can you think of a set of sufficient conditions for the before-tax real interest rate to be independent of the rate of inflation? Under your conditions, will the rate of inflation affect the optimal capital structure? What kinds of taxes can be used in your world?
14. Why is it common for a steel firm to buy coal under a long term contract? Why doesn't it just buy what it needs in the spot market?
15. Assume that depositors in a mutual thrift institution receive the same interest rates as depositors in a stock institution, and that they have no control over the affairs of the institution. Who bears the risk in a mutual institution? Who owns the equity? What are the advantages and disadvantages of the mutual format? What's the natural way to allow a conversion from mutual to stock format?
16. Assume that a firm has two classes of stock outstanding that are identical except that class A pays a cash dividend and class B pays a stock dividend with shares worth the same as the cash dividend. Assume that shares of class A and class B sell for the same price just before each ex-dividend date; but that class A shares fall on the ex-date by 70% of the amount of the dividend, while class B shares fall on the ex-date by 100% of the amount of the dividend. What interpretations of these facts are possible?

17. Suppose that there are two kinds of assets and two kinds of investors: tax-exempt and taxable. If a taxable investor holds a taxable asset, he pays a 50% tax on its return. All assets are riskless. Write R_1 and R_2 for the before-tax returns on tax-exempt and taxable assets. Assume that the government limits tax arbitrage by prohibiting short sales of (a) taxable securities or (b) tax-exempt securities. Consider the possibility for each case that the initial wealth of taxable investors (i) exceeds or (ii) falls short of the initial amount of tax-exempt securities. What will investor positions look like in each of the four cases? What will the relation between R_1 and R_2 be? Look for an equilibrium in which every investor is happy with his assets and liabilities, given R_1 , R_2 , and the constraints.
18. How will proportional trading costs affect the strategy of realizing losses immediately and gains before they become losses? How will fixed trading costs affect the strategy?
19. Assume that a regulated utility is the only source of a service in its area. Demand for the service is inelastic for the life of the utility's plant, but falls to zero when the plant is no longer useful. The operating cost of providing the service is zero. The rate at which service is provided follows a geometric random walk until the plant life is over. The regulators have mapped out a depreciation schedule for the plant, and want to choose the price of the service so that the market price of the utility's stock will always be equal to its depreciated book value. All operating revenues are to be paid out as dividends. There is no regulatory lag. How will changes in the demand for the service affect (a) the price of the service, and (b) the utility's stock price?

demand

Accounting and Economic Earnings

20. Assume that when a firm announces its earnings for the most recent accounting period, its stock price often moves up or down by an unusually large amount. In other words, the variance of the return is larger around earnings announcement dates than at other times. Is this consistent with market efficiency? Is it consistent with the notion that earnings figures don't matter? If a different set of accounting rules were in place, would the price change at the time of the earnings announcement be different?

21. Economists and financial analysts frequently define "rate of return" for a country or a firm as earnings over book value. If both book value and earnings give estimates of market value, what information is there in the rate of return for a country or firm?

Underwriting

22. Assume that underwriting syndicates compete to offer the highest price to the issuer of a security and the lowest price to the buyers. Assume that underwriters receive no indirect benefit when the issue is underpriced, sells out quickly, and goes to a premium, so the underwriters receive a maximum fee equal to the full spread. Assume that existing shareholders have the right to buy the new shares in proportion to their holdings if they wish. In this situation, how will the spread that a syndicate proposes be related to the price it offers the issuer? What other factors will influence the spread?

23. Assume that people have different information about a stock, and divide into optimists and pessimists of varying degrees. There is also a group of people ignorant about a stock who are not potential stockholders. Dumping more stock on the market drives down the price, even when the event is known, because informed investors will hold more only if the expected return goes up. Underwriting that expands the group of informed investors, even if the information given is balanced, will bring the stock closer to its correct price. When should a firm use underwriters, and when should a firm use rights? When should an investor buy the stock of a firm that habitually uses rights? Will the stockholders vote to maximize the current market price of the firm's shares, or the future market price?

Trading and Investment Companies

24. Imagine that American Airlines has long held 20% of Howard Johnson's common stock. The Federal Trade Commission has decided that this is likely to reduce potential competition between American Airlines and Howard Johnson's, and has ordered that American eliminate its holdings of Howard Johnson's within a year. The decision is appealed. A distinguished economist from the University of Pennsylvania has testified that selling this large a block of stock is likely to depress the price at least 5%, so that the shareholders of American will be cheated out of 5% of the value of their investment if forced to sell. You have been hired by the government to evaluate his testimony. Does the cost to American's shareholders depend on the strategy that American uses to eliminate its holdings of Howard Johnson's? What is the best strategy? What are the factors that might influence the effect on American's shareholders of using the best strategy?

25. Suppose that a firm has 8 million authorized and outstanding common shares, and that the outstanding short position in the stock is 2 million shares. How many votes might be cast if all proxies were sent in for the annual meeting? Why is it sometimes hard to borrow stock during a contest for control of the firm?

26. How can you automate the role of the specialist? Could a corporation with a computer take over the function of the specialist for all the stocks listed on an exchange? What rules would you use for changing the bid and asked prices as trades are executed? How would you handle orders for traders who are not information-motivated?

27. Assume that you are a "buy-and-hold" investor in a no load mutual fund. You plan to hold your shares for a very long time. Other investors in the fund, however, are doing short-term trading to take advantage of the way the fund prices its shares. It bases its calculations of net asset value per share on the most recent price at which each asset has traded, even though the asset may not have traded for several days. This means that an increase in value of the fund's shares is more likely to be followed by another increase than by a decrease. It means that the fund is issuing and redeeming shares at prices that are sometimes higher and sometimes lower than the current market value of the fund's assets per share. Are you hurt by the trading of these active investors? If you are hurt, what policies can the fund adopt to protect you? Are buy-and-hold investors in stock hurt by the actions of short term traders?

PQ

PT → QT

28. Assume that the managers of a closed end investment company generate expenses for the shareholders that are 1.5% of the assets per year, while the expenses of a comparable open end fund are 0.5% of the assets per year. The shareholders put up with this because a closed end fund does not redeem its shares continuously at net asset value per share, so the shareholders as a group can withdraw their money only as the fund pays dividends to them. Assume that the fund pays dividends of exactly 4% of the assets per year, and that tax factors are not important. What discount would you expect for this fund? (How will the value of its liabilities compare with the value of its assets?) Why?

Options and Futures

29. Make all the assumptions leading to the simple call option pricing model except: (a) assume that options can be exercised at any time; and (b) assume that the underlying stock pays quarterly dividends. At what points in time may there be early exercise or massive closing out of option positions? Assuming that you have upper bounds on all the dividends, can you give a series of lower bounds for the value of the option? Can you give a condition under which you can be sure that no early exercise will take place over the life of the option?

30. Assume that everyone is allowed to lend freely, but that each person can borrow no more than \$100. Assume that everyone can buy or sell stocks freely, and will receive the proceeds of any call options that are sold or stocks that are sold short. Except for the borrowing restriction, make all the assumptions that lead to the capital asset pricing model and the simple option pricing model. How will the pricing of options be affected by the borrowing restriction? How will the pricing of stocks be affected? What portfolios will people at different levels of risk aversion hold? How will your answers change if short sales of stocks are also restricted?

31. Assume that we are in a world with no trading costs where individuals and firms pay taxes at the same rate, and capital structure is a matter of indifference. Common stocks pay no dividends, and are not taxed. Options are not taxed either. Pension funds are not taxed on any of their income. In this world, will the interest rate in the option pricing formula be the before-tax rate or the after-tax rate? Will the gain from having bonds in the pension fund be there if the pension fund holds out-of-the-money call options too? Now do your answers change in a world with no personal taxes or bankruptcy costs, but with a maximum amount of debt for each firm derived from a maximum total risk for its stock?

32. What is the relation between forward contracts and futures contracts? Why are the exchange-traded contracts generally futures contracts? What assumptions must we make to derive a relation between forward prices (not the value of a forward contract) and futures prices?

33. Write x for the current value of a pure discount government bond with a face value of 1.0. Write t for the current time, and t^* for the maturity of the bond. What are the values of (a) a European call option, (b) an American call option, (c) a European put option, and (d) an American put option, where all the options are options on the bond that expire at time t^* and have exercise price c ? What is the futures price with maturity t^* for the bond? What are the values of options (a), (b), (c), and (d) if they are futures options rather than bond options, where the underlying futures contract calls for delivery of the bond at time t^* ?
34. For what reasons should the relation between the futures price on a market index and the currently quoted value of the index vary? For what reasons should the relation between the near and far futures prices on a market index differ from the relation between the near and far futures prices on a geometric index like Value Line's?

Corporate Assets and Liabilities

35. Assume that the market for borrowing and lending stock is fully competitive, and that there are no transaction costs or administrative costs or government restrictions. On what terms might a loan of stock be made? What would the borrower pay the lender? Who would bear the risk of default?
36. In the options market, you sell short by writing a newly created option. There is no need to borrow an option before selling it short. Could there be an analogous way to sell common stocks short? How could voting rights be handled? What changes in procedures for trading on stock exchanges would be needed?
37. A firm worth \$60 million before any debt payments has a debt payment of \$10 million due shortly, and a final payment of \$100 million due in one year. Assuming no debt payments are due, the value of an option to buy the firm for \$100 million in one year will be \$9 million if the firm's value is \$60 million. If the firm is otherwise similar but is worth only \$50 million, the value of an option to buy the firm for \$100 million in one year will be \$6 million. What will the value of the firm's equity and debt be if both debt payments are due, and (a) it is allowed to sell assets to pay the \$10 million, or (b) it is required to sell stock to pay the \$10 million.
38. Miller and Modigliani have said, under certain assumptions, that the announcement of a change in a corporation's dividend policy will not affect the price of its stock. Assume that there are no taxes, that the dividend policy change is a response to well-known events, and that investors are indifferent between dividends and capital gains, but that the firm has risky debt outstanding. Can the Miller-Modigliani theorem be true under these conditions? Consider various possible sources for the funds used to pay an increased dividend.

Banking and Monetary Theory

39. A firm maintains average working balances with a bank of \$5 million, and receives no interest on those balances, largely because the government prohibits the direct payment of interest on demand deposits. Now the firm wants to take out a loan of \$5 million (without changing its average working balances). The short term riskless interest rate is 6%, and the rate on this loan would be 8% if the firm had no balances. There are no reserve requirements. Assume that the bank wants to begin paying full interest on the firm's working balances by using them as compensating balances, and that the bank has the right to use the balances to pay off part or all of the loan if the firm is in trouble. What will the interest rate on the loan be?
40. Suppose that lenders are divided into two groups, L_1 and L_2 . Suppose that borrowers are divided into two groups, B_1 and B_2 . There are two kinds of debt securities, short-term and long-term. What pattern of expected returns will we find if (a) lenders in L_1 hold only short-term securities, lenders in L_2 hold only long term securities, borrowers in B_1 issue only short-term securities, and borrowers in B_2 issue only long-term securities; (b) lenders in both groups can hold both kinds of securities, but borrowers in B_1 and B_2 can issue only short-term and long-term securities, respectively; (c) lenders in L_1 and L_2 can hold only short-term and long-term securities, respectively, but borrowers in both groups can issue both kinds of securities; and (d) lenders in both groups can hold both kinds of securities, and borrowers in both groups can issue both kinds of securities. State the assumptions you need to reach your conclusions.
41. Suppose that the government succeeds in controlling the amount of credit in the economy through such devices as margin requirements, restrictions on the kinds of loans that can be made by banks and other intermediaries, and taxes on interest income without an equivalent subsidy for interest expense. Perhaps the government restricts the extent to which financial institutions can issue securities or borrow to raise the money to make loans, or applies non-interest-bearing reserve requirements to loans. What effects will these policies have on the pricing of risky securities? What effects will they have on the composition and total risk of the market portfolio of real assets? What effects will they have on real investment decisions by firms? What effects will they have on aggregate consumption? Focus on the effects of a tax on interest income whose proceeds are given back in a way that avoids significant redistribution of income.

42. Assume that monetary policy is passive, and that banks and other financial institutions are initially unregulated. Assume that the riskless interest rate is 8%, and that banks pay 8% on deposits transferrable by check. Now suppose (a) that the government imposes a 25% non-interest-bearing reserve requirement on such deposits. What impact will this have on deposit rates? On loan rates? On the nature of banks' funds transfer business? Alternatively, suppose (b) that the government imposes a 25% non-interest-bearing reserve requirement on all loans, whether made by banks or other firms that specialize in lending. What effects will this have on deposit rates? On loan rates? On the nature of banks' loan business?
43. Assume that money is supplied by the Federal Reserve System in the U.S. entirely passively. For example, assume that the System borrows currency or reserves freely at the federal funds rate, and lends currency or reserves freely against 110% collateral in the form of government bonds at the federal funds rate. The federal funds rate is kept in line with other interest rates. Give as many factors as you can that will influence the stock of money under these conditions. How will the rate of inflation and the short term interest rate be determined?

International Economics

44. Assume that the government has two kinds of liabilities: interest-bearing liabilities in the form of deposits at the Federal Reserve Banks, and non-interest-bearing liabilities in the form of currency. Assume that the private sector is not regulated, and that interest-bearing government deposits may be held by individuals, financial institutions, and non-financial firms. Assume that there are no government bonds, and no reserves in the sense of government deposits that don't bear interest. In this world, how can the government adopt a passive monetary policy? How should it set the interest rate on government deposits? If two governments adopt passive monetary policy in this way, is it possible for the exchange rate between the two currencies to be fixed?
45. Assume that there are two countries in the world, A and B. Country A is fixing the exchange rate between the two countries. For various reasons, the government of country A decides to devalue. A year after the devaluation, the consumer price index in A is the same as it was just before the devaluation. In other words, the devaluation did not produce inflation in country A. Give three possible reasons for this.
46. Assume that the world is divided into two countries, A and B, whose residents have the same tastes, wealth, and income, but different opportunities. The countries are identical in size, population, and total risk, but the correlation between the return on the market in country A and the return on the market in country B is zero. Even the interest rates and rates of inflation are the same in the two countries, though all trade and investment between the two countries have been banned for centuries. Now suppose that trade and investment are allowed. What kinds of flows of securities do you expect between the countries? What kinds of trade flows do you expect? Do you expect any changes in the capital asset pricing model? Do you expect any changes in real investments?

47. Suppose that country A is fixing its exchange rate with country B. Continuing to maintain the same exchange rate, country A imposes a 10% tariff on all imports, and an equivalent subsidy on all exports. This action was a surprise, but is expected to be permanent. What will happen to the price levels in countries A and B? What will happen to the trade balance? What has happened to owners of foreign capital in each country?
48. Suppose that countries A and B both produce many goods and services, some of which are traded. The exchange rate between the two countries is floating. Now suppose that country A imposes a 10% tax on both imports and exports. What is likely to happen to the trade balance between A and B? To residents of one country who own capital in the other?
49. Imagine a world of certainty in which there is just one good that is produced, consumed, and used as a factor of production. Assume that the world is divided into two countries, A and B, and that people cannot move between countries. Goods, though, can move freely between countries with no transport costs. Country A has a 5% tax on any assets in country A held by residents of country B. Country B has no such tax, and there are no other taxes. Assume that initially, the residents of country A hold some of the assets in country B. Country B's residents, however, are thriftier. They gradually become wealthier than the residents of country A. In the end, they hold some of the assets of country A. State how the relation between the interest rates in A and B changes over time.
50. What are the advantages of a fixed exchange rate regime? What are the advantages of a floating exchange rate regime?