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on
OPTIONS

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THE SEARCH FOR PUT OPTION VALUES

Put values and call values are affected by a common list of variables: (1) the stock price, (2) the strike price, (3) the option maturity, (4) the stock volatility, (5) interest rates, and (6) future dividends on the stock.

An increase in the stock price increases the value of a call and decreases the value of a put. This means that call deltas are positive and put deltas are negative. An increase in the strike price decreases the value of a call and increases the value of a put. An increase in a stock's volatility increases the values of both its calls and its puts.

Now let us look at the relation between put values and call values in the simplest case: when interest rates are zero and the underlying stock pays no dividends.

No Interest and No Dividends

With no interest and no dividends, it will almost never pay to exercise either a call or a put early. A person who holds an option will be better off selling it than exercising it early, except in very unusual tax or trading cost situations.

Either conversion or reverse conversion will be riskless. Conversion means buying a put, selling the corresponding call, and buying the stock. Reverse conversion means buying a call, selling the corresponding put, and selling the stock short. This implies an exact relation between the put price and the call price. The call price minus the put price must equal the stock price minus the exercise price.

$$\text{CALL} - \text{PUT} = \text{STOCK} - \text{STRIKE}$$

If the put price is lower than that, conversion will be profitable. If the put price is higher, reverse conversion will be profitable. So converters and reverse converters will tend to keep put and call prices in line with this relationship.

No Early Exercise

Now suppose that interest rates are positive and stocks pay dividends. But suppose that we're working with European options that cannot be exercised before maturity. Suppose further that when a reverse converter sells stock short, he receives interest on all margin money he puts up, including the proceeds of the short sale.

Both converters and reverse converters continue to have riskless positions, so making a profit means making more than the interest rate. The converter receives dividends on his stock and the reverse converter pays dividends, so dividends count toward the profit of a position. Thus, the call price minus the put price must equal the stock price minus the exercise price plus interest on the exercise price minus dividends.

CALL - PUT = STOCK - STRIKE + INTEREST - DIVIDENDS

If there is no early exercise, and if the dividends are known, this relationship should hold independently of any formula, and independently of the volatility of the stock. If it does not hold, there will be profit opportunities, at least gross of the costs of putting on and taking off positions.

Effects of Early Exercise

An option that can be exercised early will always be worth at least as much as an option that can only be exercised at maturity. Allowing early exercise normally increases the values of both calls and puts. With a call, early exercise is forced by a large dividend, and normally occurs only just before dividend ex-dates. With a put, early exercise can occur whenever the stock price is low enough.

A low stock price can cause early exercise on a put because waiting to exercise a put means losing interest on the exercise price. This is offset in part by the fact that waiting to exercise a put means gaining dividends on the stock.

Because of early exercise, neither conversion nor reverse conversion is really riskless. A converter may want to exercise his put early, or he may see the call he has written exercised early. If he closes out one side of his position, he'll probably want to close out the other side too, so the whole position may be closed out early. A position that is closed out early will generally show a profit or a loss.

The reverse converter is in a similar position. He may want to exercise his call early, or he may see the put he has written exercised early. In either case, he will close out his position with a profit or a loss.

If converters and reverse converters make early exercise decisions correctly, and if other investors do not, then converters and reverse converters will be more likely to end up with a profit than with a loss. It doesn't matter whether the other investors exercise too early or too late. Either one will give profits to the corresponding option writers.

Dividends, Interest, and Early Exercise

A higher interest rate makes calls worth more and puts worth less. Thus it tends to increase the difference between the values of at-the-money calls and at-the-money puts.

A higher dividend rate makes calls worth less and puts worth more. Thus it tends to reduce the difference between the values of at-the-money calls and at-the-money puts. If the dividends are high enough, they will outweigh the interest effects and cause at-the-money puts to be worth more than the corresponding at-the-money calls.

A higher dividend rate makes early exercise more likely (or equally likely) for calls and less likely (or equally likely) for puts. The value of a call may drop to parity with a dividend increase, which makes early exercise pay, and the value of a put may rise above parity with a dividend increase. No option worth more than parity should be exercised early.

As calendar time moves forward, two things happen to a put option with dividends. The time to expiration goes down, which tends to reduce the option value. But the dividends come closer, which tends to increase the option value. When a put option is well in the money, the second effect may dominate and the option value may go up as calendar time goes forward, if the stock price does not change.

Strategies with Puts and Calls

If puts are priced too low relative to calls, then converters will step in. If puts are priced too high relative to calls, then reverse converters who can get full use of the proceeds of a short sale of stock will step in. Some brokerage firms are in a position to do that by selling short stock in customer margin accounts.

If puts are overpriced and calls are correctly priced or overpriced, it may pay to sell combinations of puts and calls. For a neutral position, the right ratio will generally be other than 1:1. For many investors, though, this kind of position will have certain disadvantages.

A position involving writing puts and writing calls will have negative curvature: gains will occur if the stock doesn't move, and losses will occur if it moves a lot. If the position is not adjusted, potential losses are unlimited. And the margin rules for many investors who write naked puts and calls will mean tying up a good deal of capital.

On the other hand, if puts and calls are both underpriced, it may pay to buy combinations of puts and calls. This kind of position will have no extra margin requirements, potential losses will be limited, and the curvature will be positive: losses will occur if the stock doesn't move, and gains will occur if it moves a lot.

If stock volatilities go up, positions with negative curvature will lose, and positions with positive curvature will gain. Since there is no limit to how much stock volatilities can go up, this is another sense in which positions with negative curvature are more speculative, and positions with positive curvature are more conservative.

If puts are overpriced relative to calls, than it may pay to buy a call

and sell a put instead of buying a stock. Except when early exercise becomes profitable, the return from buying a call and selling a put will be very close to the return from buying the stock. If the put is enough overpriced, the return will almost always be greater than the return from buying the stock.

Similarly, if puts are overpriced relative to calls, it may pay an investor who has been buying stock and writing calls to write naked puts instead. For this to be profitable, he must be able to earn full interest on any margin money he puts up, and on the difference between what he puts up to buy stock and what he puts up to write naked put options. If the stock pays taxable dividends and the interest earned is tax exempt, there may even be tax benefits to the strategy that sells puts rather than buying stock and selling calls.

Figuring Put Values

The method I use for figuring put values makes the same assumptions as the method for figuring call values: that the stock volatility is constant, that there are no taxes or trading costs, and that there are no penalties to short selling options or stock.

There is a formula for a European put option (where there is no early exercise), and that formula can be adapted easily to include dividends. But there is no formula for an American put option (where early exercise is possible), and there's not likely to be one that would be simple enough to be helpful.

There are several methods for constructing tables of put values using a computer. Each column of such a table would represent a different date, and each row would represent a different stock price. We start by stating the value of a put at maturity for each different stock price. From that

column, we can create a column of values for the put on a date just before maturity. We continue working backward, constructing columns for earlier and earlier dates, until we come to the present.

I chose the method developed by Michael Parkinson, because it seemed the simplest, most efficient, and most accurate of the methods for constructing tables on a computer. It is described in the January, 1977 issue of the Journal of Business. His paper does not tell how to handle dividends, but it's not hard to change the method to include dividends.

If what you want is to construct a table of values, then the Parkinson method does the job. But if what you want is to find a single value, it's very costly to have to construct a whole table of values just to get the one value you want. There had to be an approximate method for finding a single value.

Dan McFadden (of Monchik-Weber Associates) and I spent many months looking for a good approximate method for American puts. We had help and advice from John Cox, Steven Grossman, and Robert Merton. In the end, we came up with a relatively simple method. In fact, it is as fast or faster to compute put values using this method as it is to compute call values using the call option formula.

The general method we decided on was to construct a group of tables that can be used for all options, rather than a separate table for each option. Parkinson has a group of tables in his paper that can be used for this purpose, but he assumes no dividends on the stock. So we had to construct a group of tables like his for each of several possible dividend rates.

When we want to value a particular option, we generally find that it doesn't appear in any of the tables. So we look for similar options that do appear

in the tables, and average their values, giving greater weight to the options that are most similar. Normally, we will average the values of 16 options that appear in the tables to get an estimate of the value of the option we want.

Each table assumes that the stock pays dividends continuously. But real companies don't pay small dividends every day; they pay larger dividends at quarterly or greater intervals. The hardest problem that McFadden and I faced was in figuring how to use tables for continuous dividends without significant errors.

An obvious possibility is to assume that all the dividends between now and an option's maturity are spread evenly over the life of the option, with a little paid each day. This works well when there are several dividends, or when there's a dividend near the end of the option's life, and when the dividends are not too large. It does not work well when there's a large dividend in the near future, especially when the option is well in the money.

With a big dividend coming up, it generally won't pay to exercise a put option now, even if it is far in the money. It's better to wait till after the ex-dividend date to get the benefit of the dividend. After the stock price falls as it goes ex-dividend, it may well pay to exercise the option. Spreading the dividend out and moving it to the future may completely eliminate the benefit of the dividend because it may suggest immediate early exercise. Just moving the dividend to the future may reduce the benefit of the dividend by as much as interest on the exercise price for the time over which the dividend is moved.

Another reason to be careful in moving dividends is that moving a dividend changes the effective volatility of the stock. When a dividend is paid, the dollar volatility of a stock goes down because the value of the stock goes down. Thus pushing a dividend to the future causes a change in effective

volatility that tends to increase the value of a put.

However, these two effects tend to cancel. Moving a dividend to the future tends to decrease the value of a put because it delays the effect of the dividend on the stock price, but it tends to increase the value of a put because it delays the effect of the dividend on the stock's effective volatility.

Balancing all these factors, we came up with a method of approximating the dividends on a stock in two parts: (1) an initial dividend, which we subtract from the stock price, and (2) a continuous dividend, which we use in deciding which tables to choose option values from. The two parts add up to the total dividends that we project for the stock over the life of the option.

We find the initial dividend by adding up the amounts by which each dividend exceeds interest on the exercise price between that dividend and the one before it. For the first dividend, it's the amount by which the dividend exceeds interest on the exercise price in the time till the first dividend.

One reason this works is that if the dividends are greater than interest on the exercise price, they will largely eliminate the possibility of early exercise. Any added dividends can be moved to the present without changing the early exercise pattern significantly.

Overall, this approximation for the put dividends is probably better than the approximation I have been using for the call dividends. We're working on the possibility of using a similar method for calls.