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TAXES AND CAPITAL MARKET EQUILIBRIUM

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INTRODUCTION

Much of the research now being done in finance is on the pricing of securities and other assets under conditions of capital market equilibrium, and the implications of market equilibrium for corporate investment and financial policy. For an overview of this work, see Sharpe (1970). Most of this research assumes a world in which there are no corporate or personal taxes.

Some of the early work on capital market equilibrium and corporate financial policy was done by Modigliani and Miller (1958, 1963). They explicitly considered the effects of corporate taxes, and noted that in a world in which there are corporate taxes but no personal taxes, a corporation can increase the value of its shares by increasing its debt-equity ratio.

Farrar and Selwyn (1967) and Myers (1967) extend the Modigliani and Miller analysis by introducing personal income and capital gains taxes along with corporate taxes. They note that there are circumstances under which a corporation would not want to have as much debt as possible in its financial structure. But they do not explore the changes in the nature of market equilibrium caused by the introduction of personal taxes.

Pye (1969) explores the nature of market equilibrium when there are securities with different degrees of tax exemption and investors in different marginal tax brackets. He does not look into the implications of this market equilibrium for corporate financial policy.

Finally, Brennan (1970) examines the nature of market equilibrium under uncertainty in the presence of personal income and capital gains taxes, and the implications for corporate financial policy. He shows that if short positions of any size are allowed in any security, the basic equations for the prices and expected returns of all assets continue to have the same form as in the case in which there are no taxes. He notes that under these assumptions, the desirability of increased corporate debt depends on the relation between the corporate tax rate and an aggregate personal income tax

rate. (The capital gains tax rate is also relevant, if capital gains are realized.) However, the assumption that short positions of any size are allowed in any security does not seem appropriate in a market in which securities are taxed at different rates and in which investors are in different tax brackets. A high tax bracket investor will sell highly taxed securities short, and buy lightly taxed securities, and a low tax bracket investor will do the reverse. If there are investors (such as municipalities) that are fully tax-exempt and that can issue tax-exempt securities, this process will create a form of tax arbitrage that will eliminate all taxes. Brennan avoids this extreme result by assuming that all riskless securities are fully taxable, and that it is not possible to create two portfolios that covary perfectly and that are taxed differently. But it seems more realistic to assume that there are constraints on this kind of tax arbitrage (as he does assume in the case of share repurchase), and in the presence of such constraints, as we will see below, the nature of capital market equilibrium changes markedly.

When tax arbitrage is limited because short sales are restricted, or because the losses and gains on short positions are not treated symmetrically with the gains and losses on long positions, the analysis of equilibrium becomes quite complicated, especially under uncertainty. Thus we will limit our analysis in this paper to the case of certainty.

ASSETS AND INVESTORS

We will take a universe of assets that includes both investments in productive activities and securities issued by firms that make such investments. For some purposes, we will assume that investors may make investments directly instead of buying securities of firms that make the investments. An investor may buy a share in an investment by participating in a partnership that makes the investment.

Each asset, whether an investment or a security, will be assumed to have a future rate of return that is known to all investors. The rate of return may vary over time, but the rates for all future points in time are known.

(If we assume that investors may trade shares in an investment or a firm, at any time, with zero cost, then it is enough to say that the rates of return are known for the immediate future only.)

We will assume that a given fraction of the return on an asset is taxable, and the rest is tax-exempt. This fraction will generally lie between zero and one, but may be negative (for assets that generate losses for tax purposes) or greater than one (for assets on which taxable income is offset by tax-exempt losses). The fraction may vary over time.

The idea that part of the return from an asset is fully taxable and that part is fully tax-exempt is meant to abstract from a large number of factors that affect the degree to which the return from an asset is taxed. For example, if the asset is an investment, both accounting practices and business practices will affect the extent to which its return is taxed. The use of accelerated depreciation will not change the total return, but will change the division of the gross return between taxable and non-taxable income. The capitalization of research and development expenses will convert non-taxable income into an equivalent amount of taxable income. Or a change from selling products to renting them may convert taxable income into non-taxable income.

Also, the nature of the investment has a strong impact on the fraction of its return that is effectively taxable. An investment in a "growth" area is likely to give low current income, and thus low taxable income, but high growth in income, and thus high tax-exempt returns. The tax-exempt portion of the return on such an investment is the increase in the value of the investment, so long as the investment is not going to be sold. Certain investments, in areas such as real estate and natural resource exploration, have negative taxable returns in the early years, which are offset by substantial tax-exempt returns.

In the area of securities, the taxable portion of the return usually comes in the form of dividend or interest payments, plus a portion of the appreciation. We can think of securities in general as having some part of their appreciation

in the form of realized capital gains, and the rest in the form of unrealized capital gains. Since realized capital gains are generally taxed at lower rates than dividend or interest income, we can think of the taxable part of the appreciation on securities as being a fraction of their realized gains. The interest payments on municipal bonds, on the other hand, are generally tax-exempt, while the realized capital gains are taxable. The taxable portion of the appreciation on municipal bonds can be thought of as a fraction of the realized capital gains.

We will represent all of these influences by saying that a certain fraction of the return from any asset is taxable. We can call this fraction the asset's "tax bracket." Similarly, we will represent an individual's marginal tax bracket by a fraction representing the tax he pays on each additional dollar of taxable income. Thus the tax the individual pays on a given asset is equal to the return on the asset, times the asset's tax bracket, times the individual's tax bracket. We will assume that a corporation has a tax bracket similar to that of an individual.

TAX ARBITRAGE AND CORPORATE DEBT POLICY

In a world of certainty in which assets and individuals are in different tax brackets, we cannot assume that an individual can hold a short position of any size in any asset, and subtract the taxable income on his short holdings from the taxable income on his long holdings. Investors in high tax brackets would sell assets in high tax brackets to investors in low tax brackets, and would buy assets in low tax brackets from investors in low tax brackets. This would increase the after-tax income of both groups, and, if some investors are in a zero tax bracket, would reduce all taxes to zero.

We will assume that every investor has a limit on the amount he can sell short. So long as tax arbitrage is possible, it will be advantageous for every investor to sell short the full amount that he is allowed to sell short. If corporations exist, it will be advantageous for them to sell short to their limits, as well. Selling short a fully taxable security can be called borrowing. Thus in this

world, it is advantageous for every taxable individual to borrow as much as he can, and in addition for every taxable corporation to borrow as much as it can. In a world of certainty, corporate borrowing is not a substitute for individual borrowing, it is a supplement to individual borrowing.

Both Farrar and Selwyn (1967) and Brennan (1970) found that corporations should not always have as much debt in their capital structure as possible. But they implicitly assumed away the possibility of tax arbitrage. When tax arbitrage is possible, within limits, as in a world of certainty with assets and investors in different tax brackets, corporations will use as much debt in their capital structure as they can. In this world, the Modigliani and Miller conclusion (1963) for the case in which there are no personal taxes carries over to the case where there are personal taxes.

Since we must assume some limit for the amount of borrowing or other short selling that an individual or corporation can do, it will be convenient for the remainder of the paper to assume that the limit is zero.

CORPORATE DIVIDEND POLICY

In this world, corporations have no reason to pay dividends, so we can assume that they will never pay dividends. Paying dividends simply puts the corporation's stock into a higher tax bracket, and thus increases taxes for all of its taxable shareholders. If an individual is so irrational as to prefer taxable income to tax-exempt income, he can always buy assets in high tax brackets.

Brennan (1970) assumes that corporations would like to reduce their dividends but are prevented from doing so by the laws taxing "unreasonable" accumulations of profits in a corporation. Making this assumption would not change our analysis: it would simply imply that a corporation should reduce its dividend as much as possible. However, it is worth noting here that these laws are unlikely to be effective in preventing a corporation from reducing its dividend. If a corporation has an accumulation of profits, it can make new investments to expand its business, or it can buy other companies, or it can buy a portfolio

of shares in other companies. Under certain circumstances, it can even repurchase its own shares. It is not difficult for a corporation to find profitable uses for its money when it has both securities and direct investments to choose from.

In other words, it is much easier to limit borrowing and short selling than to force corporations to pay dividends. Brennan concentrated on the wrong constraint.

CORPORATIONS AND PARTNERSHIPS

Corporations have several potential advantages over partnerships, but under certain circumstances, these advantages are minimal, and partnerships have significant tax advantages.

The principal advantage of the corporate form is the ease with which its shares may be transferred. The fact that corporate shareholders have limited liability for the obligations of the corporation seems less important, since some partners in a partnership can be limited partners, and since the general partners can buy insurance to cover them against all but the most extreme forms of liability.

If corporations pay no dividends, however, then corporate shares themselves will be in a very low tax bracket (zero if they are never sold), and any investor in a tax bracket higher than the corporate tax bracket is better off holding assets through a corporation than holding them directly through a partnership. The existence of corporations, in this world, means that the highest effective tax bracket of any investor is the corporate tax bracket.

On the other hand, any investor in a tax bracket below that of a corporation is better off from a tax point of view if he holds investments directly through a partnership than if he holds corporate shares.

In analyzing market equilibrium, then, we can assume that all investors in high tax brackets are represented by corporations in the corporate tax bracket, and that all investors in lower tax brackets than the corporate tax bracket make investments directly.

CAPITAL MARKET EQUILIBRIUM UNDER CERTAINTY

We are now ready to derive the nature of capital market equilibrium when there are assets in different tax brackets and investors in different tax brackets, and when borrowing and short selling are prohibited. In equilibrium, the rates of return on assets in different tax brackets must be related in such a way that every asset is held by some investor.

It is clear that the higher an asset's tax bracket, the higher its return must be. If an asset in a higher tax bracket had a lower return than an asset in a lower tax bracket, then every investor would prefer the asset in the lower tax bracket, and no investor would be willing to hold the asset in the higher tax bracket.

It is also clear that investors in higher tax brackets must hold assets in lower tax brackets. This suggests that we can line up the assets from the highest tax bracket to the lowest tax bracket, and the investors from the lowest tax bracket to the highest tax bracket, and match them up. The returns on the assets in any tax bracket must be such that the investors in the corresponding tax bracket find that the assets in that tax bracket have a higher after-tax return than the assets in any other tax bracket.

Let us write s for an asset's tax bracket, and t for an investor's tax bracket. Thus the fractional tax that an investor in bracket t pays on the return on an asset in bracket s is st. Let us assume that the fraction of the total outstanding value of assets that lie at or below tax bracket s is $x(s)$, and that the fraction of the total wealth of individuals at or below tax bracket t is $y(t)$. Both $x(s)$ and $y(t)$ lie between 0 and 1, and increase monotonically with s and t.

Then if we line up the assets and investors to establish a correspondence as suggested above, we can write $t(s)$ for the tax bracket of the investors that hold assets in tax bracket s. Since each investor will invest all of his wealth in assets in the tax bracket that is best for him, and since high

tax bracket investors hold low tax bracket assets, we can write:

(1) $$1 - x(s) = y(t(s))$$

Solving equation (1) for $t(s)$, we have:

(2) $$t(s) = y^{-1}(1-x(s))$$

Now let us write $r(s)$ for the before-tax return on assets in tax bracket s. In equilibrium, the function $r(s)$ must be such that investors in tax bracket $t(s)$ find that assets in tax bracket s maximize their after-tax returns. Writing u for the tax bracket of assets being considered by an investor in bracket $t(s)$, the after-tax return on the assets is:

(3) $$(1-t(s)u)r(u)$$

The function $r(u)$ must be such that the maximum of expression (3) occurs at $u=s$. Setting the derivative with respect to u of expression (3) equal to zero, and substituting s for u, we have:

(4) $$-t(s)r(s) + (1-t(s)s)r'(s) = 0$$

Rearranging terms, and substituting for $r'(s)/r(s)$, we have:

(5) $$\frac{\partial}{\partial s} \log (r(s)) = t(s)/(1-t(s)s)$$

Integrating equation (5) we get:

(6) $$\log (r(s)) - \log (r(0)) = \int_0^s t(u)/(1-t(u)u)du$$

Equation (6) may also be written as follows:

(7) $$r(s) = r(0)e^{\int_0^s t(u)/(1-t(u)u)du}$$

Equation (7), then, gives the return $r(s)$ on assets in tax bracket s, in terms of the correspondence $t(s)$ between investors' and assets' tax brackets defined by equation (2).

Note that $r(0)$, which is the return on tax exempt assets, is a parameter that adjusts the function $r(s)$ to the general level of interest rates. Equation (7) says that the ratio between the returns on assets in two different tax brackets is independent of the general level of interest rates.

THE COST OF CAPITAL

The function $r(s)$ which gives the equilibrium return on assets in tax bracket s, may also be interpreted as the cost of capital for investments in tax bracket s. The cost of capital increases steadily as the tax bracket of potential investments increases.

CAPITAL MARKET EQUILIBRIUM UNDER UNCERTAINTY

Under uncertainty, there are two kinds of potential tax arbitrage. There may be riskless assets in different tax brackets, or there may be pairs of portfolios of risky assets whose returns are perfectly correlated but that are in different tax brackets. Because of the possibility of these kinds of tax arbitrage, and for other reasons, borrowing and short selling are restricted.

The analysis of market equilibrium under uncertainty when assets and investors are in different tax brackets and when short sales are restricted becomes quite complicated. The work of Black and Scholes (1970) on the effects of dividends on stock prices, however, suggests that tax factors may not be very important in determining the equilibrium expected returns on assets that may differ in risk characteristics as well as in tax bracket.

If we assume that borrowing and short selling are completely prohibited, then an investor will choose a portfolio of assets that fits his risk preferences

and that is as well-diversified as possible. If he tries to emphasize assets in high tax brackets (because he is in a low tax bracket) or assets in low tax brackets (because he is in a high tax bracket), he may find that the loss in diversification of his portfolio and the cost of seeking out risky assets in particular tax brackets outweigh the extra after-tax expected return that he is able to achieve. It may be optimal for most investors to ignore tax factors in choosing their portfolios, and thus the effects of tax factors on the cost of capital for risky assets in different tax brackets may be minimal.

This is not meant as a rigorous analysis of the case of uncertainty, but rather is meant to suggest that the conclusions reached for the case of certainty may not carry over to the case of uncertainty, except in a general, qualitative way.

CONCLUSIONS

We have found, then, that under conditions of certainty, the steps necessary to prevent tax arbitrage lead to a market equilibrium that has rather different properties than those found by recent writers such as Farrar and Selwyn (1967), Myers (1967), and Brennan (1970).

Under these conditions, both taxable investors and taxable corporations should borrow as much as they can, and corporations should pay out as little in dividends as they can. When possible, investors in tax brackets below the corporate tax bracket should make investments directly through partnerships, and investors in tax brackets above the corporate tax bracket should hold only corporate shares.

Given the aggregate values of assets in different tax brackets, and the aggregate wealth of investors in different tax brackets, we were able to find a formula (equation (7)) for the equilibrium return on an asset as a function of its tax bracket. The introduction of uncertainty complicates the analysis considerably, and may make tax factors relatively unimportant in determining asset returns and the cost of capital in equilibrium.

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