Fischer Black
on
OPTIONS

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ALL THAT'S WRONG WITH THE BLACK-SCHOLES MODEL

When we figure option values using the Black-Scholes model, and compare them with option prices, there's usually a difference. It is rare that the value of an option comes out exactly equal to the price at which it trades on an exchange.

One possible reason for the difference between value and price is that we have made a mistake in figuring the value. We may be looking at the wrong date, or using a volatility estimate that we meant to use for a different stock, or using a stock price that was reported incorrectly.

Leaving aside errors like these, there are three kinds of reasons for a difference between value and price: (1) we may have the correct value, and the option price may be out of line; (2) we may have used the wrong inputs to the Black-Scholes formula; and (3) the Black-Scholes formula itself may be wrong. These are not meant to conflict with one another: normally, they will all play a part in explaining a difference between value and price.

The fact that the option price may be out of line with value is what may make it possible to trade profitably using the formula. Some people would say, however, that transaction costs will wipe out any possible trading profits in options (no matter what formula is used).

The main input that may be wrong is the volatility. The stock price can be observed at a different time from the option price, or the interest rate we use may be outdated, but these errors can be detected and corrected if they are large enough to make correction worthwhile. The volatility of the stock over the life of the option, though, must be estimated.

Different people will make different estimates. When the option price is higher than the value we figure, that may mean primarily that others in the market have higher volatility estimates than we do. Sometimes the market will be closer, and sometimes we'll be closer.

I want to focus in this letter on the third kind of reason for a difference between value and price: on the fact that the Black-Scholes formula is wrong. We know some specific problems with the formula; we know how some of these problems affect the values that come out; and we know a little about how to create a better formula. There will be a series of models developed over time that are better than the original Black-Scholes model.

In the original derivation of the formula, Myron Scholes and I made the following unrealistic assumptions:

(a) A stock's volatility is known, and never changes.

(b) The short-term interest rate never changes.

(c) Anyone can borrow or lend as much as he wants at a single interest rate, so long as he provides a portfolio as collateral with a value that exceeds any borrowing he may do.

(d) An investor who sells a security short will have the use of all the proceeds of the sale, and will receive any returns from investing these proceeds, even if the proceeds are used as collateral.

(e) There are no transaction costs for either stock or options.

(f) An investor's trades do not affect the taxes he pays.

(g) Stocks pay no dividends, and investors are not allowed to exercise options early.

Let's look now at how the values might change if we substitute more realistic assumptions for these.

Volatility Changes

In fact, the volatility of a stock is not constant. The fact that the volatility can change may have a major impact on the values of certain options, especially way-out-of-the-money options. For example, if we use a volatility estimate of .20 for the annual standard deviation of a 6 month call option with a $40 exercise price on a $28 stock, and if we take the interest rate to be zero, we get a value of $0.00884 using the original formula. Keeping everything else the same, but doubling the volatility to .40, we get a value of $0.465. For this out-of-the-money option, doubling the volatility estimate multiples the option value by a factor of 53.

If we think that the volatility is .20 now, but that there is some chance it will change to .40 in the near future, we will want to use a higher value than $0.00884. We will want to give some weight to the possibility that the value will shortly be much higher than that. One way to do this is to assign probability estimates to various volatility figures, and to use these probabilities to weight the resulting option values. Thus if we think there's a .50 chance that the volatility will be .20, and a .50 chance that it will be .40, we'll get a value in the above example of $0.237.

Taking possible changes in volatility into account will generally increase the values of all options, but it will increase out-of-the-money option values the most. It will make writing such options look less attractive.

In part, the volatility of a stock changes in unexplainable ways. But in part, it changes in ways related to changes in the price of the stock. This relationship seems to be quite strong. A decline in the stock price implies a substantial increase in volatility, while an increase in the stock price implies a substantial decline in volatility. The effect is so strong that it's even possible that a stock with a price of \$20 and a typical daily move of \$0.50 will start having a typical daily move of only \$0.375 if the stock price doubles to \$40.

The fact that the stock price and the volatility generally change in opposite directions can be used in making estimates of volatility, and it also means that we should be using a different basic formula. John Cox and Stephen Ross have come up with two possible alternative formulas. Their work is reported in a paper in the January/March, 1976, issue of the Journal of Financial Economics.

To see the effects of using one of their formulas on the pattern of option values for at-the-money and out-of-the-money options, let's look at the values using both Black-Scholes and Cox-Ross formulas for a 6 month call option on a \$40 stock, taking the interest rate as zero and the volatility as .20 per year. For various exercise prices, the values are:

Exercise Price	Black-Scholes	Cox-Ross
40	2.26	2.26
50	.155	.088
57.1	.0126	.0020

Thus the Cox-Ross formula implies lower values for out-of-the-money options than the Black-Scholes formula. But this will be offset, at least in part, by the adjustment discussed above for general uncertainty about the volatility.

In addition to showing changes in volatility in general and changes in volatility related to changes in stock price, a stock may have jumps. There may be a major news development that causes a sudden large change in the stock price, often accompanied by a temporary suspension of trading in the stock. Jumps may be thought of as momentary large increases in a stock's volatility.

Robert Merton, writing along with Cox and Ross in the January/March, 1976, issue of the Journal of Financial Economics, shows that taking jumps into account will tend to increase the relative values of both out-of-the-money and in-the-money options, and will decrease the relative values of at-the-money options. This will be true, at least, when the jumps tend to be specific to individual stocks. The effects of jumps on option values will be greatest on way-out-of-the-money options.

Merton's formula handles jumps but does not handle general or stock-price-related changes in volatility. The Cox-Ross formulas handle stock-price-related volatility changes, but do not handle jumps or general changes in volatility. But Cox and Ross also give a method that should allow several effects of this kind to be taken into account simultaneously, at least in certain cases.

Finally, the fact that a stock's volatility changes means that what seems like a close-to-riskless hedge isn't really. Suppose that a call option moves $0.50 for a $1.00 move in the underlying stock, and you set up a position that is short two option contracts and long one round lot of stock. This position will be fairly well protected against stock price changes in the short run. But if the stock's volatility increases, you will lose. The option will go up even if the stock price stays where it is. Because it may be impossible to diversify away risks like this so that investors don't care about them, options may be priced so that those who take these risks are paid by those who take the opposite side, or vice versa. These payments would not be explicit, but would be built into the prices of various kinds of options according to their exposure to risk of changes in volatility. If the direction and size of this effect could be estimated, it would imply further changes in the formulas used to value options.

Interest Rate Changes

A stock's volatility changes over time, and so does the interest rate. The volatility of a stock can't be observed; it can only be estimated. Interest rates, though, can be observed. This makes interest rate changes much easier to handle than changes in volatility.

Robert Merton has shown, in a paper in the Spring, 1973, Bell Journal of Economics and Management Science, that when the interest rate is changing one can sometimes simply substitute the interest rate on a bond with no coupons and a maturity equal to the option maturity for the short term interest rate in the original option formula. Strictly speaking, this works only when the volatility of the stock is not changing. When both the volatility and the interest rate are changing, a more complicated adjustment must be made.

One might also want to take possible changes in the interest rate into account when trying to set up a close-to-riskless hedged position. One might buy long bonds, and add them to a position that is long options and short stock, or to a position that is long out-of-the-money options and short more in-the-money options. Or one might sell long bonds short, to go with a position that is short options and long stock, or a position that is short out-of-the-money options and long more in-the-money options.

In general, though, the effects of interest rate changes on option values do not seem nearly as great as the effects of volatility changes.

Borrowing Penalties

In fact, the rate at which an investor can borrow, even with securities as collateral, is higher than the rate at which he can lend. Sometimes an investor's borrowing rate is substantially higher than his lending rate. And sometimes, margin requirements or restrictions put on by lenders limit the amount that he can borrow.

High borrowing rates and limits on borrowing amounts may cause a general increase in option values, because options provide leverage that can substitute for borrowing. If this happens, investors subject to borrowing limits may still want to buy options, but investors who can borrow freely at a rate close to the lending rate may want to get leverage by borrowing rather than by buying options. Investors who can borrow on favorable terms and investors who don't want to borrow may also find that they can make consistent profits writing options against stock positions. It's not clear how large those profits will tend to be, however.

Short Selling Penalties

Short selling penalties are generally even worse than borrowing penalties. On a stock, an investor sometimes can't sell on a downtick. He must go to the expense of borrowing stock if he wants to sell it short. Part of this expense is that he has to put up cash collateral with the person who lends the stock, and he generally gets no interest or interest well below market rates on this collateral. In addition, he may have to put up margin with his broker in cash, and he may not receive interest on cash balances with his broker.

For options, the penalties tend to be much less severe. An investor who does not meet a "suitability test" may not be allowed to write naked options. An investor may have to put up cash margin on which he receives no interest. And the broker may not encourage him to invest the money he gets from writing naked options. But the investor does not have to borrow an option in order to sell it short, and there is no "downtick rule" for options. Also, professional option traders and brokerage firms find that the penalties to writing naked options generally do not affect them.

Penalties on short selling of stock may allow options to be somewhat mispriced at times. For example, well-in-the-money call options often sell at parity (stock price minus exercise price) or below. You can make a profit from this situation if you can buy the option and sell the stock short without penalty and without transaction costs. Indeed, some brokers are able to do this. But most investors cannot make a profit net of expenses from this kind of position.

Since buying put options is equivalent to selling stock short, penalties on short selling of stock may tend to increase the prices of put options. But there are some ways of taking advantage of this kind of mispricing, as I said in my options letter of February 9, 1976, so I expect it to be minimal.

Transaction Costs

An "outside investor" must pay brokerage charges on his options and stock trades. An "inside investor" must pay floor brokerage charges or must execute the trade himself. He must pay clearing charges, and the costs of any exchange memberships he may have. These transaction costs are often substantial in relation to any potential profits to be made because options are mispriced. But they are clearly more of a barrier for outside investors than for inside investors.

Because of transaction costs, it is not literally possible to maintain a neutral hedge continuously, changing the ratio of your option position to your stock position as the stock price and other factors change. The fact that stock prices sometimes jump to a higher or lower level without a chance for trades to take place also makes it impossible to maintain a neutral hedge all the time. It's also impossible to maintain a neutral spread between one option on a stock and another, for the same reason.

But hedging and spreading are not the only forces tending to keep option prices in line. A person who wants a long position may choose between the option and the stock based partly on whether the option is underpriced or not. And an investor might put together a diversified portfolio with long positions in underpriced options and short positions in overpriced options that tends to stay low in risk even though he doesn't adjust his positions continuously.

Transaction costs may limit an investor's ability to take advantage of any options formula, but may not have too great an effect on option prices in practice.

Taxes

In the United States, current tax laws have the effect of reducing option prices in general, and of making it possible for both high and low tax bracket investors to make money at the expense of the IRS. Low bracket investors should tend to buy call options more often, and high bracket investors should tend to write call options more often. For a more detailed analysis of this, see my paper in the July/August, 1975, Financial Analysts Journal. It is not yet clear how puts or straddles will be taxed when they start trading on options exchanges, so I have nothing to say on the effects of taxation of puts.

Taxes also can make early exercise (or early closing of a position) pay, even for a stock that pays no dividends. At times, however, early exercise occurs when there is no apparent reason for it. The effect of unexpected early exercise on an option writer who is taking taxes into account may be to limit the influence that taxes have on the pricing of options and on strategies for using options.

Finally, taxes can have an impact on the strategies of brokers and market makers. This and other effects of taxes will be discussed in a forthcoming paper by Myron Scholes in the May, 1976, Journal of Finance.

Dividends

The original Black-Scholes formula does not take account of dividends. But dividends reduce the values of call options and increase the values of put options, at least if there is no offsetting adjustment in the terms of an option. They make early exercise of a call option more likely; and they make early exercise of a put option less likely.

We now have several ways to modify option formulas to take account of dividends. Some of these are discussed in the various papers referred to above. None of them are exact, in part because option prices depend on future dividends, and future dividends are never known for sure. To find an exact solution, we need to know not only what the possible future dividends are, but also how the amount of any future dividend depends on factors that also affect the stock price.

Conclusions

With all of these problems, it's remarkable that option formulas sometimes give values that are very close to the prices at which options trade in the market. The most serious problems have to do with changes in volatility and uncertainty about the future volatility of a stock.

In spite of all these problems, it seems that the Black-Scholes formula gives at least a rough approximation to the formula we would use if we knew how to take all these factors into account. Further modifications of the Black-Scholes formula will presumably move it in the direction of that hypothetical perfect formula.

If you have any questions or comments, please call me at 617-253-6691, or write me at 50 Memorial Drive, Cambridge, MA 02139.