Correction to an Auction Model in Chapter 12 of *Games and Information*, Third Edition

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Eric Rasmusen

Abstract

The Third Edition of *Games and Information* is unclear about the underlying assumptions used in the model of private-value auctions with uncertainty over a player's own valuation of the object being auctioned. These notes clarify.

Rasmusen: Professor of Business Economics and Public Policy and Sanjay Subhedar Faculty Fellow, Indiana University, Kelley School of Business, BU 456, 1309 East 10th Street, Bloomington, Indiana, 47405- 1701. (812) 855-9219. Fax: 812- 855-3354. Erasmuse@indiana.edu. Http: //Php.indiana.edu/∼erasmuse. Copies of these notes can be found at
Http: //Php.indiana.edu/∼erasmuse/GI/pvauctions.pdf.

(1) On page 330, the quick fix is to change

"Suppose there are N bidders, each with a private value, in an ascending open cry auction."

by adding a clarification:

"Suppose there are N bidders, each with a private value, in an ascending open cry auction and diffuse priors on the other bidders' values (that is, all values on the real line are equally likely)."

Diffuse priors have two properties: 1. no value has more probability than any other value. 2. there are no bounds on the support.

If that is not true, then the example needs to be more complicated, because the expected value is not simply the signal that the bidder observes, but will involve regression to the mean.

(2) The term "diffuse priors" is from Bayesian decision theory and is based on the viewpoint of the decisionmakers rather than on the world having reality that is reflected in their viewpoints. I feel more comfortable doing the following:

Each bidder's value is v_i,but what he observes is a signal of that value, y_i. The true values v are distributed uniformly on [0,100]. If the bidder's signal y_i is more than 100, he knows the true value is $y_i - x$. If it is less than 0, he knows it is $y_i + x$. If it is between $100 - x$ and 100, his estimate is $y_i - x/2$, since he knwos it cannot be a negative error. If y_i is between 0 and x, his estimate is $y_i + x/2$.

For my example, suppose the observed value y happens to be between x and $100 - x$. (The other cases are a bit more complicated, but the conclusion that risk is bad for the seller is the same.)

(4) Using a non-uniform distribution involves something like regression to the mean (exactly that, if it is single-peaked). Take a weighted average of $y_i - x$, y_i and $y_i + x$, with the weights being the probabilities (or densities, if it is a continuous distribution) of the three values. Those probabilities are derived using Bayes's Rule, since we know that the true value must take one of those three values, even though we don't know which one. Thus, we get

$$\begin{aligned}\hat{v} \;=\; & \frac{Prob(v=y_i-x)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}(y_i - x)+ \\ & \frac{Prob(v=y_i)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}(y_i)+ \qquad (1) \\ & \frac{Prob(v=y_i+x)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}(y_i + x)\end{aligned}$$

Note that bidder i's valuation will not change even if he is told that he is the highest bidder. He will adjust his valuation down from his signal if the signal is high, but he will do that whether he knows he is the highest bidder or not. Thus, there is no Winner's Curse here, just regression to the mean.

Equation (1) also applies to diffuse priors and the uniform distribution, but there the complicated probability terms are all equal (at least for the middle value if the distribution is uniform), and so $\hat{v} = y$.

The rest of the example needs to be modified if the distribution is not uniform. Bidder i's expected utility is, if he bids b,

$$\begin{aligned} E(utility) &= \frac{Prob(v=y_i-x)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}U(y_i-x-b)+ \\ & \frac{Prob(v=y_i)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}U(y_i-b)+ \\ & \frac{Prob(v=y_i+x)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}U(y_i+x-b) \end{aligned} \tag{2}$$

Will he bid $b = \hat{v}$? Yes, if he is risk neutral, but not if he is risk averse. If he is risk neutral, do $U(z) = z$, then equation (2) boils down to

$$\begin{aligned} E(utility) &= \frac{Prob(v=y_i-x)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}(y_i-x)+ \\ & \frac{Prob(v=y_i)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}(y_i)+ \\ & \frac{Prob(v=y_i+x)}{Prob(v=y_i-x)+Prob(v=y_i)+Prob(v=y_i+x)}(y_i+x)-b \\ &= \hat{v}-b \end{aligned} \tag{3}$$

In that case, the bidder's expected utility is zero if he wins with a bid of $b = \hat{v}$ and positive for any smaller bid, so he is willing to bid all the way up to $\hat{v}$. The effect of regression to the mean is that he bids more than his signal if his signal is below the mean, but less if it is above the mean. An interesting implication is that the winning bid is likely to be less than the signal of the winning bidder, since the winning bidder probably got an unusually high signal (and high true value too).

What if the bidder is risk averse? Then $U'' < 0$ and we can't use the simplification in equation (3). The utility of fair gamble– such as bidding $b = \hat{v}$– is negative for a risk averse person. This is true even if the fair gamble is unbalanced, giving him a greater probability of winning than of losing, but a bigger loss when he does lose, as is the case when the signal y is above the mean and so $\hat{v} < y$. Thus, the bidder should never bid all the way up to $b = \hat{v}$. The bid is lower, to the detriment of the seller, than it would be if the bidder knew his value with certainty. This is true whether the signal y is above or below the mean of v.

HERE IS WHAT IS IN THE BOOK, 3RD EDITION:

In a private value auction, does it matter what the seller does, given the Revenue Equivalence Theorem? Yes, because of risk aversion, which invalidates the Theorem. Risk aversion makes it important which auction rule is chosen, because the seller should aim to reduce uncertainty, even in a private value auction. (In a common value auction, reducing uncertainty has the added benefit of ameliorating the winner's curse.)

Consider the following question:

If the seller can reduce bidder uncertainty over the value of the object being auctioned, should he do so?

We must assume that this is a precommitment on the part of the seller, since otherwise he would like to reveal favorable information and conceal unfavorable information. But it is often plausible that the seller can set up an auction system which reduces uncertainty– say, by a regular policy of allowing bidders to examine the goods before the auction begins. Let us build a model to show the effect of such a policy.

Suppose there are N bidders, each with a private value, in an ascending open cry auction. Each measures his private value v with an independent error. This error is with equal probability $-x, +x$ or 0. Let us denote the measured value by $\hat{v}$, which is an unbiased estimate of v. What should bidder i bid up to?

If bidder i is risk neutral, he should bid up to $\hat{v}$. His expected utility is, if he pays$\hat{v}$,

$$Ew = \frac{(\hat{v}+x-\hat{v})}{3} + \frac{(\hat{v}-\hat{v})}{3} + \frac{(\hat{v}-x-\hat{v})}{3} = 0. \tag{4}$$

If bidder i is risk averse, however, and wins with bid v_{bid}, his expected utility is

$$EU(w) = \frac{U(\hat{v}+x-v_{bid})}{3} + \frac{U(\hat{v}-v_{bid})}{3} + \frac{U(\hat{v}-x-v_{bid})}{3} \tag{5}$$

Note that if the utility function U is concave,

$$\frac{U(\hat{v}+x-v_{bid})}{3} + \frac{U(\hat{v}-x-v_{bid})}{3} < \frac{2}{3}U(\hat{v}-v_{bid}). \tag{6}$$

The implication is that a fair gamble of x has less utility than no gamble. This means that the middle term in equation (12.5) must be positive if it is to be true that $EU(w) = U(0)$, which means that $\hat{v} - v_{bid} > 0$. In other words, bidder i will have a negative expected payoff unless his maximum bid is strictly less than his valuation.