PART I GAME THEORY

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1 The Rules of the Game

1.1 Definitions

Describing a Game

The essential elements of a game are **players**, **actions**, **payoffs**, and **information**– PAPI, for short. These are collectively known as the **rules of the game**.

Players *are the individuals who make decisions. Each player's goal is to maximize his utility by choice of actions.*

In the Dry Cleaners Game, let us specify the players to be NewCleaners and OldCleaners.

Nature *is a pseudo-player who takes random actions at specified points in the game with specified probabilities.*

In the Dry Cleaners Game, we will model the possibility of recession as a move by Nature. With probability.3, Nature decides that there will be a recession, and with probability.7 there will not.

An **action** *or* **move** *by player i, denoted* a_i*, is a choice he can make.*

Player i's **action set**, $A_i = \{a_i\}$, *is the entire set of actions available to him.*

An **action combination** *is an ordered set* $a = \{a_i\}$, $(i = 1, \ldots, n)$ *of one action for each of the n players in the game.*

Newcleaners' action set can be modelled very simply as $\{Enter, Stay\ Out\}$. We will also specify Oldcleaners' action set to be simple: it is to choose price from $\{Low, High\}$.

By player i's **payoff** $\pi_i(s_1, \ldots, s_n)$*, we mean either:*
(1) The utility player i receives after all players and Nature have picked their strategies and the game has been played out; or
(2) The expected utility he receives as a function of the strategies chosen by himself and the other players.

Table 1.1 The Dry Cleaners Game

		OldCleaners	
		Low Price	*High Price*
NewCleaners:	*Enter*	-100, -50	100, 100
	Stay Out	0,50	0,300

Payoffs to: (NewCleaners,OldCleaners) in thousands of dollars (normal econom

It is convenient to lay out information and actions together in an **order of play**. Here is the order of play we have specified for the Dry Cleaners Game:

(1) Newcleaners chooses its entry decision from $\{Enter, Stay\ Out\}$.
(2) Oldcleaners chooses its price from $\{Low, High\}$.
(3) Nature picks demand, D, to be *Recession* with probability.3 or $Normal$ with probability.7.

The **outcome** *of the game is a set of interesting elements that the modeller picks from the values of actions, payoffs, and other variables after the game is played out.*

One way to define the outcome of the Dry Cleaners Game would be as either *Enter* or *Stay Out.* Another way, appropriate if the model is being constructed to help plan NewCleaners' finances, is as the payoff that NewCleaners realizes, which is, from Table 1.1, one element of the set { *0, 100, -100, 40, -160* } .

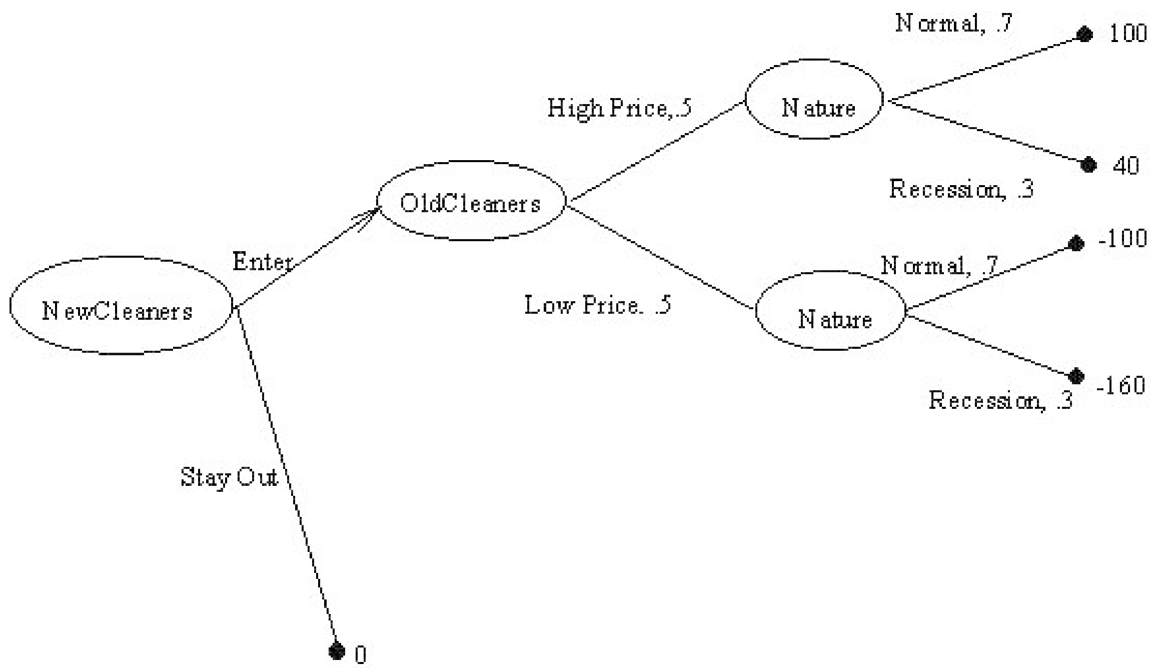

Figure 1.1 The Dry Cleaners Game as a Decision Tree

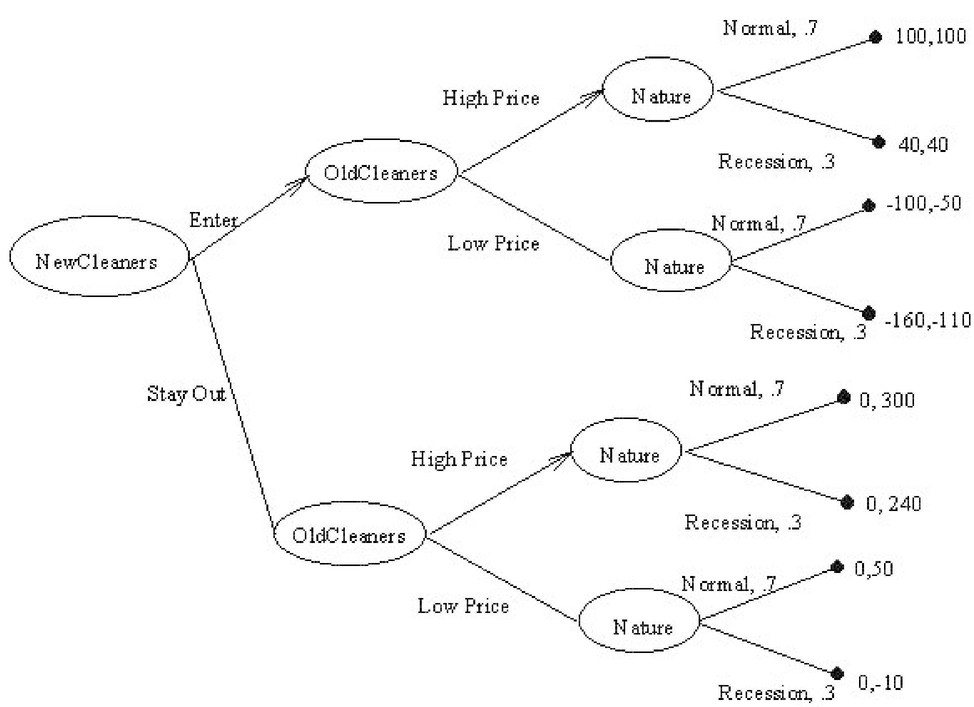

Figure 1.2 The Dry Cleaners Game as a Game Tree

Player i's **strategy** s_i *is a rule that tells him which action to choose at each instant of the game, given his information set.*

Player i's **strategy set** *or* **strategy space** $S_i = \{s_i\}$ *is the set of strategies available to him.*

A **strategy combination** $s = (s_1, \ldots, s_n)$ *is an ordered set consisting of one strategy for each of the* n *players in the game.*

$$\left\{ \begin{array}{l} \textit{High Price if NewCleaners Enters, Low Price if NewCleaners Stays Out} \\ \textit{Low Price if NewCleaners Enters, High Price if NewCleaners Stays Out} \\ \textit{High Price No Matter What} \\ \textit{Low Price No Matter What} \end{array} \right\}$$

Equilibrium

In the Dry Cleaners Game, the single outcome of *NewCleaners Enters* would result from either of the following two strategy combinations:

$$\left\{ \begin{array}{l} \textit{High Price if NewCleaners Enters, Low Price if NewCleaners Stays Out} \\ \textit{Enter} \end{array} \right\}$$

$$\left\{ \begin{array}{l} \textit{Low Price if NewCleaners Enters, High Price if NewCleaners Stays Out} \\ \textit{Enter} \end{array} \right\}$$

An **equilibrium** $s^* = (s_1^*, \ldots, s_n^*)$ *is a strategy combination consisting of a best strategy for each of the* n *players in the game.*

The **equilibrium strategies** are the strategies players pick in trying to maximize their individual payoffs, as distinct from the many possible strategy combinations obtainable by arbitrarily choosing one strategy per player.

An **equilibrium concept** *or* **solution concept** $F : \{S_1, \ldots, S_n, \pi_1, \ldots, \pi_n\} \rightarrow s^*$ *is a rule that defines an equilibrium based on the possible strategy combinations and the payoff functions.*

1.3 Dominant Strategies: The Prisoner's Dilemma

For any vector $y = (y_1, \ldots, y_n)$*, denote by* y_{-i} *the vector* $(y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$*, which is the portion of* y *not associated with player* i*.*

Using this notation, s_{-Smith}, for instance, is the combination of strategies of every player except player $Smith$. That combination is of great interest to Smith, because he uses it to help choose his own strategy, and the new notation helps define his best response.

Player i*'s* **best response** *or* **best reply** *to the strategies* s_{-i} *chosen by the other players is the strategy* s_i^* *that yields him the greatest payoff; that is,*

$$\pi_i(s_i^*, s_{-i}) \geq \pi_i(s_i', s_{-i}) \quad \forall s_i' \neq s_i^*. \tag{1}$$

The best response is strongly best if no other strategies are equally good, and weakly best otherwise.

The strategy s_i^* is a **dominant strategy** *if it is a player's strictly best response to* any *strategies the other players might pick, in the sense that whatever strategies they pick, his payoff is highest with s_i^*. Mathematically,*

$$\pi_i(s_i^*, s_{-i}) > \pi_i(s_i', s_{-i}) \;\; \forall s_{-i}, \;\; \forall s_i' \neq s_i^*. \qquad (2)$$

His inferior strategies are **dominated strategies.**

A **dominant strategy equilibrium** *is a strategy combination consisting of each player's dominant strategy.*

Table 1.2 The Prisoner's Dilemma

		Column		
		Deny		*Confess*
	Deny	-1,-1	→	-10, 0
Row:		↓		↓
	Confess	0,-10	→	**-8,-8**

Payoffs to: (Row,Column).

Table 1.3 The Battle of the Bismarck Sea

			Imamura	
		North		South
	North	**2,-2**	$\leftrightarrow$	$2, -2$
Kenney		$\uparrow$		$\downarrow$
	South	$1, -1$	$\leftarrow$	$3, -3$

Payoffs to Kenney, Imamura.

Strategy s'_i *is* **weakly dominated** *if there exists some other strategy* s''_i *for player* i *which is possibly better and never worse, yielding a higher payoff in some strategy profile and never yielding a lower payoff. Mathematically,* s'_i *is weakly dominated if there exists* s''_i *such that*

$$\begin{array}{l} \pi_i(s''_i, s_{-i}) \geq \pi_i(s'_i, s_{-i}) \quad \forall s_{-i}, \text{ and} \\ \pi_i(s''_i, s_{-i}) > \pi_i(s'_i, s_{-i}) \quad \text{for some } s_{-i}. \end{array} \tag{3}$$

One might define a **weak dominant strategy equilibrium** as the strategy profile found by deleting all the weakly dominated strategies of each player.

An **iterated dominance equilibrium** *is a strategy profile found by deleting a weakly dominated strategy from the strategy set of one of the players, recalculating to find which remaining strategies are weakly dominated, deleting one of them, and continuing the process until only one strategy remains for each player.*

Table 1.4 The Swiss Cheese Game

		Jones		
		Left		*Right*
	Up	**0,0**	↔	**0,0**
Smith:		↕		↕
	Down	**0,0**	↔	**0,0**

Payoffs to: (Smith, Jones).

Table 1.5 The Iteration Path Game

		Column		
		c_1	c_2	c_3
	r_1	**2,12**	1,10	**1,12**
Row:	r_2	0,12	0,10	0,11
	r_3	0,12	1,10	0,13

Payoffs to: (Row, Column).

Table 1.6 Boxed Pigs

		Small Pig		
		Press		Wait
	Press	5, 1	→	**[4], [4]**
Big Pig		↓		↑
	Wait	[9], −1	→	0, [0]

Payoffs to: (Big Pig, Small Pig).

The strategy combination s^* *is a* **Nash equilibrium** *if no player has incentive to deviate from his strategy given that the other players do not deviate. Formally,*

$$\forall i, \;\; \pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i', s_{-i}^*), \;\; \forall s_i'. \tag{4}$$

Table 1.7 The Modeller's Dilemma

		Column		
		Deny		*Confess*
Row:	*Deny*	0, 0	↔	−10, 0
		↕		↓
	Confess	0,-10	→	**-8**,**-8**

Payoffs to: (Row, Column).

(Confess, Confess) is an iterated dominant strategy equilibrium, and it is a strong Nash equilibrium as well. So the case for *(Confess, Confess)* still being the equilibrium outcome seems very strong.

There is, however, another Nash equilibrium in the Modeller's Dilemma: *(Deny, Deny)*, which is a weak Nash equilibrium , but *(Deny, Deny)* has the advantage that its outcome is pareto-superior: (0,0) is uniformly greater than $(-8, -8)$.

Table 1.8 The Battle of the Sexes

		Woman		
		Prize Fight		*Ballet*
	Prize Fight	**2,1**	←	0, 0
Man		↑		↓
	Ballet	0, 0	→	**1,2**

Payoffs to: (Man, Woman)

Table 1.9 Ranked Coordination

		Jones		
		Large		*Small*
	Large	**2,2**	←	$-1, -1$
Smith		↑		↓
	Small	$-1, -1$	→	**1,1**

Payoffs to: (Smith, Jones)

Table 1.10 Dangerous Coordination

		Jones		
		Large		*Small*
	Large	**2,2**	←	$-1000, -1$
Smith		↑		↓
	Small	$-1, -1$	→	**1,1**

Payoffs to: (Smith, Jones)

1.6 Focal Points

Thomas Schelling's book, *The Strategy of Conflict*.

(1) Circle one of the following numbers: 100, 14, 15, 16, 17, 18.

(2) Circle one of the following numbers 7, 100, 13, 261, 99, 666.

(3) Name Heads or Tails.

(4) Name Tails or Heads.

(5) You are to split a pie, and get nothing if your proportions add to more than 100 percent.

(6) You are to meet somebody in New York City. When? Where?