September 6, 1999.

Eric Rasmusen, Erasmuse@indiana.edu **8 Further Topics in Moral Hazard: OVERHEADS**

8.1 Efficiency Wages (formerly Section 8.4)

The Lucky Executive Game

Players
A corporation and an executive.

The Order of Play
(1) The corporation offers the executive a contract which pays $w(q) \geq 0$ depending on profit, q.
(2) The executive accepts the contract, or rejects it and receives his reservation utility of $\overline{U} = 5$
(3) The executive exerts effort e of either 0 or 10.
(4) Nature chooses profit according to Table 8.1.

Payoffs
Both players are risk neutral. The corporation's payoff is $q - w$. The executive's payoff is $w - e$ if he accepts the contract.

Table 8.1 Output in the Lucky Executive Game

	Probability of Outputs		
Effort	0	400	Total
Low ($e = 0$)	0.5	0.5	1
High ($e = 10$)	0.1	0.9	1

Since both players are risk neutral, you might think that the first-best can be achieved by selling the store, putting the entire risk on the agent. The participation constraint if the executive exerts high effort is

$$0.1[w(0) - 10] + 0.9[w(400) - 10] \geq 5, \tag{1}$$

so his expected wage must equal 15. The incentive compatility constraint is

$$0.5w(0) + 0.5w(400) \leq 0.1w(0) + 0.9w(400) - 10, \tag{2}$$

which can be rewritten as $w(400) - w(0) \geq 25$, so the gap between the executive's wage for high output and low output must equal at least 25.

A contract that satisfies both constraints is $\{w(0) = -345, w(400) = 55\}$. But this contract is not feasible, because the game requires $w(q) \geq 0$. This is an example of the common and realistic **bankruptcy constraint**; the principal cannot punish the agent by taking away more than the agent owns in the first place. The worst the boss can do is fire the worker. (In fact, the same problem would arise in a slavery regime that allowed the owner to kill his slaves– there, the worst the boss can do is kill the worker.) So what can be done?

What can be done is to use the carrot instead of the stick and abandon satisfying the participation constraint as an equality. All that is needed from constraint (2) is a gap between the high wage and the low wage of 25. Setting the low wage as low as is feasible, the corporation can use the contract $\{w(0) = 0, w(400) = 25\}$, and this will induce high effort. Notice, however, that the executive's expected utility will be $.1(0) + .9(25) - 10 = 12.5$, more than double his reservation utility of 5. He is very happy in this equilibrium– but the corporation is reasonably happy, too. The corporation's payoff is $337.5 (= 0.1(0-0) + 0.9(400-25)$, compared with the $195 (= 0.5(0-5) + 0.5(400-5))$ it would get if it paid a lower expected wage.

8.2 Tournaments (formerly Section 8.5)

Let firm Apex have two possible production techniques, $Fast$ and $Careful$. Independently for each technique, Nature chooses production cost $c = 1$ with probability θ and $c = 2$ with probability $1 - \theta$. The manager can either choose a technique at random or investigate the costs of both techniques at a utility cost to himself of α. The shareholders can observe the resulting production cost, but not whether the manager investigates. If they see the manager pick $Fast$ and a cost of $c = 2$, they do not know whether he chose it without investigating, or investigated both techniques and found they were both costly. The wage contract is based on what the shareholders can observe, so it takes the form (w_1, w_2), where w_1 is the wage if $c = 1$ and w_2 if $c = 2$. The manager's utility is log w if he does not investigate, log w $-\alpha$ if he does, and the reservation utility of log $\bar{w}$ if he quits.

If the shareholders want the manager to investigate, the contract must satisfy the self-selection constraint

$$U(\text{not investigate}) \leq U(\text{investigate}). \tag{3}$$

If the manager investigates, he still fails to find a low-cost technique with probability $(1-\theta)^2$, so (8.3) is equivalent to

$$\theta \log w_1 + (1-\theta)\log w_2 \leq [1-(1-\theta)^2]\log w_1 + (1-\theta)^2 \log w_2 - \alpha. \tag{4}$$

$$\theta(1-\theta)\log \frac{w_1}{w_2} = \alpha. \tag{5}$$

The participation constraint, which is also binding, is $U(\bar{w}) = U(\text{investigate})$, or

$$\log \bar{w} = [1-(1-\theta)^2]\log w_1 + (1-\theta)^2 \log w_2 - \alpha. \tag{6}$$

Solving equations (8.5) and (8.

refe7) together for w_1 and w_2 yields

$$\begin{aligned} w_1 = & \ \bar{w}e^{\alpha/\theta}. \\ w_2 = & \ \bar{w}e^{-\alpha/(1-\theta)}. \end{aligned} \tag{7}$$

The expected cost to the firm is

$$[1-(1-\theta)^2]\bar{w}e^{\alpha/\theta} + (1-\theta)^2\bar{w}e^{-\alpha/(1-\theta)}. \tag{8}$$

If the parameters are $\theta = 0.1$, $\alpha = 1$, and $\bar{w} = 1$, the rounded values are $w_1 = 22,026$ and $w_2 = 0.33$, and the expected cost is $4,185$. Quite possibly, the shareholders decide it is not worth making the manager investigate.

But suppose that Apex has a competitor, Brydox, in the same situation. The shareholders of Apex can threaten to boil their manager in oil if Brydox adopts a low-cost technology and Apex does not. If Brydox does the same, the two managers are in a prisoner's dilemma, both wishing not to investigate, but each investigating from fear of the other. The forcing contract for Apex specifies $w_1 = w_2$ to fully insure the manager, and boiling-in-oil if Brydox has lower costs than Apex. The contract need satisfy only the participation constraint that $\log w - \alpha = \log \bar{w}$, so $w = 2.72$ and the cost of learning to Apex is only 2.72, not $4,185$. Competition raises efficiency, not through the threat of firms going bankrupt but through the threat of managers being fired.

*8.4 Renegotiation: The Repossession Game

"The Repossession Game"

Players
A bank and a consumer.

The Order of Play
(1) The bank can do nothing or it can offer the consumer an auto loan which allows him to buy a car that costs 11, but requires him to pay back L or lose possession of the car to the bank.
(2) The consumer accepts or rejects the loan.
(3) The consumer chooses to $Work$, for an income of 15, or $Play$, for an income of 8. The disutility of work is 5.
(4) The consumer repays the loan or defaults.
(4a) In one version of the game, the bank offers to settle for an amount S and leave possession of the car to the consumer.
(4b) The consumer accepts or rejects the settlement S.
(5) If the bank has not been paid L or S, it repossesses the car.

Payoffs
If the bank does not make any loan or the consumer rejects it, both players' payoffs are zero. The value of the car is 12 to the consumer and 7 to the bank, so the bank's payoff if a loan is made is

$$\pi_{bank} = \begin{cases} L - 11 & \text{if the original loan is repaid} \\ S - 11 & \text{if a settlement is made} \\ 7 - 11 & \text{if the car is repossessed.} \end{cases}$$

If the consumer chooses $Work$ his income W is 15 and his disutility of effort D is -5. If he chooses $Play$, then $W = 8$ and $D = 0$. His payoff is

$$\pi_{consumer} = \begin{cases} W + 12 - L - D & \text{if the original loan is repaid} \\ W + 12 - S - D & \text{if a settlement is made} \\ W - D & \text{if the car is repossessed.} \end{cases}$$

Repossession Game I The first version of the game does not allow renegotiation, so moves (4a) and (4b) are dropped from the game. In equilibrium, the bank will make the loan at a rate of $L = 12$, and the consumer will choose $Work$ and repay the loan. Working back from the end of the game in accordance with sequential rationality, the consumer is willing to repay because by repaying 12 he receives a car worth 12.[1] He will choose $Work$ because he can then repay the loan and his payoff will be 10 $(= 15 + 12 - 12 - 5)$, but if he chooses $Play$ he will not be able to repay the loan and the bank will repossess the car, reducing his payoff to 8 $(= 8 - 0)$. The bank will offer a loan at $L = 12$ because the consumer will repay it and that is the maximum repayment to which the consumer will agree. The bank's equilibrium payoff is 1 $(= 12 - 11)$.

[1] As usual, we could change the model slightly to make the consumer strongly desire to repay the loan, by substituting a bargaining subgame that splits the gains from trade between bank and consumer rather than specifying that the bank make a take-it-or-leave-it offer. See Section 4.3.

Repossession Game II The second version of the game does allow renegotiation, so moves (4a) and (4b) are added back into the game. Renegotiation turns out to be harmful, because it results in an equilibrium in which the bank refuses to make a loan, reducing the payoffs of bank and consumer to (0,10) instead of (1,10); the gains from trade are lost.

The equilibrium in Repossession Game I breaks down because the consumer would deviate by choosing *Play*. In Repossession Game I, this would result in the bank repossessing the car, and in Repossession Game II, the bank still has the right to do this, for a payoff of -4 ($= 7 - 11$). If the bank chooses to renegotiate and offer $S = 8$, however, this settlement will be accepted by the consumer, since in exchange he gets to keep a car worth 12, and the payoffs of bank and consumer are -3 ($= 8 - 11$) and 12 ($= 8 + 12 - 8$). Thus, the bank will renegotiate, and the consumer will have increased his payoff from 10 to 12 by choosing *Play*. Looking ahead to this from move (1), however, the bank will see that it can do better by refusing to make the loan, resulting in the payoffs (0,10). The bank cannot even break even by raising the loan rate L. If $L = 30$, for instance, the consumer will still happily accept, knowing that when he chooses *Play* and defaults the ultimate amount he will pay will be just $S = 8$.

In the game as a whole, however, renegotiation reduces efficiency by preventing players from using punishments to deter inefficient actions.

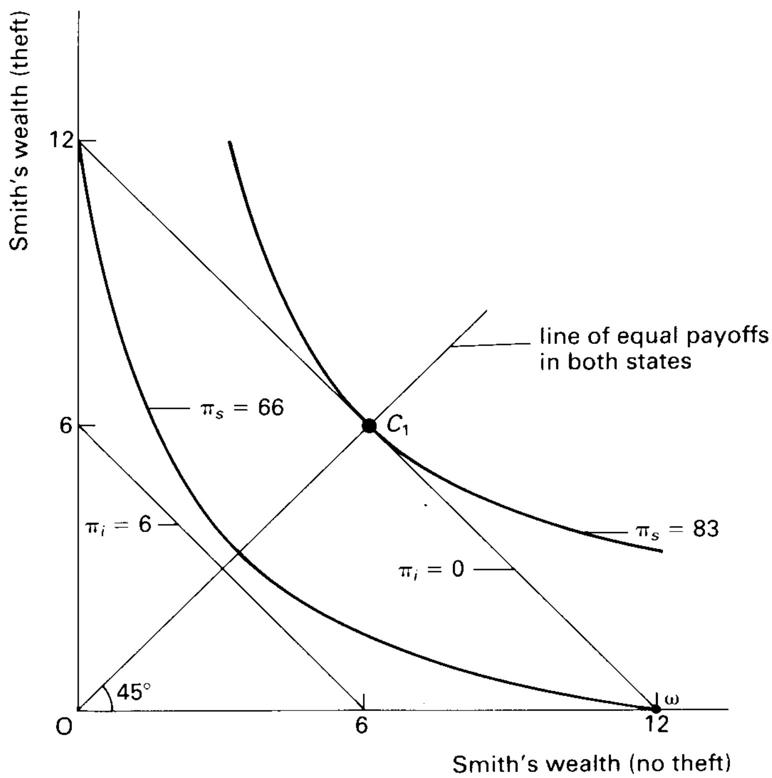
The renegotiation problem also comes up in principal-agent models because of risk bearing by a risk-averse agent when the principal is risk neutral. Optimal contracts impose risk on risk-averse agents to provide incentives for high effort or self selection.

This game also illustrates the difficulty of deciding what "bargaining power" means.

*8.5 State-Space Diagrams: Insurance Games I and II
(formerly Section 7.5)

Figure 8.1: Insurance Game I

Figure 7.4 ''Insurance Game I''



Insurance Game I: Observable Care

Players
Smith and two insurance companies.

The Order of Play
(1) Smith chooses to be either *Careful* or *Careless*, observed by the insurance company.
(2) Insurance company 1 offers a contract (x, y), in which Smith pays premium x and receives compensation y if there is a theft.
(3) Insurance company 2 also offers a contract of the form (x, y).
(4) Smith picks a contract.
(5) Nature chooses whether there is a theft, with probability 0.5 if Smith is $Careful$ or 0.75 if Smith is $Careless$.

Payoffs
Smith is risk averse and the insurance companies are risk neutral. The insurance company not picked by Smith has a payoff of zero.
Smith's utility function U is such that $U' > 0$ and $U'' < 0$. If Smith picks contract (x, y), the payoffs are:

If Smith chooses *Careful*,

$$\pi_{Smith} = 0.5U(12 - x) + 0.5U(0 + y - x)$$
$$\pi_{company} = 0.5x + 0.5(x - y), \text{ for his insurer.}$$

If Smith chooses *Careless*,

$$\pi_{Smith} = 0.25U(12 - x) + 0.75U(0 + y - x) + \epsilon$$
$$\pi_{company} = 0.25x + 0.75(x - y), \text{ for his insurer.}$$

Figure 8.2: Insurance Game II with Full and Partial Insurance

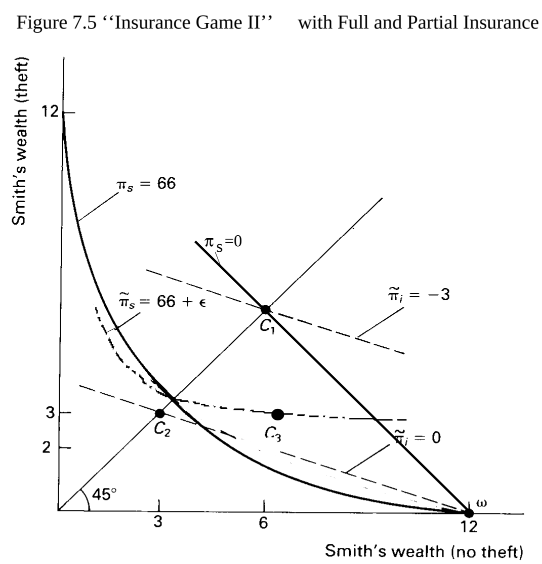


Figure 8.2 shows that no full insurance contract will be offered.

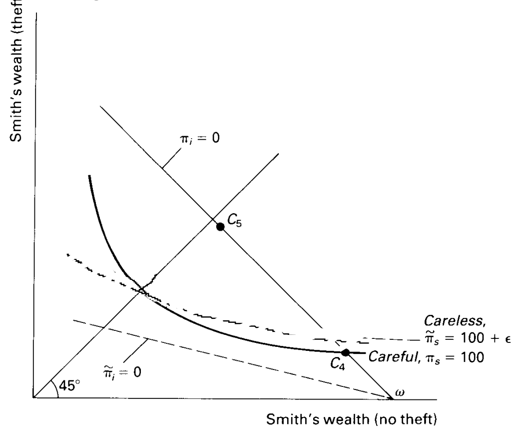
Although no full insurance contract such as C_1 or C_2 is mutually agreeable, other contracts can be used. Consider the partial insurance contract C_3 in Figure 8.2, which has a premium of 6 and a payout of 8. Smith would prefer C_3 to his endowment of $\omega = (12, 0)$ whether he chooses *Careless* or *Careful*. We can think of C_3 in two ways:

1. Full insurance except for a **deductible** of four. The insurance company pays for all losses in excess of four.
2. Insurance with a **coinsurance** rate of one-third. The insurance company pays two-thirds of all losses.

Figure 8.3 illustrates effort choice under partial insurance.

Figure 8.3 More on Partial Insurance in Insurance Game II

Figure 7.6 More on Partial Insurance in ''Insurance Game II''



*8.6 Joint Production by Many Agents: The Holmstrom Teams Model (formerly Section 8.7)

A **team** *is a group of agents who independently choose effort levels that result in a single output for the entire group.*

"Teams"
(Holmstrom [1982])

Players
A principal and n agents.

The Order of Play
(1) The principal offers a contract to each agent i of the form $w_i(q)$, where q is total output.
(2) The agents decide whether or not to accept the contract.
(3) The agents simultaneously pick effort levels e_i, $(i = 1, \ldots, n)$.
(4) Output is $q(e_1, \ldots e_n)$.

Payoffs
If any agent rejects the contract, all payoffs equal zero. Otherwise,

$$\begin{array}{ll} \pi_{principal} & = q - \sum_{i=1}^n w_i; \\ \pi_i & = w_i - v_i(e_i), \text{ where } v_i' > 0 \text{ and } v_i'' > 0. \end{array}$$

Denote the efficient vector of actions by e^*. An efficient contract is

$$w_i(q) = \begin{cases} b_i & \text{if } q \geq q(e^*) \\ 0 & \text{if } q < q(e^*) \end{cases} \qquad (9)$$

where $\sum_{i=1}^n b_i = q(e^*)$ and $b_i > v_i(e_i^*)$.

Proposition 8.1. *If there is a budget-balancing constraint, no differentiable wage contract $w_i(q)$ generates an efficient Nash equilibrium.*