10 Signalling OVERHEADS

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Eric Rasmusen, Erasmuse@indiana.edu

10.1 The Informed Player Moves First: Signalling

"Education I"

Players
A worker and two employers.

The Order of Play
(0) Nature chooses the worker's ability $a \in \{2, 5.5\}$, the *Low* and *High* ability each having probability 0.5. The variable a is observed by the worker, but not by the employers.
(1) The worker chooses education level $s \in \{0, 1\}$.
(2) The employers each offer a wage contract $w(s)$.
(3) The worker accepts a contract, or rejects both of them.
(4) Output equals a.

Payoffs
The worker's payoff is his wage minus his cost of education, and the employer's is his profit.

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w. \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{for the employer whose contract is accepted.} \\ 0 & \text{for the other employer.} \end{cases}$$

Pooling Equilibrium 1.1 $\begin{cases} s(Low) = s(High) = 0 \\ w(0) = w(1) = 3.75 \\ Prob(a = Low|s = 1) = 0.5 \end{cases}$

Separating Equilibrium 1.2 $\begin{cases} s(Low) = 0, s(High) = 1 \\ w(0) = 2, w(1) = 5.5 \end{cases}$

The participation constraints for the employers require that

$$w(0) \leq a_L = 2 \text{ and } w(1) \leq a_H = 5.5. \quad (1)$$

Competition between the employers makes the expressions in (10.1) hold as equalities. The self-selection constraint of the *Lows* is

$$U_L(s = 0) \geq U_L(s = 1), \quad (2)$$

which in Education I is

$$w(0) - 0 \geq w(1) - \frac{8}{2}. \quad (3)$$

Since in Separating Equilibrium 1.2 the separating wage of the *Low*'s is 2 and the separating wage of the *High*'s is 5.5 from (10.1), the self-selection constraint (10.3) is satisfied.

The self-selection constraint of the *High*'s is

$$U_H(s = 1) \geq U_H(s = 0), \quad (4)$$

which in Education I is

$$w(1) - \frac{8}{5.5} \geq w(0) - 0. \quad (5)$$

Constraint (10.5) is satisfied by Separating Equilibrium 1.2.

There is another conceivable pooling equilibrium for Education I, in which $s(Low) = s(High) = 1$, but this turns out not to be an equilibrium, however, because the *Low*'s would deviate to zero education. Even if such a deviation caused the employer to believe they were low-ability with probability 1 and reduce their wage to 2, the low-ability workers would still prefer to deviate, because

$$U_L(s = 0) = 2 \geq U_L(s = 1) = 3.75 - \frac{8(1)}{2}. \quad (6)$$

Education II: Modelling Trembles so Nothing is Out of Equilibrium

The pooling equilibrium of Education I required the modeller to specify the employers' out-of-equilibrium beliefs. An equivalent model constructs the game tree to support the beliefs instead of introducing them via the equilibrium concept. This approach was briefly mentioned in connection with the game of PhD Admissions in Section 6.2. The advantage is that the assumptions on beliefs are put in the rules of the game along with the other assumptions. So let us replace Nature's move in Education I and modify the payoffs as follows.

"Education II"

The Order of Play
(0) Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5. (a is observed by the worker, but not by the employer.) With probability 0.001, Nature endows a worker with free education. ...

Payoffs

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w \text{ (ordinarily)} \\ w & \text{if the worker accepts contract } w \text{ (with free education)} \\ 0 & \text{if the worker does not accept a contract} \end{cases}$$

With probability 0.001 the worker receives free education regardless of his ability. If the employer sees a worker with education, he knows that the worker might be one of this rare type, in which case the probability that the worker is *Low* is 0.5. Both $s = 0$ and $s = 1$ can be observed in any equilibrium and Education II has almost the same two equilibria as Education I, without the need to specify beliefs.

Education III: No Separating Equilibrium, Two Pooling Equilibria

Pooling Equilibrium 3.1 $\left\{ \begin{array}{l} s(Low) = s(High) = 0 \\ w(0) = w(1) = 7 \\ Prob(a = Low|s = 1) = 0.5 \text{ (passive conjectu} \end{array} \right.$

Pooling Equilibrium 3.2 $\left\{ \begin{array}{l} s(Low) = s(High) = 1 \\ w(0) = 2, w(1) = 7 \\ Prob(a = Low|s = 0) = 1 \end{array} \right.$

Education IV: Continuous Signals and Continua of Equilibria

Let us now return to Education I, with one change that education s take any level on the continuum between 0 and infinity.

$$\textbf{Pooling Equilibrium 4.1} \quad \begin{cases} s(Low) = s(High) = s^* \\ w(s^*) = 3.75 \\ w(s \neq s^*) = 2 \\ Prob(a = Low | s \neq s^*) = 1 \end{cases}$$

The critical value $\overline{s}$ can be discovered from the incentive compatibility constraint of the *Low* type, which is binding if $s^* = \overline{s}$. The most tempting deviation is to zero education, so that is the deviation that appears in the constraint.

$$U_L(s=0) = 2 \leq U_L(s = \overline{s}) = 3.75 - \frac{8\overline{s}}{2}. \tag{7}$$

Equation (10.e10.1a) yields $\overline{s} = \frac{7}{16}$. Any value of s^* less than $\frac{7}{16}$ will also support a pooling equilibrium. Note that the incentive-compatibility constraint of the *High* type is not binding. If a *High* deviates to $s = 0$, he, too, will be thought to be a *Low*, so

$$U_H(s=0) = 2 \leq U_H(s = \frac{7}{16}) = 3.75 - \frac{8\overline{s}}{5.5} \approx 3.1. \tag{8}$$

Separating Equilibrium 4.2 $\begin{cases} s(Low) = 0, \quad s(High) = s^* \\ w(s^*) = 5.5 \\ w(s \neq s^*) = 2 \\ Prob(a = Low | s \notin \{0, s^*\}) = 1 \end{cases}$

The critical value $\overline{s}$ can be discovered from the incentive-compatibility constraint of the *Low*, which is binding if $s^* = \overline{s}$.

$$U_L(s=0) = 2 \geq U_L(s = \overline{s}) = 5.5 - \frac{8\overline{s}}{2}. \tag{9}$$

Equation (10.e10.9) yields $\overline{s} = \frac{7}{8}$. Any value of s^* greater than $\frac{7}{8}$ will also deter the *Low* workers from acquiring education. If the education needed for the wage of 5.5 is too great, the *High* workers will give up on education too. Their incentive compatibility constraint requires that

$$U_H(s=0) = 2 \leq U_H(s = \overline{\overline{s}}) = 5.5 - \frac{8\overline{\overline{s}}}{5.5}. \tag{10}$$

Equation (10.e10.1d) yields $\overline{\overline{s}} = \frac{77}{32}$. s^* can take any lower value than $\frac{77}{32}$ and the *High*'s will be willing to acquire education.

Productive Signalling

Even if education is largely signalling, we might not want to close the schools. Signalling might be wasteful in a pooling equilibrium like Pooling Equilibrium 3.2, but in a separating equilibrium it can be second-best efficient for at least three reasons.

First, it allows the employer to match workers with jobs suited to their talents.

Second, signalling keeps talented workers from moving to jobs where their productivity is lower but their talent is known.

Third, if ability is endogenous— moral hazard rather than adverse selection— signalling encourages workers to acquire ability.

"Education V: Screening with a Discrete Signal"

Players
A worker and two employers.

The Order of Play
(0) Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5.
Employers do not observe ability, but the worker does.
(1) Each employer offers a wage contract $w(s)$.
(2) The worker chooses education level $s \in \{0, 1\}$.
(3) The worker accepts a contract, or rejects both of them.
(4) Output equals a.

Payoffs

$$\pi_{worker} = \begin{cases} w - \frac{8s}{a} & \text{if the worker accepts contract } w. \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{for the employer whose contract is accepted.} \\ 0 & \text{for the other employer.} \end{cases}$$

Separating Equilibrium 5.1 $\left\{ \begin{array}{l} s(Low) = 0, s(High) = 1 \\ w(0) = 2, w(1) = 5.5 \end{array} \right\}$

"Education VI: Screening with a Continuous Signal"

Players
A worker and two employers.

The Order of Play
(0) Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5.
Employers do not observe ability, but the worker does.
(1) Each employer offers a wage contract $w(s)$.
(2) The worker choose education level $s \in [0,1]$.
(3) The worker chooses a contract, or rejects both of them.
(4) Output equals a.

Payoffs.

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w. \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{for the employer whose contract is accepted.} \\ 0 & \text{for the other employer.} \end{cases}$$

Separating Equilibrium 6.1 $\begin{cases} s(Low) = 0, s(High) = 0.875 \\ w = \begin{cases} 2 & \text{if } s < 0.875 \\ 5.5 & \text{if } s \geq 0.875 \end{cases} \end{cases}$

The participation constraints for the employers require that

$$w(0) \leq a_L = 2 \text{ and } w(s^*) \leq a_H = 5.5, \tag{11}$$

where s^* is the separating value of education that we are trying to find. Competition turns the inequalities in (10.e10.11) into equalities. The self selection constraint for the low-ability workers is

$$U_L(s = 0) \geq U_L(s = s^*), \tag{12}$$

which in Education VI is

$$w(0) - 0 \geq w(s^*) - \frac{8s^*}{2}. \tag{13}$$

Since the separating wage is 2 for the *Low*'s and 5.5 for the *High*'s, constraint (10.e10.13) is satisfied as an equality if $s^* = 0.875$, which is the crucial education level in Separating Equilibrium 6.1.

Similarly, competition in offering attractive contracts rules out pooling contracts. The nonpooling constraint, required by competition between employers, is

$$U_H(s = s^*) \geq U_H(pooling), \tag{14}$$

which, for Education VI, is, using the most attractive possible pooling contract,

$$w(s^*) - \frac{8s^*}{5.5} \geq 3.75. \tag{15}$$

Since the payoff of *High*'s in the separating contract is 4.23 $(= 5.5 - 8 \cdot 0.875/5.5$, rounded), the nonpooling constraint is satisfied.

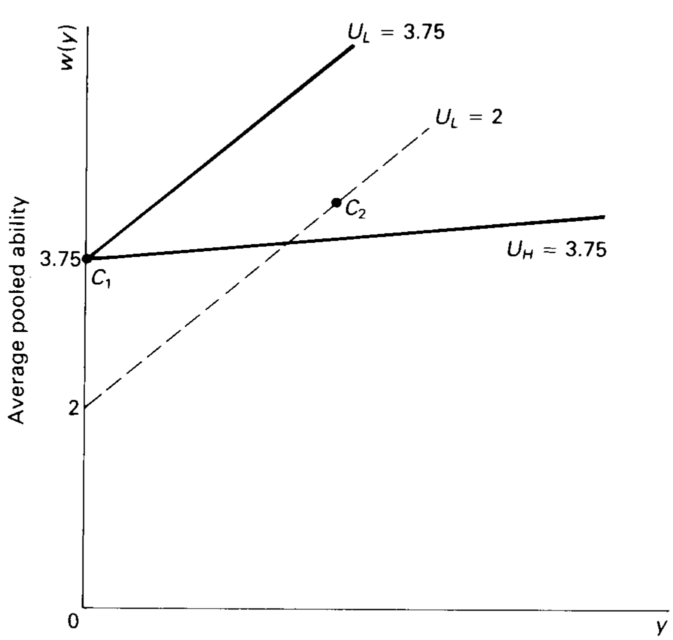
No Pooling Equilibrium in Education VI

Education VI lacks a pooling equilibrium, which would require the outcome $\{s = 0, w(0) = 3.75\}$, shown as C_1 in figure 10.1. If one employer offered a pooling contract requiring more than zero education (such as the inefficient Pooling Equilibrium 3.2), the other employer could make the more attractive offer of the same wage for zero education. The wage is 3.75 to ensure zero profits. The rest of the wage function— the wages for positive education levels— can take a variety of shapes, so long as the wage does not rise so fast with education that the *High*'s are tempted to become educated.

But no equilibrium has these characteristics. In a Nash equilibrium, no employer can offer a pooling contract, because the other employer could always profit by offering a separating contract paying more to the educated. One such separating contract is C_2 in figure 10.1, which pays 5 to workers with an education of $s = 0.5$ and yields a payoff of 4.89 to the *High*'s ($= 5 - [8 \cdot 0.5]/5.5$, rounded) and 3 to the *Low*'s ($= 5 - 8 \cdot 0.5/2$). Only *High*'s prefer C_2 to the pooling contract C_1, which yields payoffs of 3.75 to both *High* and *Low*, and if only *High*'s accept C_2, it yields positive profits to the employer.

Figure 10.1: Education VI: No Pooling Nash Equilibrium

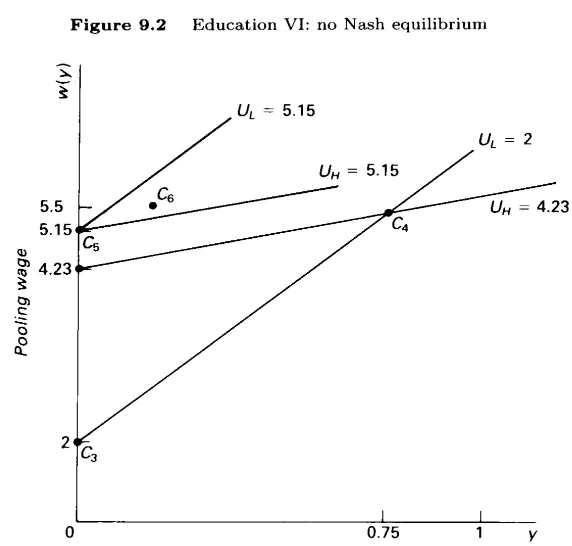
Figure 9.1 Education V: no pooling Nash equilibrium



Education VII: No Nash Equilibrium in Pure Strategies

In Education VI we showed that screening models have no pooling equilibria. In Education VII the parameters are changed a little to eliminate even the separating equilibrium. Let the proportion of *High*'s be 0.9 instead of 0.5, so the zero-profit pooling wage is 5.15 ($= 0.9[5.5] + 0.1[2]$) instead of 3.75. Consider the separating contracts C_3 and C_4, shown in figure 10.2, calculated in the same way as Separating Equilibrium 5.1. The pair (C_3, C_4) is the most attractive pair of contracts that separates *High*'s from *Low*'s by satisfying constraint (7). *Low* workers accept contract C_3, obtain $s = 0$, and receive a wage of 2, their ability. *High*'s accept contract C_4, obtain $s = 0.875$, and receive a wage of 5.5, their ability. Education is not attractive to *Low*'s because the *Low* payoff from pretending to be *High* is 2 ($= 5.5 - 8 \cdot 0.875/2$), no better than the *Low* payoff of 2 from C_3 ($= 2 - 8 \cdot 0/2$).

Figure 10.2: Education VII: No Nash equilibrium



The wage of the pooling contract C_5 is 5.15, so that even the *High*'s strictly prefer C_5 to (C_3, C_4). But our reasoning that no pooling equilibrium exists is still valid; some contract C_6 would attract all the *High*'s from C_5. No Nash equilibrium in pure strategies exists, either separating or pooling.

*10. 5. Two Signals: Game of Underpricing New Stock Issues

"Underpricing New Stock Issues"
(Grinblatt & Hwang [1989])

Players
The entrepreneur and many investors.

The Order of Play
(See Figure 2.3a for a time line.)
(0) Nature chooses the expected value (μ) and variance (σ^2) of a share of the firm using some distribution F.
(1) The entrepreneur retains fraction α of the stock and offers to sell the rest at a price per share of P_0.
(2) The investors decide whether to accept or reject the offer.
(3) The market price becomes P_1, the investors' estimate of μ.
(4) Nature chooses the value V of a share using some distribution G such that μ is the mean of V and σ^2 is the variance. With probability θ, V is revealed to the investors and becomes the market price.
(5) The entrepreneur sells his remaining shares at the market price.

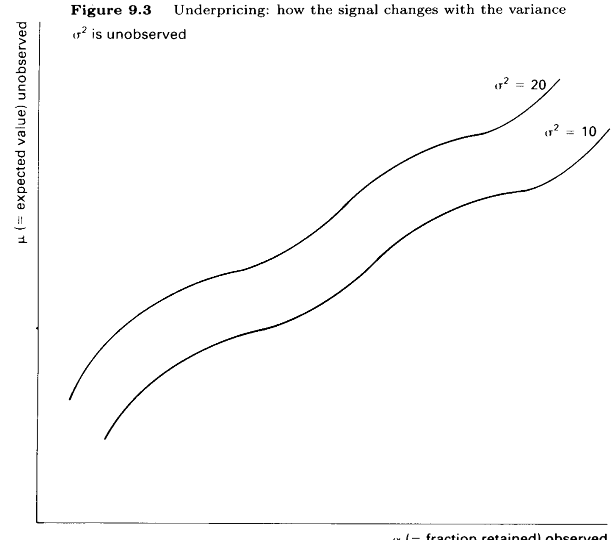
Payoffs

$$\pi_{entrepreneur} = U([1-\alpha]P_0 + \alpha[\theta V + (1-\theta)P_1]), \text{ where } U' > 0 \text{ and } U'' < 0.$$

$$\pi_{investors} = (1-\alpha)(V - P_0) + \alpha(1-\theta)(V - P_1).$$

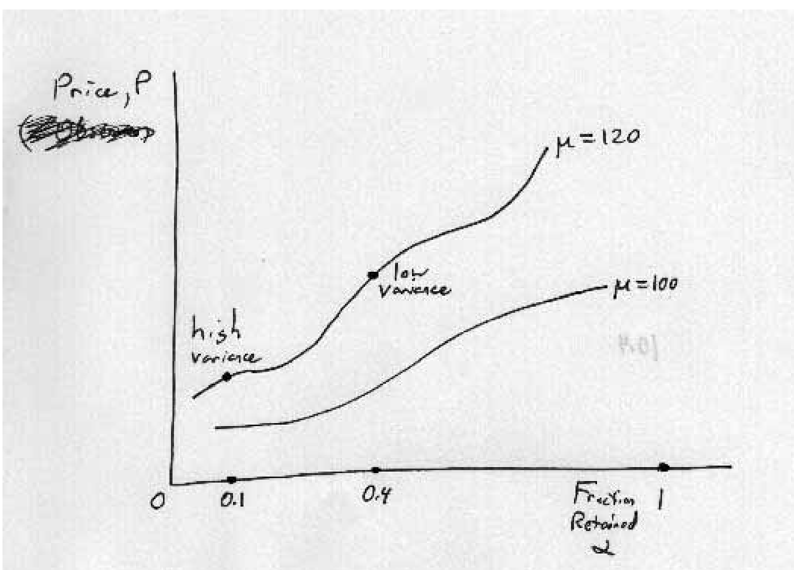
Figure 10.3: How the Signal Changes with the Variance

Figure 9.3 Underpricing: how the signal changes with the variance



I will use specific numbers for concreteness. The entrepreneur could signal that the stock has the high mean value, $\mu = 120$, in two ways: (a) retaining a high percentage, $\alpha = 0.4$, and making the initial offering at a high price of $P_0 = 90$, or (b) retaining a low percentage, $\alpha = 0.1$, and making the initial offering at a low price, $P_0 = 80$. Figure 10.4 shows the different combinations of initial price and fraction retained that might be used. If the stock has a high variance, he will want to choose behavior (b), which reduces his risk. Investors deduce that the stock of anyone who retains a low percentage and offers a low price actually has $\mu = 120$ and a high variance, so stock offered at the price of 80 rises in price. If, on the other hand, the entrepreneur retained $\alpha = .1$ and offered the high price $P_0 = 90$, investors would conclude that μ was lower than 120 but that variance was low also, so the stock would not rise in price. The low price conveys the information that this stock has a high mean and high variance rather than a low mean and low variance.

Figure 10.4: Different Ways to Signal a Given μ.



*10.6 Signal Jamming and Limit Pricing

"Limit Pricing as Signal Jamming"

Players
The incumbent and the rival.

The Order of Play
(0) Nature chooses the market size M to be M_{Small} with probability θ and M_{Large} with probability $(1-\theta)$, observed only by the incumbent.
(1) The incumbent chooses the signal S to equal s_0 or s_1 for the first period if the market is small, s_1 or s_2 if it is large. This results in monopoly profit $\mu f(S) - C$, where $\mu > 1$. Both players observe the value of S.
(2) The rival decides whether to be *In* or *Out* of the market.
(3) If the rival chooses *In*, each player incurs cost C in the second period and they each earn the duopoly profit $M - C$. Otherwise, the incumbent earns $\mu M - C$.

Payoffs
If the rival does not enter, the payoffs are $\pi_{incumbent} = (\mu f(S) - C) + (\mu M - C)$ and $\pi_{rival} = 0$.
If the rival does enter, the payoffs are $\pi_{incumbent} = (\mu f(S) - C) + (M - C)$ and $\pi_{rival} = M - C$.
Assume that $f(s_0) < f(s_1) = M_{Small} < f(s_2) = M_{Large}$, $M_{Large} - C > 0$, and $M_{Small} - C < 0$.

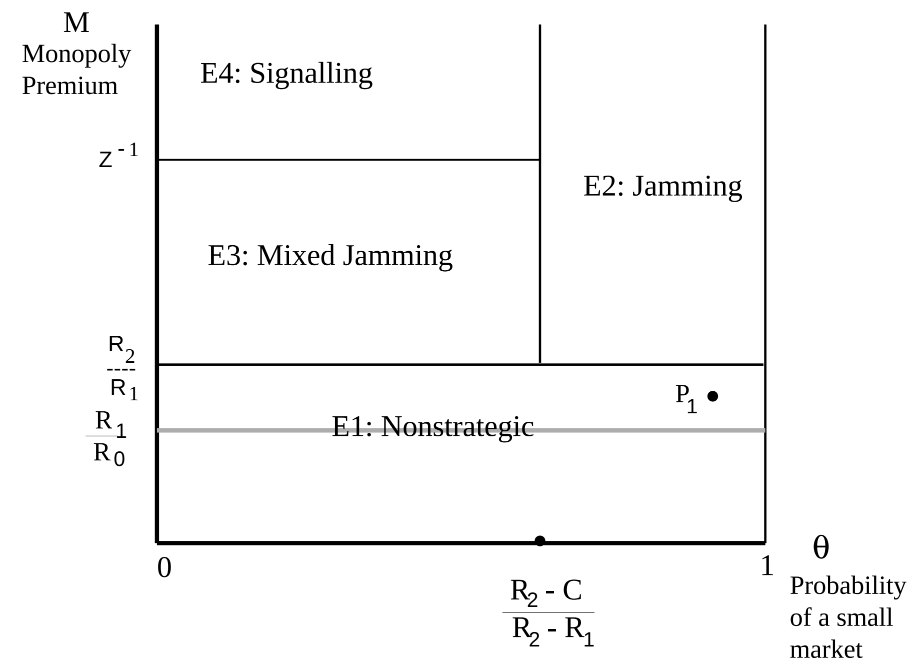


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Figure 10.5 Signal Jamming

There are four equilibria, each appropriate to a different parameter region in Figure 10.5. If the parameter μ, which shows the value to being a monopoly, is small enough, a nonstrategic equilibrium exists, in which the incumbent simply maximizes profits in each period separately. This equilibrium is: (E1: Nonstrategic. $s_2|Large$, $s_1|Small$, $Out|s_0$, $Out|s_1$, $In|s_2$). The incumbent's equilibrium payoff in a large market is $\pi_I(s_2|Large) = (\mu M_{Large} - C) + (M_{Large} - C)$, compared with the deviation payoff of $\pi_I(s_1|Large) = (\mu M_{Small} - C) + (\mu M_{Large} - C)$. The incumbent has no incentive to deviate if $\pi_I(s_2|Large) - \pi_I(s_1|Large) = (1+\mu)M_{Large} - \mu(M_{small} + M_{Large}) \geq 0$, which is equivalent to

$$\mu \leq \frac{M_{Large}}{M_{small}}, \tag{16}$$

as shown in Figure 10.5. The rival will not deviate, because the incumbent's choice fully reveals the size of the market.

Signal jamming occurs if monopoly profits are somewhat higher, and if the rival would refrain from entering the market unless he decides it is more profitable than his prior beliefs would indicate. The equilibrium is (E2: Pure Signal-Jamming. $s_1|Large$, $s_1|Small$, $Out|s_0$, $Out|s_1$, $In|s_2$). The rival's strategy is the same as in E1, so the incumbent's optimal behavior remains the same, and he chooses s_1 if the opposite of condition (10.16) is true. As for the rival, if he stays out, his second-period payoff is 0, and if he enters, its expected value is $\theta(M_{small} - C) + (1-\theta)(M_{Large} - C)$. Hence, as shown in Figure 10.5, he will follow the equilibrium behavior of staying out if

$$\theta \geq \frac{M_{Large} - C}{M_{Large} - M_{small}}. \qquad (17)$$

A mixed form of signal jamming occurs if the probability of a small market is unlikely, so if the signal of first-period revenues was jammed completely, the rival would enter anyway. This equilibrium is (E3: Mixed Signal Jamming. ($s_1|Small$, $s_1|Large$ with probability α, $s_2|Large$ with probability $(1-\alpha)$, $Out|s_0$, $In|s_1$ with probability β, $Out|s_1$ with probability $(1-\beta)$, $In|s_2$). If the incumbent played $s_2|Large$ and $s_1|Small$, the rival would interpret s_1 as indicating a small market—an interpretation which would give the incumbent incentive to play $s_1|Large$. But if the incumbent always plays s_1, the rival would enter even after observing s_1, knowing there was a high probability that the market was really large. Hence, the equilibrium must be in mixed strategies, which is equilibrium E3, or the incumbent must convince the rival to stay out by playing s_0, which is equilibrium E4.

For the rival to mix, he must be indifferent between the second-period payoffs of $\pi_E(In|s_1) = \frac{\theta}{\theta+(1-\theta)\alpha}(M_{small} - C) + \frac{(1-\theta)\alpha}{\theta+(1-\theta)\alpha}(M_{Large} - C)$ and $\pi_E(Out|s_1) = 0$. Equating these two payoffs and solving for α yields $\alpha = \left(\frac{\theta}{1-\theta}\right)\left(\frac{C-M_{small}}{M_{Large}-C}\right)$, which is always non-negative, but avoids equalling one only if condition (10.17) is false.

For the incumbent to mix when the market is large, he must be indifferent between $\pi_I(s_2|Large) = (\mu M_{Large} - C) + (M_{Large} - C)$ and $\pi_I(s_1|Large) = (\mu M_{small} - C) + \beta(M_{Large} - C) + (1-\beta)(\mu M_{Large} - C)$. Equating these two payoffs and solving for β yields $\beta = \frac{\mu M_{small} - M_{Large}}{(\mu-1)M_{Large}}$, which is strictly less than one, and which is non-negative if condition (10.17) is false.

If the market is small, the incumbent's alternative payoffs are the equilibrium payoff of $\pi_I(s_1|Small) = (\mu M_{small} - C) + \beta(M_{small} - C) + (1-\beta)(\mu M_{small} - C)$ and the deviation payoff of $\pi_I(R_0|Small) = (MR_0 - C) + (MM_{small} - C)$. The difference is

$$\pi_I(s_1|Small) - \pi_I(R_0|Small) = [MM_{small} + \beta M_{small} + (1-\beta)MM_{small}] - [MR_0 + MM_{sm} \qquad (18)$$

Expression (10.18) is non-negative under either of two conditions, both of which are found by substituting the equilibrium value of β into expression (10.18). The first is if R_0 is small enough, a sufficient condition for which is

$$R_0 \leq M_{small}\left(1 - \frac{M_{small}}{M_{Large}}\right). \qquad (19)$$

The second is if M is no greater than some amount Z^{-1} defined so that

$$M \leq \left(\frac{M_{small}}{M_{Large}} - 1 + \frac{R_0}{M_{small}}\right)^{-1} = Z^{-1}. \qquad (20)$$

If condition (10.19) is false, then $Z^{-1} > \frac{M_{Large}}{M_{small}}$, because $Z < \frac{M_{small}}{M_{Large}}$ and $Z > 0$. Thus, we can draw region E3 as it is shown in Figure 10.5.

It follows that if condition (10.20) is replaced by its converse, the unique equilibrium is for the incumbent to choose $s_0|Small$, and the equilibrium is (E4: Signalling. $s_0|Small$, $s_2|Large$, $Out|s_0$, $In|s_1$, $In|s_2$). Passive conjectures will support this pooling signalling equilibrium, as will the out-of-equilibrium belief that if the rival observes s_1, he believes the market is large with probability $\frac{(1-\theta)\alpha}{\theta+(1-\theta)\alpha}$, as in equilibrium E3.

The signalling equilibrium is also an equilibrium for other parameter regions outside of E4, though less reasonable beliefs are required. Let the out-of-equilibrium belief be $Prob(Large|s_1) = 1$. The equilibrium payoff is $\pi_I(s_0|Small) = (\mu f(s_0) - C) + (\mu M_{small} - C)$ and the deviation payoff is $\pi_I(s_1|Small) = (\mu M_{small} - C) + (M_{small} - C)$. The signalling equilibrium remains an equilibrium so long as $\mu \geq \frac{M_{small}}{f(s_0)}$.