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notes on procurement

Varian.

Reading for Monday.

Data assignment this weeekend.

Procurement I: Perfect Information

The Order of Play

0 Nature determines whether the firm has special problems that add costs of x, which has probability θ, or no special problems, which has probability $(1-\theta)$. We will call these "special" and "normal" firms, with the understanding that "special" problems may be the norm in engineering projects. The government and the firm both observe this move.
1 The government offers a contract agreeing to cover the firm's cost c of producing a cruise missile and specifying an additional price $p(c)$ for each cost level that the firm might report.
2 The firm accepts or rejects the contract.
3 If the firm accepts, it chooses effort level e, unobserved by the government.
4 The firm finishes the cruise missile at a cost of $c = c_0 + x - e$ or $c = c_0 - e$ which is observed by the government, plus an additional cost $f(e - c_0)$ that the government does not observe. The government reimburses c and pays $p(c)$.

Payoffs

Both firm and government are risk neutral and both receive payoffs of zero if the firm rejects the contract. If the firm accepts, its payoff is

$$\pi_{firm} = p - f(e - c_0), \tag{1}$$

where $f(e - c_0)$, the cost of effort, is increasing and convex, so $f' > 0$ and $f'' > 0$. Assume, too, for technical convenience, that f is increasingly convex, so $f''' > 0$.[1] The government's payoff is

$$\pi_{government} = B - (1 + \lambda)c - \lambda p - f, \tag{2}$$

where B is the benefit of the cruise missile and λ is the deadweight loss from the taxation needed for government spending.[2]

[1]The argument of f is normalized to be $(c_0 - e)$ rather than just e to avoid clutter in the algebra later. The assumption that $f''' > 0$ allows the use of first-order conditions by making concave the maximand in (13), which is a difference of two concave functions. See p. 58 of Laffont & Tirole (1993).

[2]Hausman & Poterba (1987) estimate this loss to be around \$0.30 for each \$1 of tax revenue raised at the margin for the United States.

The model differs from other principal-agent models in this book because the principal cares about the welfare of the agent. If the government cared only about the value of the cruise missile and the cost to taxpayers, its payoff would be $[B - (1+\lambda)c - (1+\lambda)p]$. Instead, the payoff function maximizes social welfare, the sum of the welfares of the taxpayers and the firm. The welfare of the firm is $(p - f)$, and summing the two welfares yields equation (2). Either kind of government payoff function may be realistic, depending on the political balance in the country being modelled, and the model will have similar properties whichever one is used.

In this first variant of the game, whether the firm has special problems is observed by the government, which can therefore specify a contract conditioned on the type of the firm. The government pays prices of p_N to a normal firm with the cost $\underline{c}$, p_S to a special firm with the cost $\overline{c}$, and a price of $p = 0$ to a firm that does not achieve its appropriate cost level.

The participation constraints will be binding for both types of firms, and to make a firm's payoff zero the government will provide prices that exactly cover the firm's disutility of effort. Since there is no uncertainty we can invert the cost equation and write it as $e = c_0 + x - c$ or $e = c_0 - c$. The prices will be $p_S = f(x - \overline{c})$ and $p_N = f(-\underline{c})$.

Suppose the government knows the firm has special problems. Substituting the subsidy into the government's payoff function, equation (2), yields

$$\pi_{government} = B - (1+\lambda)\overline{c} - \lambda f(c_0 + x - \overline{c}) - f((c_0 + x - \overline{c}) - c_0). \quad (3)$$

Since $f'' > 0$, the government's payoff function is concave, and standard optimization techniques can be used. The first-order condition for $\overline{c}$ is

$$\frac{\partial \pi_{government}}{\partial \overline{c}} = -(1+\lambda) + (1+\lambda) f'(x - \overline{c}) = 0, \quad (4)$$

so

$$f'(x - \overline{c}) = 1. \quad (5)$$

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Since $f'(x-\overline{c}) = f'([c_0+x-\overline{c}]-c_0)$ and $c_0+x-\overline{c} = e$, equation (5) says that $\overline{c}$ should be chosen so that $f'(e-c_0) = 1$; at the optimal effort level, the marginal disutility of effort equals the marginal reduction in cost because of effort. This is the first-best efficient effort level, which we will denote by $e^* \equiv e : \{f'(e - c_0) = 1\}$.

Exactly the same is true for the normal firm, so $f'(x-\overline{c}) = f'(-\underline{c}) = 1$ and $\underline{c} = \overline{c} - x$. The cost targets assigned to each firm are $\overline{c} = c_0 + x - e^*$ and $\underline{c} = c_0 - e^*$. Since both types must exert the same effort, e^*, to achieve their different targets, $p_S = f(e^* - c_0) = p_N$. The two firms exert the same efficient effort level and are paid the same price to compensate for the disutility of effort. Let us call this price level p^*.

The assumption that B is sufficiently large can now be made more specific: it is that $B - (1+\lambda)\overline{c} - \lambda f(e^* - c_0) - f(e^* - c_0) \geq 0$, which requires that $B - (1+\lambda)(c_0 + x - e^*) - (1+\lambda)p^* \geq 0$.

Procurement II: Incomplete Information

In the second variant of the game, the existence of special problems is not observed by the government, which must therefore provide incentives for the firm to volunteer its type if the normal firm is to produce at lower cost than the firm with special problems.

The government could use a pooling contract, simply providing a price of p^* for a cost of $c = c_0 + x - e^*$, enough to compensate the firm with special problems for its effort, with $p = 0$ for any other cost. Both types would accept this, but the normal firm could exert effort less than e^* and still get costs down enough to receive the price. (Notice that this is the cheapest possible pooling contract; any cheaper contract would be rejected by the firm with special problems.) Thus, if the government would build the cruise missile under full information knowing that the firm has special problems, it would also build it under incomplete information, when the firm might have special problems.

The pooling contract, however, is not optimal. Instead, the government could offer a choice between the contract $(p^*, c = c_0 + x - e^*)$ and a new contract that offers a higher price but requires reimbursable costs to be lower. By definition of e^*, $f'(c_0 + x - e^* - c_0) = 1$, so $f'(c_0 - e^* - c_0) < 1$, which is to say that the normal firm's marginal disutility of effort when it exerts just enough effort to get costs down to $c = c_0 + x - e^*$ is less than 1. This means that if the government can offer a new contract with slightly lower c but slightly higher p that will be acceptable to the normal firm but will have a lower combined expense of $(p + c)$. This tell us that a separating contract exists that is superior to the pooling contract.

Let us therefore find the optimal contract with values $(\underline{c}, p_N)$ and $(\overline{c}, p_S)$ and $p = 0$ for other cost levels. It will turn out that the $(\overline{c}, p_S)$ part of the optimal separating contract will not be the same as the pooling contract in the previous paragraph, because to find the optimal separating contract it is not enough to find the optimal "new contract;" we need to find the optimal *pair* of contracts, and by finding a new contract for the special-problems firm too, we will be able to reduce the government's expense from the normal firm's contract.

A separating contract must satisfy participation constraints and incentive compatibility constraints for each type of firm. The firm with special problems exerts effort $e = c_0 + x - \overline{c}$, achieves $c = \overline{c}$, generating unobserved effort disutility $f(e - c_0) = f(x - \overline{c})$ and participation constraint

$$p_S - f(x - \overline{c}) \geq 0. \tag{6}$$

Similarly, in equilibrium the normal firm exerts effort $e = c_0 - \underline{c}$, so its participation constraint is

$$p_N - f(-\underline{c}) \geq 0. \tag{7}$$

The incentive compatibility constraint for the firm with special problems is

$$p_S - f(x - \overline{c}) \geq p_N - f(x - \underline{c}), \tag{8}$$

and for the normal firm it is

$$p_N - f(-\underline{c}) \geq p_S - f(-\overline{c}). \tag{9}$$

Since the normal firm can achieve the same cost level as the special firm with less effort, inequality (9) tells us that if we are to have $\underline{c} < \overline{c}$, as is necessary for us to have a separating equilibrium, we need $P_N > P_S$. The second half of inequality (9) must be positive, If the special-firm participation constraint, inequality (8), is satisfied, then $p_S - f(-\overline{c}) > 0$. This, in turn implies that (7) is a strong inequality; the normal firm's participation constraint is nonbinding.

The special firm's participation constraint, (6), will be binding (and therefore satisfied as an equality), because the government will reduce the subsidy as much as possible in order to avoid the deadweight loss of taxation. The incentive compatibility constraint for the normal firm must also be binding, because if the pair $(\overline{c}, p_N)$ were strictly more attractive for the normal firm, the government could reduce the subsidy p_N. Constraint (9) is therefore satisfied as an equality.[3] Knowing that constraints (6) and (9) are binding, we can write from constraint (6),

$$p_S = f(x - \overline{c}) \tag{10}$$

and, making use of both (6) and (9),

$$p_N = f(-\underline{c}) + f(x - \overline{c}) - f(- - \overline{c}). \tag{11}$$

[3]The same argument does not hold for the special firm, because if p_S were reduced, the participation constraint would be violated.

From (2), the government's maximization problem under incomplete information is

$$\underset{\underline{c},\overline{c},p_N,p_S}{Maximize} \quad \theta\left[B-(1+\lambda)\overline{c}-\lambda p_S-f(x-\overline{c})\right]+[1-\theta]\left[B-(1+\lambda)\underline{c} \right. \tag{12}$$

Substituting for p_S and p_N from (10) and (11) reduces the problem to

$$\underset{\underline{c},\overline{c}}{Maximize} \quad \theta[B-(1+\lambda)\overline{c}-\lambda(f(x-\overline{c})-f(x-\overline{c})]+[1-\theta][B-(1+$$

$$-\lambda f(-\underline{c})-\lambda f(x-\overline{c})+\lambda f(-\overline{c})-f(-\underline{c})]. \tag{13}$$

$$\underset{\underline{c},\, \overline{c}}{Maximize} \quad \theta[B - (1+\lambda)\overline{c} - \lambda(f(x-\overline{c}) - f(x-\overline{c})] + [1-\theta][B - (1+$$

$$-\lambda f(-\underline{c}) - \lambda f(x-\overline{c}) + \lambda f(-\overline{c}) - f(-\underline{c})].$$

(1) The first-order condition with respect to $\underline{c}$ is

$$(1-\theta)[-(1+\lambda) + \lambda f'(-\underline{c}) + f'(-\underline{c})] = 0, \qquad (14)$$

which simplifies to

$$f'(-\underline{c}) = 1. \qquad (15)$$

Thus, as earlier, $f'_N = 1$. The normal firm chooses the efficient effort level e^* in equilibrium, and $\underline{c}$ takes the same value as it did in Procurement I. Equation (11) can be rewritten as

$$p_N = p^* + f(x-\overline{c}) - f(-\overline{c}). \qquad (16)$$

Because $f(x-\overline{c}) > f(-\overline{c})$, equation (16) shows that $p_N > p^*$. Incomplete information increases the subsidy to the normal firm, which earns more than its reservation utility in the game with incomplete information. Since the firm with special problems will earn exactly its reservation, this means that the government is on average providing its supplier with an above-market rate of return, not because of corruption or political influence, but because that is the way to induce normal suppliers to reveal that they do not have special costs. This should be kept in mind as an alternative to the product quality model of Chapter 5 and the efficiency wage model of Section 8.1 for why above- average rates of return persist.

$$\underset{\underline{c},\,\overline{c}}{Maximize} \quad \theta[B-(1+\lambda)\overline{c}-\lambda(f(x-\overline{c})-f(x-\overline{c})]+[1-\theta][B-(1+\lambda$$

$$-\lambda f(-\underline{c})-\lambda f(x-\overline{c})+\lambda f(-\overline{c})-f(-\underline{c})].$$

(2) The first-order condition with respect to $\overline{c}$ is

$$\theta\left[-(1+\lambda)+\lambda f'(x-\overline{c})+f'(x-\overline{c})\right]+[1-\theta]\left[\lambda f'(x-\overline{c})+f'(-\overline{c})\right]= \tag{17}$$

This can be rewritten as

$$f'(x-\overline{c})=1-\left(\frac{1-\theta}{\theta(1+\lambda)}\right)\left[\lambda f'(x-\overline{c})+f'(-\overline{c})\right]. \tag{18}$$

Since the right-hand-side of equation (18) is less than one, the special firm has a lower level of f' than the normal firm, and must be exerting effort less than e^* since $f''>0$. Perhaps this explains the expression "good enough for government work". Also since the special firm's participation constraint, (6), is satisfied as an equality, it must also be true that $p_S<p^*$. The special firm's price is lower than under full information, although since its effort is also lower, its payoff stays the same.

We must also see that the incentive compatibility constraint for the firm with special problems is satisfied as a weak inequality; the firm with special problems is not near being tempted to pick the normal firm's contract. This is a bit subtle. Setting the left-hand-side of the incentive compatibility constraint (8) equal to zero because the participation constraint is binding for the firm with special problems, substituting in for p_N from equation (11) and rearranging yields

$$f(x - \underline{c}) - f(-\underline{c}) \geq f(x - \overline{c}) - f(-\overline{c}). \tag{19}$$

This is true, and true as a strict inequality, because $f'' > 0$ and the arguments of f on the left-hand-side of equation (19) take larger values than on the right-hand side, as shown in Figure 10.7.

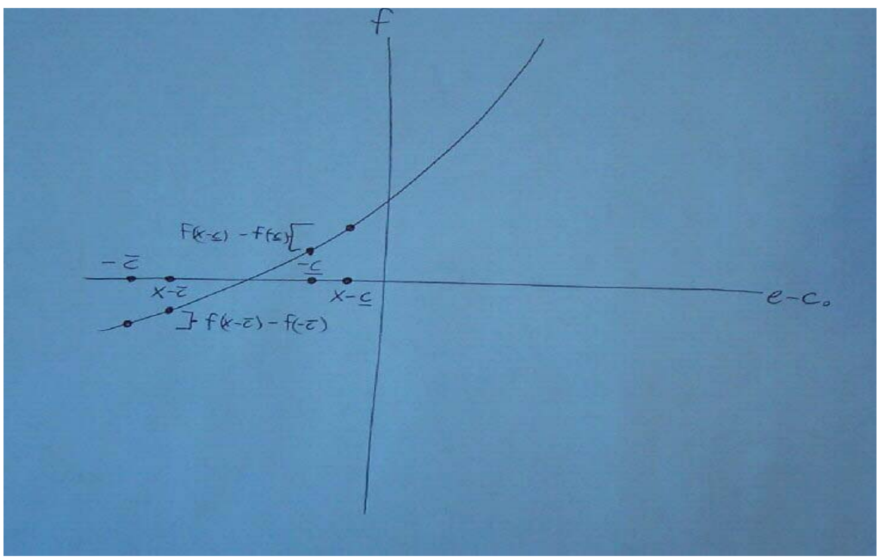

Figure 10.7: The Disutility of Effort

To summarize, the government's optimal contract will

(1)induce the normal firm to exert the first-best efficient effort level and achieve the first-best cost level,

(2) will yield that firm a positive profit.

(3) will induce the firm with special costs to exert something less than the first-best effort level

(4) result in a special-firm cost level higher than the first-best, but its profit will be zero.

There is a tradeoff between the government's two objectives of inducing the correct amount of effort and minimizing the subsidy to the firm.

Even under complete information, the government cannot provide a subsidy of zero, or the firms will refuse to build the cruise missile. Under incomplete information, not only must the subsidies be positive but the normal firm earns **informational rents**; the government offers a contract that pays the normal firm with more then under complete information to prevent it from mimicking a firm with special problems by choosing an inefficiently low effort. The firm with special problems, however, does choose an inefficiently low effort, because if it were assigned greater effort it would have to be paid a greater subsidy, which would tempt the normal firm to imitate it. In equilibrium, the government has compromised by having some probability of an inefficiently high subsidy ex post, and some probability of inefficiently low effort.

Procurement III: Moral Hazard with Hidden Information

The Order of Play

1 The government offers a contract agreeing to cover the firm's cost c of producing a cruise missile and specifying an additional price $p(c)$ for each cost level that the firm might report.
2 The firm accepts or rejects the contract.
3 Nature determines whether the firm has special problems that add costs of x, which has probability θ, or no special problems, which has probability $(1-\theta)$. We will call these "special" and "normal" firms, with the understanding that "special" problems may be the norm in engineering projects. The government and the firm both observe this move.
4 If the firm accepts, it chooses effort level e, unobserved by the government.
5 The firm finishes the cruise missile at a cost of $c = c_0 + x - e$ or $c = c_0 - e$ which is observed by the government, plus an additional cost $f(e - c_0)$ that the government does not observe. The government reimburses c and pays $p(c)$.

The contract must satisfy one overall participation constraint and and two incentive compatibility constraints, one for each type of firm. The participation constraint is

$$\theta[p_S - f(x - \overline{c})] + [1 - \theta][p_N - f(-\underline{c})] \geq 0. \tag{20}$$

The incentive compatibility constraints are the same as before: for the special firm,

$$p_S - f(x - \overline{c}) \geq p_N - f(-x - \underline{c}), \tag{21}$$

and for the normal firm,

$$p_N - f(-\underline{c}) \geq p_S - f(-\overline{c}). \tag{22}$$

As before, constraint (20) will be binding (and therefore satisfied as an equality), because the government will reduce the price as much as possible in order to avoid the deadweight loss of taxation. The normal firm's incentive compatibility constraint must also be binding, because if the pair $(\overline{c}, p_N)$ were strictly more attractive for the normal firm, the government could reduce the price p_N. Constraint (22) is therefore satisfied as an equality.[4] Knowing that constraints (20) and (22) are binding, we can write from constraint (20),

$$p_S = f(x - \overline{c}) - \frac{[1-\theta][p_N - f(-\underline{c})]}{\theta}. \tag{23}$$

Substituting from (23) for p_S into (22), we get

$$p_N - f(-\underline{c}) = f(x - \overline{c}) - \frac{[1-\theta][p_N - f(-\underline{c})]}{\theta} - f(-\overline{c}). \tag{24}$$

This can be solved for p_N to yield

$$p_N = \theta[f(x - \overline{c}) - f(-\overline{c})] + f(-\underline{c}), \tag{25}$$

which when substituted into (23) yields

$$p_S = [1-\theta][f(x - \overline{c}) - f(-\overline{c})]. \tag{26}$$

[4] The same argument does not hold for the firm with special costs, because if p_S were reduced, the participation constraint would be violated.xxx check this

From (2), the government's maximization problem under incomplete information is

$$\underset{\underline{c}, \overline{c}, p_N, p_S}{Maximize} \quad \theta \left[B - (1+\lambda)\overline{c} - \lambda p_S - f(x-\overline{c})\right] + [1-\theta]\left[B - (1+\lambda)\underline{c} \right. \tag{27}$$

Substituting for p_N and p_S from (25) and (26) reduces the problem to

$$\begin{aligned} \underset{\underline{c}, \overline{c}, p_N, p_S}{Maximize} \quad & \theta \left\{B - (1+\lambda)\overline{c} - \lambda[1-\theta][f(x-\overline{c}) - f(-\overline{c})] - f(x-\overline{c})\right\} \\ & + [1-\theta]\left[B - (1+\lambda)\underline{c} - \lambda\{\theta[f(x-\overline{c}) - f(-\overline{c})] + f(-\underline{c})\}\right. \end{aligned} \tag{28}$$

$$\underset{\underline{c}, \overline{c}, p_N, p_S}{Maximize} \quad \theta \{B - (1+\lambda)\overline{c} - \lambda[1-\theta][f(x-\overline{c}) - f(-\overline{c})] - f(x-\overline{c})\}$$
$$+ [1-\theta] [B - (1+\lambda)\underline{c} - \lambda\{\theta[f(x-\overline{c}) - f(-\overline{c})] + f(-\underline{c})\}$$

(1) The first-order condition with respect to $\underline{c}$ is

$$(1-\theta)[-(1+\lambda) + \lambda f'(-\underline{c}) + f'(-\underline{c})] = 0, \tag{29}$$

just as under adverse selection, which simplifies to

$$f'(-\underline{c}) = 1. \tag{30}$$

Thus, as earlier, $f'_N = 1$. The normal firm chooses the efficient effort level e^* in equilibrium, and $\underline{c}$ takes the same value as it did in Procurement I. Equation (24) can be rewritten as

$$p_N = p^* + f(x-\overline{c}) - f(-\overline{c}). \tag{31}$$

Because $f(x-\overline{c}) > f(-\overline{c})$, equation (31) shows that $p_N > p^*$. The normal firm earns more than its reservation utility, even under complete information. The special firm must therefore earn less than its reservation utility, so that the overall participation constraint will be satisfied as an equality.

$$\underset{\underline{c}, \overline{c}, p_N, p_S}{Maximize} \; \theta\left\{B - (1+\lambda)\overline{c} - \lambda[1-\theta][f(x-\overline{c}) - f(-\overline{c})] - f(x-\overline{c})\right\}$$
$$+ [1-\theta]\left[B - (1+\lambda)\underline{c} - \lambda\{\theta[f(x-\overline{c}) - f(-\overline{c})] + f(-\underline{c})\}\right.$$

(2) The first-order condition with respect to $\overline{c}$ is

$$\theta\left\{-(1+\lambda) - \lambda(1-\theta)[-f'(x-\overline{c}) + f'(-\overline{c})] + f'(x-\overline{c})\right\} + \lambda\,[1-\theta]\,[\quad (32)$$

This can be rewritten as

$$xcxcvxcvcxf'(x-\overline{c}) = 1 - sdfsfsdfsdfdsf \quad (33)$$

The ultimate effect: the participation constraint is binding, and total cost of p and c is less for the government than under incomplete information, so the deadweight loss of taxation is less too. The general features of the contract are similar, but the special firm now earns a loss, rather than breaking even.

Additional ways to alter the Procurement Game.

What if the firm discovers its costs only after accepting the contract? (we did this)

What if two firms bid against each other for the contract?

What if the firm can bribe the government?

What if the firm and the government bargain over the gains from the project instead of the government being able to make a take-it-or-leave-it contract offer?

What if the game is repeated, so the government can use the information it acquires in the second period?

If it is repeated, can the government commit to long-term contracts?

Can it commit not to renegotiate?

See Spulber (1989) and Laffont & Tirole (1993) if these questions interest you.