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9a.1 The Revelation Principle and Moral Hazard with Hidden Knowledge

"Production Game VII: Hidden Knowledge"

Players
The principal and the agent.

The Order of Play
(1) The principal offers the agent a wage contract of the form $w(q, m)$, where q is output and m is a message to be sent by the agent.
(2) The agent accepts or rejects the principal's offer.
(3) Nature chooses the state of the world θ, according to probability distribution $F(\theta)$. The agent observes θ, but the principal does not.
(4) If the agent accepts, he exerts effort e and sends a message m, both observed by the principal.
(5) Output is $q(e, \theta)$.

Payoffs
If the agent rejects the contract, $\pi_{agent} = \bar{U}$ and $\pi_{principal} = 0$.
If the agent accepts the contract, $\pi_{agent} = U(e, w, \theta)$ and $\pi_{principal} = V(q - w)$.

[1] Overheads made September 28, 1999.

Unravelling the Truth when Silence is the Only Alternative

Suppose that Nature uses the uniform distribution to assign the variable θ some value in the interval $[0, 10]$ and the agent's payoff is increasing in the principal's estimate of θ.

Usually we assume that the agent can lie freely, sending a message m taking any value in $[0, 10]$, but let us assume instead that he cannot lie but he can conceal information. Thus, if $\theta = 2$, he can send the uninformative message $m \geq 0$ (equivalent to no message), or $m \geq 1$, or $m = 2$, but not the lie that $m \geq 4$.

When $\theta = 2$ the agent might as well send a message that is the exact truth: "$m = 2$." If he were to choose the message "$m \geq 1$" instead, the principal's first thought might be to estimate θ as the average value of the interval $[1, 10]$, which is 5.5. But the principal would realize that no agent with a value of θ greater than 5.5 would want to send that message "$m \geq 1$" if that was the resulting deduction. This realization restricts the possible interval to [1, 5.5], which in turn has an average of 3.25. But then no agent with $\theta > 3.25$ would send the message "$m \geq 1$." The principal would continue this process of logical **unravelling** to conclude that $\theta = 1$. The message "$m \geq 0$" would be even worse, making the principal believe that $\theta = 0$. In this model, "No news is bad news." The agent would therefore not send the message "$m \geq 1$" and he would be indifferent between "$m = 2$" and "$m \geq 2$" because the principal would make the same deduction from either message.

The Revelation Principle. *For every contract $w(q,m)$ that leads to lying (that is, to $m \neq \theta$), there is a contract $w^*(q,m)$ with the same outcome for every θ but no incentive for the agent to lie.*

Applied to concrete examples, the revelation principle is obvious. Suppose we are concerned with the effect on the moral climate of cheating on income taxes, but anyone who makes $70,000 a year can claim he makes $50,000 and we do not have the resources to catch him. The revelation principle says that we can rewrite the tax code to set the tax to be the same for taxpayers earning $70,000 and for those earning $50,000, and the same amount of taxes will be collected without anyone having incentive to lie.

(3) **Truthtelling.** The equilibrium contract makes the agent willing to choose $m = \theta$.

The revelation principle says that a truthtelling equilibrium exists, but not that it is unique. It may well happen that the equilibrium is a weak Nash equilibrium in which the optimal contract gives the agent no incentive to lie but also no incentive to tell the truth. This is similar to the open-set problem discussed in Section 4.3; the optimal contract may satisfy the agent's participation constraint but makes him indifferent between accepting and rejecting the contract.

"The Salesman Game"

Players
A manager and a salesman.

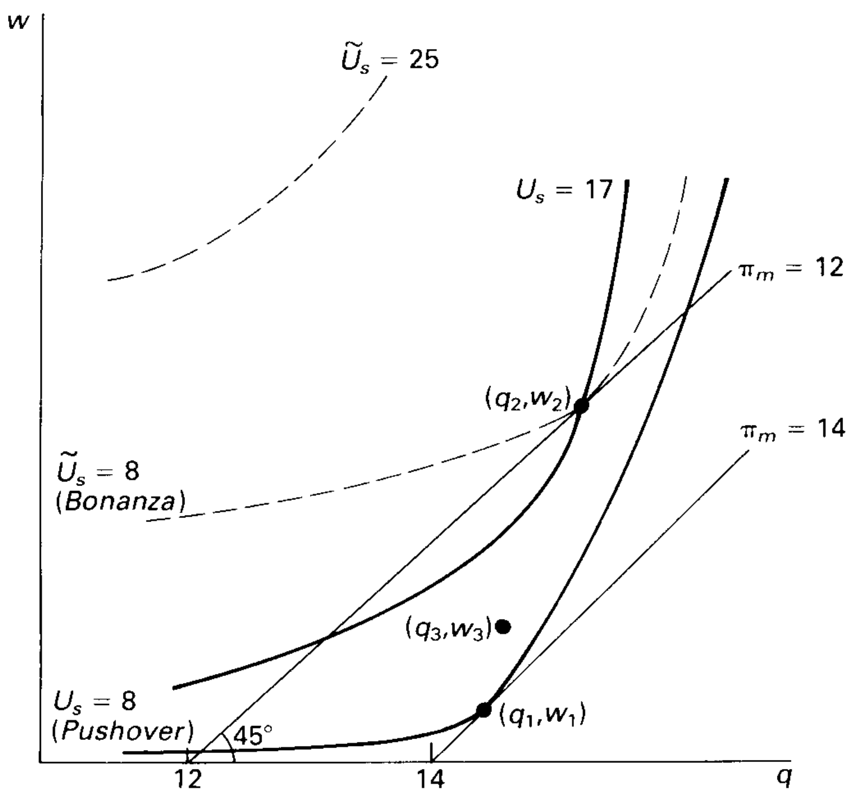
The Order of Play
(1) The manager offers the salesman a contract of the form $w(q, m)$, where q is sales and m is a message.
(2) The salesman decides whether or not to accept the contract.
(3) Nature chooses whether the customer is a *Bonanza* or a *Pushover* with probabilities 0.2 and 0.8. Denote the state variable "customer status" by θ. The salesman observes the state, but the manager does not.
(4) If the salesman has accepted the contract, he chooses his sales level q, which implicitly measures his effort.

Payoffs
The manager is risk neutral and the salesman is risk averse. If the salesman rejects the contract, his payoff is $\bar{U} = 8$ and the manager's is zero. If he accepts the contract, then

$$\begin{aligned} \pi_{manager} &= q - w \\ \pi_{salesman} &= U(q, w, \theta), \text{ where } \frac{\partial U}{\partial q} < 0, \frac{\partial^2 U}{\partial q^2} < 0, \frac{\partial U}{\partial w} > 0, \frac{\partial^2 U}{\partial w^2} < 0 \end{aligned}$$

Figure 9a.1 The Salesman Game with Curves for Pooling Equilibrium

Figure 8.1 ''The Salesman Game'' with Curves for a Pooling Equilibrium



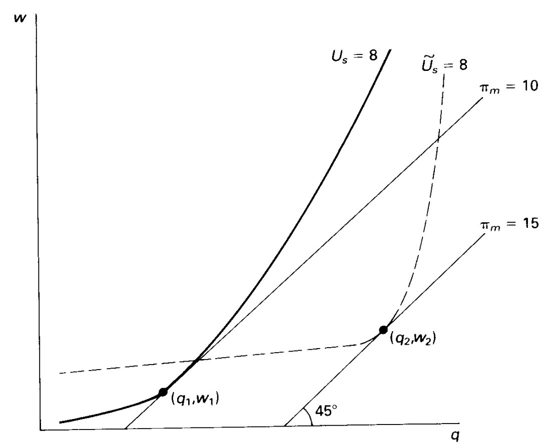
If it were common knowledge that the customer was a *Bonanza*, the principal could choose w_2 so that $U(q_2, w_2, Bonanza) = 8$ and offer the forcing contract

$$w = \begin{cases} 0 & \text{if } q < q_2. \\ w_2 & \text{if } q \geq q_2. \end{cases} \tag{1}$$

The salesman would accept the contract and choose $q = q_2$. But if the customer were actually a *Pushover*, the salesman would still choose $q = q_2$, an inefficient outcome that does not maximize profits. But high sales would be inefficient.

Figure 9a.2 Indifference Curves for a Separating Equilibrium

Figure 8.2 ''The Salesman Game'' with Curves for a Pooling Equilibrium



$$\text{Separating Contract}\begin{cases} \text{Agent announces } \textit{Pushover}: & w = \begin{cases} 0 \text{ if } q < q_1. \\ w_1 \text{ if } q \geq q_1. \end{cases} \\ \\ \text{Agent announces } \textit{Bonanza}: & w = \begin{cases} 0 \text{ if } q < q_2. \\ w_2 \text{ if } q \geq q_2. \end{cases} \end{cases} \quad (2)$$

Again, we know from the revelation principle that we can narrow attention to contracts that induce the salesman to tell the truth. With the indifference curves of Figure 9a.2, contract (9a.2) induces the salesman to be truthful and the incentive compatibility constraint is satisfied. If the customer is a *Bonanza*, but the salesman claims to observe a *Pushover* and chooses q_1, his utility is less than 8 because the point (q_1, w_1) lies below the $\tilde{U_S} = 8$ indifference curve. If the customer is a *Pushover* and the salesman claims to observe a *Bonanza*, then although (q_2, w_2) does yield the salesman a higher wage than (q_1, w_1), the extra income is not worth the extra effort, because (q_2, w_2) is far below the indifference curve $U_S = 8$.

*9a.3 Price Discrimination

Pigou was a contemporary of Keynes at Cambridge who usefully divided price discrimination into three types in 1920, but who named them so obscurely that I relegate his terms to the endnotes and will use better names here:

(a) Interbuyer Price Discrimination. This is when the seller can charge different prices to different buyers. Smith's price for a hamburger is $4 perburger, but Jones's is $6.

(b) Nonlinear Pricing or **Interquantity Price Discrimination**. This is when the seller can charge different unit prices for different quantities. Anyone can buy their first sausage for $9, their second sausage for $4, and their third sausage for $3. Rather than paying the "linear" total price of $9 for one sausage, $18 for two, and $27 for three, they thus pay a nonlinear price of $9 for one sausage, $13 for two, and $16 for three, the concave price path shown in Figure 9a.3.

(c) Perfect Price Discrimination. This combines interbuyer and interquantity price discrimination. When the seller does have perfect information and can charge each buyer that buyer's reservation price for each unit bought, Smith might end up paying $50 for his first hot dog and $20 for his second, while next to him Jones pays $4 for his first and $3 for his second.

Figure 9a.3 Linear and Nonlinear Pricing

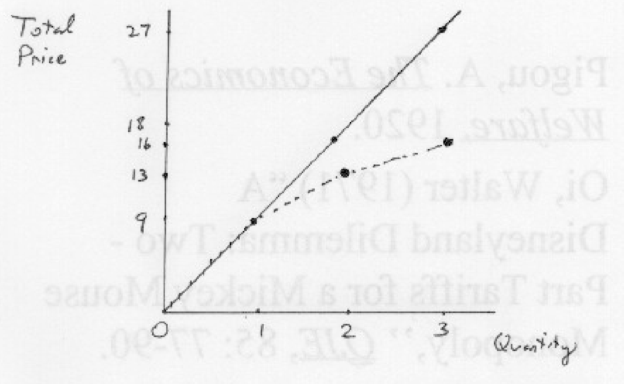


Figure 9a.3 : Linear and Nonlinear Pricing

"Varian's Nonlinear Pricing Game"

Players
One seller and two buyers, Smith and Jones.

The Order of Play
(0) Nature assigns one of the two buyers to be Unenthusiastic with utility function u and the other to be Valuing with utility function v, Smith and Jones having equal probabilities of filling each role. The seller does not observe Nature's move.
(1) The seller offers a price mechanism $r(x)$ under which a buyer can buy amount x for total price $r(x)$.
(2) The buyers simultaneously choose quantities x_u and x_v to buy.

Payoffs
The seller's payoff is $r(x_u) + r(x_v) - c(x_u + x_v)$. The buyers' payoffs are $u(x_u) - r(x_u)$ and $v(x_v) - r(x_v)$ if x is positive, and 0 if $x = 0$, with $u', v' > 0$ and $u'', v'' < 0$. The total and marginal willingnesses to pay are greater for V: for all x,

$$\begin{array}{ll} (a) & u(x) < v(x) \text{and} \\ (b) & u'(x) < v'(x). \end{array} \qquad (3)$$

Condition (9a.3b) is known as the **single crossing property**, since it implies that the indifference curves of the two agents cross at most one time. Combined with Condition (9a.3a), it means they never cross– the Valuing buyer always has stronger demand.

Perfect Price Discrimination

The game would allow perfect price discrimination if the seller did know which buyer had which utility function. He can then just maximize profit subject to the participation constraints for the two buyers:

$$\overset{Maximize}{r(x_u), r(x_v), x_u, x_v} \quad r(x_u) + r(x_v) - c(x_u + x_v). \tag{4}$$

subject to

$$\begin{array}{ll} (a) & u(x_u) - r(x_u) \geq 0 \text{and} \\ (b) & v(x_v) - r(x_v) \geq 0. \end{array} \tag{5}$$

The constraints will be satisfied as equalities, since the seller will charge all that the buyers will pay. Substituting for $r(x_u)$ and $r(x_v)$ into the maximand, the first order conditions become

$$\begin{array}{ll} (a) & u'(x_u*) - c = 0 \text{and} \\ (b) & v'(x_v*) - c = 0. \end{array} \tag{6}$$

Thus, the seller will choose quantities so that each buyer's marginal utility equals the marginal cost of production, and will choose prices so that the entire consumer surpluses are eaten up: $v * (x_u) = u(x_u*)$ and $v * (x_v) = v(x_v*)$.

Interbuyer Price Discrimination

The interbuyer price discrimination problem arises when the seller knows which utility functions Smith and Jones have and can sell to them separately but he must charge each buyer a single price per unit and let the buyer choose the quantity. The seller's problem is

$$\underset{x_u, x_v, p_u, p_v}{Maximize} \quad p_u x_u + p_v x_v - c(x_u + x_v), \tag{7}$$

subject to

$$\begin{array}{ll} (a) & u(x_u) - p_u x_u \geq 0 \text{and} \\ (b) & v(x_v) - p_v x_v \geq 0 \end{array} \tag{8}$$

and

$$\begin{array}{ll} (a) & x_u = argmax[u(x_u) - p_u x_u] \text{and} \\ (b) & x_v = argmax[v(x_v) - p_v x_v] \geq 0. \end{array} \tag{9}$$

This should remind you of moral hazard. It is very like the problem of a principal designing two incentive contracts for two agents to induce appropriate effort levels given their different disutilities of effort.

The agents will solve their quantity choice problems in (9a.9) to yield

$$\begin{array}{ll} (a) & u'(x_u) - p_u = 0 \text{and} \\ (b) & v'(x_v) - p_v = 0. \end{array} \tag{10}$$

Thus, we can simplify the original problem in (9a.7) to

$$\underset{x_u, x_v}{Maximize} \quad u'(x_u)x_u + v'(x_v)x_v - c(x_u + x_v), \tag{11}$$

subject to

$$\begin{array}{ll} (a) & u(x_u) - p_u x_u \geq 0 \text{and} \\ (b) & v(x_v) - p_v x_v \geq 0. \end{array} \tag{12}$$

The first order conditions are

$$\begin{array}{ll} (a) & u''(x_u)x_u + u' = c \text{and} \\ (b) & v''(x_v)x_v + v' = c. \end{array} \tag{13}$$

These are just the Marginal Revenue equals Marginal Cost condition that any monopolist uses, but one for each buyer instead of one for the entire market.

Back to Nonlinear Pricing

Neither of those two problems are mechanism design problems, since the seller is perfectly informed about the types of the buyers and has no need to worry about designing incentives to separate them. In the original game, however, separation is the seller's main concern. He must satisfy not just the participation constraints, but self selection constraints. The seller's problem is

$$\underset{x_u, x_v, r(x_u), r(x_v)}{Maximize} \quad r(x_u) + r(x_v) - c(x_u + x_v). \tag{14}$$

subject to the participation constraints,

$$\begin{array}{ll} (a) & u(x_u) - r(x_u) \geq 0 \\ (b) & v(x_v) - r(x_v) \geq 0, \end{array} \tag{15}$$

and the self selection constraints,

$$\begin{array}{ll} (a) & u(x_u) - r(x_u) \geq u(x_v) - r(x_v) \\ (b) & v(x_v) - r(x_v) \geq v(x_u) - r(x_u). \end{array} \tag{16}$$

Let us start with the premise that a constraint is binding and see if we can use our data– principally assumptions (9a.3a) and (9a.3b)– to find a contradiction. Assume that the Valuing participation constraint, (9a.15b), is binding. Then $v(x_v) = r(x_v)$. Substituting for $v(x_v)$ in the self selection constraint (9a.16b) then yields

$$r(x_v) - r(x_v) \geq v(x_u) - r(x_u), \tag{17}$$

so $r(x_u) \geq v(x_u)$. It follows from assumption (9a.3a), which says $u(x) < v(x)$, that $r(x_u) \geq u(x_u)$. But the Unenthusiastic participation constraint, (9a.15a), says that $r(x_u) \leq u(x_u)$, and since these are compatible only when $r(x_u) = u(x_u)$ and we have assumed that (9a.15b) is the binding participation constraint, we have arrived at a contradiction. Our starting point must be false, and it is in fact (9a.15a) that is the binding participation constraint.

We could next start with the premise that self selection constraint (9a.16a) is binding and derive a contradiction using assumption (9a.3b). But the reasoning above showed that if the participation constraint is binding for one type of agent then the self selection constraint will be binding for the other, so we can jump to the conclusion that it is in fact self selection constraint (9a.16b) that is binding.

Rearranging our two binding constraints and setting them out as equalities yields:

$$\begin{array}{ll} (9a.15a') & r(x_u) = u(x_u) \\ & and \\ (9a.16b') & r(x_v) = r(x_u) - v(x_u) + v(x_v) \end{array}$$

This allows us to reformulate the seller's problem from (9a.14) as

$$\underset{x_u, x_v}{Maximize} \quad u(x_u) + u(x_u) - v(x_u) - v(x_v) - c(x_u + x_v), \tag{18}$$

which has the first-order conditions

$$\begin{array}{ll} (a) & u'(x_u) - c + [u'(x_u) - v'(x_u)] = 0 \\ (b) & v'(x_v) - c = 0 \end{array} \tag{19}$$

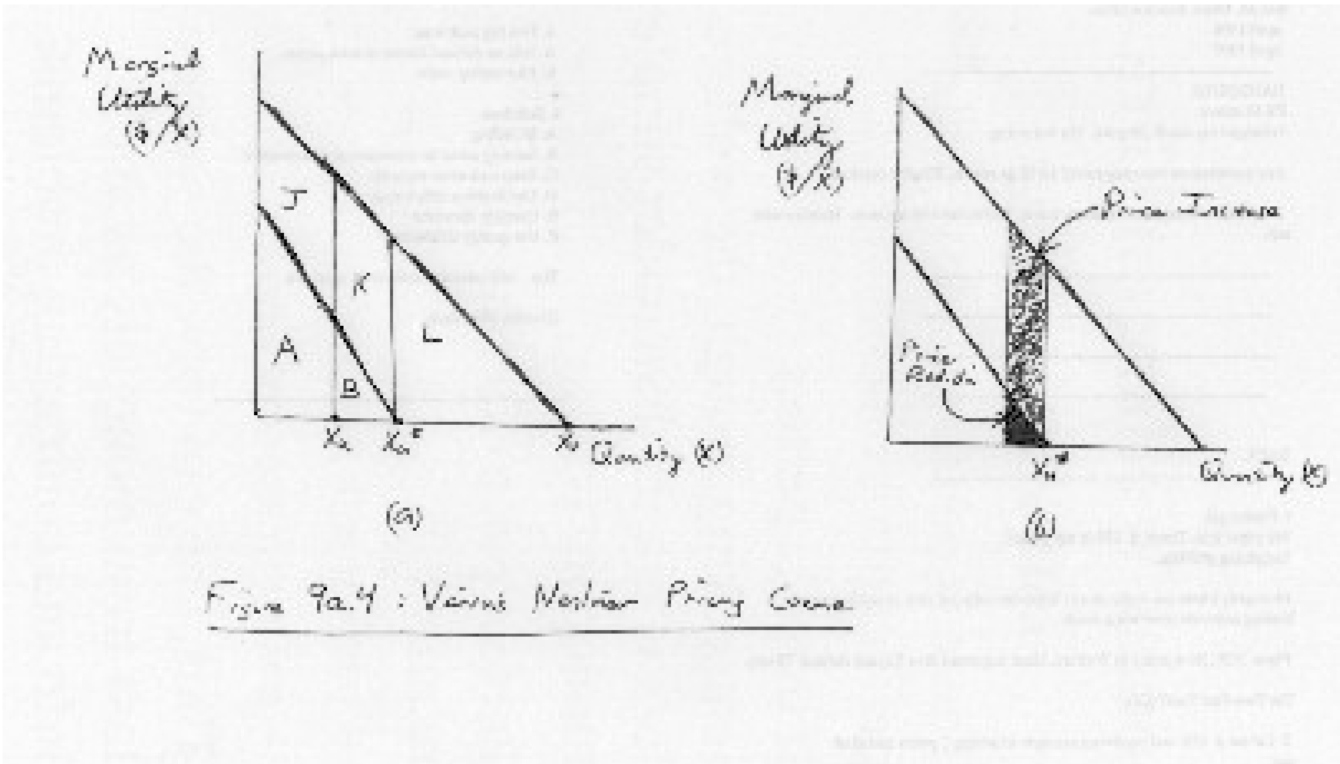
These first-order conditions could be solved for exact values of x_u and x_v if we chose particular functional forms, but they are illuminating even if we do not. Note from (9a.19b) that the Valuing buyer buys a quantity such that his last unit's marginal utility exactly equals the marginal cost of production; his consumption is at the efficient level. The Unenthusiastic type, however, buys less than his first-best amount, something we can deduce using the single-crossing property, assumption (9a.3b), that $u'(x) < v'(x)$, which implies from (9a.19a) that $u'(x_u) - c > 0$ and the Unenthusiastic type has not bought enough to drive his marginal utility down to marginal cost. The intuition is that the seller must sell less than first-best optimal to the Unenthusiastic type so as not to make that contract too attractive to the Valuing type. On the other hand, making the Valuing type's contract more valuable to him actually helps separation, so x_v is chosen to maximize social surplus.

The single-crossing property has another very important implication. Substituting from first-order condition (9a.19b) into first-order condition (9a.19a) yields

$$[u'(x_u) - v'(x_v)] + [u'(x_u) - v'(x_u)] = 0 \tag{20}$$

The second term in square brackets is negative by the single-crossing property. Thus, the first term must be positive. But since the single-crossing property tells us that $[u'(x_u) - v'(x_u)] < 0$, it must be true, since $v'' < 0$, that if $x_u \geq x_v$ then $[u'(x_u) - v'(x_v)] < 0$– that is, that the first term is negative. We cannot have that without contradiction, so it must be that $x_u < x_v$– the Unenthusiastic buyer buys strictly less than the Valuing buyer. This accords with our intuition, and also lets us know that the equilibrium is separating, not pooling. (We still have not proven that the equilibrium involves both players buying a positive amount, something hard to prove elegantly since one player buying zero would be a corner solution to our maximization problem.)

Figure 9a.4 The Varian Nonlinear Pricing Game



A Graphical Approach to the Same Problem

Under perfect price discrimination, the seller would charge $r_u = A + B$ and $r_v = J + K + L$ to the two buyers for quantities x_u^* and x_v, as shown in Figure 9a.4a. An attempt to charge $r(x_u^*) = A + B$ and $r(x_v) = J + K + L$, however, would simply lead to both buyers choosing to buy x_u^*, which would yield the Valuing buyer a payoff of $J + K$ rather than the 0 he would get as a payoff from buying x_v.

The seller could separate the two buyers by charging $r(x_u^*) = A+B$ and $r(x_v) = A + B$, since the Unenthusiastic buyer would have no reason to switch to the greater quantity, but that would not increase his profits any over pooling. Figure 9a.4b shows that the seller would do better to slightly reduce the quantity sold to the Unenthusiastic buyer and reduce the price by the amount of the dark shading, while selling x_u^* to the Valuing buyer and raising the price to him by the light shaded area. The Valuing buyer will still not be tempted to buy the smaller quantity at the lower price.

The profit-maximizing mechanism found earlier is shown in Figure 9a.4a by $r(x_u) = A$ and $r(x_v) = A + B + K + L$. The Unenthusiastic buyer is left with a binding participation constraint, because $r(x_u) = A = u(x_u)$. The Valuing buyer has a non-binding participation constraint, because $r(x_v) = A + B + K + L < v(x_v) = A + B + J + K + L$. But the Valuing buyer does have a binding self selection constraint, because he is exactly indifferent between buying x_u and x_v— $v(x_u) - r(x_u) = (A + J) - A$ and $v(x_v) - r(x_v u) = (A+B+J+K+L) - (A+B+K+L)$. Thus, the diagram replicates the algebraic conclusions.

*9a.4 Rate-of-Return Regulation and Government Procurement "Government Procurement"

Players The government and the firm.

The Order of Play
(0) Nature assigns the firm ability $a \in \{a_l, a_h\}$,where low ability has probability θ and high ability has probability $(1-\theta)$.
(1) The government offers a contract $s(c)$ agreeing to cover the firm's cost c of producing a cruise missile and specifying a subsidy s for each cost level that the firm might report.
(2) The firm accepts or rejects the contract.
(3) If the firm accepts, it chooses effort level e, unobserved by the government.
(4) The firm finishes the cruise missile at a cost of $c = c_0 - a - e$ which is observed by the government. The government reimburses the cost and pays the appropriate subsidy.

Payoffs
Both firm and government are risk neutral and both receive payoffs of zero if the firm rejects the contract. If the firm accepts, its payoff is

$$\pi_{firm} = s - f(e), \tag{21}$$

where $f(e)$, the disutility of effort, is increasing and convex, so $f' > 0$ and $f'' > 0$. Assume, too, for technical convenience, that f is increasingly convex, so $f''' > 0$.[2] The government's payoff is

$$\pi_{government} = B - (1+\lambda)c - \lambda s - f(e), \tag{22}$$

where B is the benefit from the cruise missile and λ is the deadweight loss from the taxation needed for government spending. This loss is estimated to be around \$0.30 for each \$1 of tax revenue raised, at the margin for the United States [Hausman & Poterba [1987]).

[2] The assumption that $f''' > 0$ allows the use of first order conditions by making the maximand in (9a.33)– a difference of two concave functions– concave. See p. 58 of Laffont & Tirole (1993).

Assume for the moment that B is large enough that the government definitely wishes to build the missile (how large will become apparent later.) Cost, not output, is the focus of this model. The optimal output is one cruise missile regardless of agency problems, but the government wants to minimize the cost of producing the missile .

The model differs from other principal-agent models in this book because the principal cares about the welfare of the agent. If the government cared only about the value of the cruise missile and the cost to taxpayers, its payoff would be $B - (1+\lambda)c - (1+\lambda)s$. Instead, the payoff function maximizes social welfare, the sum of the welfares of the taxpayers and the firm; the welfare of the firm is $s - f(e)$, and summing the two welfares yields equation (9a.22). Either kind of government payoff function may be realistic, depending on the political balance in the country being modelled, and the model will have similar properties whichever one is used.

Government Procurement I: Symmetric Information In the first variant of the game, the firm's ability, a, is observed by the government, which can therefore specify a contract $s(a, c)$, in effect assigning different contracts to the two types of firms. The government pays subsidies of s_h to a firm with ability a_h ("Firm H") for the low cost $\underline{c}$, s_l to a firm with ability a_l ("Firm L") for the high cost $\overline{c}$, and a boiling-in-oil subsidy of $s = -\infty$ to a firm that does not choose the contract designed for it.

The participation constraints will be binding for both types of firms, and to make a firm's payoff zero the government will provide subsidies that exactly cover the firm's disutility of effort. Since there is no uncertainty we can invert the cost equation and write it as $e = c_0 - a - c$. The subsidies will be $s_l = f(c_0 - a_l - \underline{c})$ and $s_h = f(c_0 - a_h - \overline{c})$. Suppose the government knows the firm has low ability. Substituting the subsidy into the government's payoff function yields

$$\pi_{government} = B - (1+\lambda)\overline{c} - \lambda f(c_0 - a_l - \overline{c}) - f(c_0 - a_l - \overline{c}). \quad (23)$$

Since $f'' > 0$, the government's payoff function is concave, and standard optimization technique can be used. The first order condition for the optimal level of the cost assigned to the low-ability firm, $\overline{c}$, is

$$\frac{\partial \pi_{government}}{\partial \overline{c}} = -(1+\lambda) + \lambda f'(c_0 - a_l - \overline{c}) + f'(c_0 - a_l - \overline{c}) = 0, \quad (24)$$

so

$$f'(c_0 - a_l - \overline{c}) = 1. \quad (25)$$

Equation (9a.25) says that at the efficient effort level, the marginal disutility of effort equals the marginal reduction in cost because of effort. Exactly the same is true for firm H, so $c_0 - a_h - \underline{c} = c_0 - a_l - \overline{c}$, and $f'(c_0 - a_h - \underline{c}) = 1$. Thus, $s_l = s_h$. The cost targets assigned to each firm are $\overline{c} = c_0 - a_l - e^*$ and $\underline{c} = c_0 - a_h - e^*$. The two firms exert the same efficient effort level and are paid the same positive subsidy to compensate for the disutility of effort. Let us call this effort level e^* and the subsidy level s^*. The assumption that B is sufficiently large can now be made more specific: it is that $B - (1+\lambda)(c_0 - a_l - e^* - f(e^*)) \geq 0$.

Government Procurement II: Asymmetric Information In the second variant of the game, ability is not observed by the government, which must therefore provide incentives for the firm to volunteer its type if firm H is to produce at lower cost than firm L.

Let us find the optimal contract with values $(\underline{c}, s_h)$ and $(\overline{c}, s_l)$ and heavy punishments for other cost levels. Although we have not yet established that the optimal contract is separating, we will go through the analysis for two separate contracts, and if a pooling contract is optimal we will find that $\underline{c} = \overline{c}$.

The contract must satisfy participation constraints and incentive compatibility constraints for each type of firm. The participation constraint for firm L is

$$s_l - f(c_0 - a_l - \overline{c}) \geq 0, \tag{26}$$

and for firm H it is

$$s_h - f(c_0 - a_h - \underline{c}) \geq 0. \tag{27}$$

The incentive compatibility constraint for firm L is

$$s_l - f(c_0 - a_l - \overline{c}) \geq sh - f(c_0 - a_l - \underline{c}), \tag{28}$$

and for firm H it is

$$s_h - f(c_0 - a_h - \underline{c}) \geq s_l - f(c_0 - a_h - \overline{c}). \tag{29}$$

Since the high-ability firm can achieve the same cost level as the low-ability firm with less effort, if constraint (9a.26) is satisfied, so is (9a.27). Constraint (9a.26) will be binding (and therefore satisfied as an equality), because the government will reduce the subsidy as much as possible in order to avoid the deadweight loss of taxation. The incentive compatibility constraint for firm H must also be binding, because if the pair $(\overline{c}, s_h)$ were strictly more attractive for firm H, the government could reduce the subsidy s_h. Constraint (9a.29) is therefore satisfied as an equality.[3] Knowing that constraints (9a.26)

[3]The same argument does not hold for firm L, because if s_l were reduced, the participation constraint would be violated.

and (9a.29) are binding, we can write from constraint (9a.26),

$$s_l = f(c_0 - a_l - \overline{c}) \tag{30}$$

and, making use of both (9a.26) and (9a.29),

$$s_h = f(c_0 - a_h - \underline{c}) + f(c_0 - a_l - \overline{c}) - f(c_0 - a_h - \overline{c}). \tag{31}$$

From (9a.22), the government's maximization problem under incomplete information is

$$\underset{\underline{c}, \overline{c}, s_h, s_l}{Maximize} \quad \theta\left[B - (1+\lambda)\overline{c} - \lambda s_l - f(c_0 - a_l - \overline{c})\right] + [1-\theta]\left[B - (1+\lambda)\underline{c} - \lambda s_h - f(c \tag{32}$$

Substituting for s_l and s_h from (9a.30) and (9a.31) reduces the problem to

$$\underset{\underline{c}, \overline{c}}{Maximize} \quad \theta[B - (1+\lambda)\overline{c} - \lambda(f(c_0 - a_l - \overline{c}) - f(c_0 - a_l - \overline{c})] + [1-\theta][B - (1+\lambda)\underline{c}$$

$$-\lambda f(c_0 - a_h - \underline{c}) - \lambda f(c_0 - a_l - \overline{c}) + \lambda f(c_0 - a_h - \overline{c}) - f(c_0 - a_h - \underline{c})]. \tag{33}$$

The first order condition with respect to $\underline{c}$ is

$$(1-\theta)[-(1+\lambda) + \lambda f'(c_0 - a_h - \underline{c}) + f'(c_0 - a_h - \underline{c})] = 0, \tag{34}$$

which simplifies to

$$f'(c_0 - a_h - \underline{c}) = 1. \tag{35}$$

Thus, as earlier, $f_h'(e) = 1$. The high-ability firm chooses the efficient effort level e^* in equilibrium, and $\underline{c}$ takes the same value as it did in Government Procurement I. From the definition of $s^* = f(e^*)$ in that game, equation (9a.31) can be rewritten as

$$s_h = s^* + f(c_0 - a_l - \overline{c}) - f(c_0 - a_h - \overline{c}). \tag{36}$$

Because $f(c_0 - a_l - \overline{c}) > f(c_0 - a_h - \overline{c})$, equation (9a.36) shows that $s_h > s^*$. Incomplete information increases the subsidy to the high-ability firm, which earns more than its reservation utility in the game with incomplete information. Since the low-ability firm will earn exactly its reservation, this means that the government is on average providing its supplier with an above-market rate of return, not because of corruption or political influence, but because that is the way to induce high-ability suppliers to reveal that its ability is high. This should be kept in mind as an alternative to the product quality model of Chapter 5 and the efficiency wage model of Chapter 7 for why above-average rates of return persist.

Turning now to the contract alternative to be chosen by the high-ability firm, the first order condition for maximizing the government payoff (9a.33) with respect to $\overline{c}$ is

$$\begin{aligned}&\theta\left[-(1+\lambda)+\lambda f'(c_0-a_l-\overline{c})+f'(c_0-a_l-\overline{c})\right]+\\&[1-\theta]\left[\lambda f'(c_0-a_l-\overline{c})+f'(c_0-a_h-\overline{c})\right]=0.\end{aligned} \tag{37}$$

This can be rewritten as

$$f'(c_0-a_h-\overline{c})=1-\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{\theta}{1-\theta}\right)\left[f'(c_0-a_h-\overline{c})+f'(c_0-a_l-\overline{c})\right]. \tag{38}$$

Since the right-hand-side of equation (9a.38) is less than one, firm L has a lower level of f' than firm H, and must be exerting effort less than e^*, since $f''>0$. Perhaps this explains the expression "good enough for government work". Also since the participation constraint, (9a.26), is satisfied as an equality, it must also be true that $s_l<s^*$. The low-ability firm's subsidy is lower than under full information, although since its effort is also lower, its payoff stays the same.

We must also see that the incentive compatibility constraint for firm L is satisfied as a weak inequality; the low-ability firm is not near being tempted to pick the high-ability firm's contract. This is a bit subtle. Setting the left-hand-side of the incentive compatibility constraint (9a.28) equal to zero because the participation constraint is binding for firm L, substituting in for s_h from equation (9a.31) and rearranging yields

$$f(c_0-a_h-\underline{c})-f(c_0-a_l-\underline{c})\geq f(c_0-a_h-\overline{c})-f(c_0-a_l-\overline{c}). \tag{39}$$

This is true, and true as a strict inequality, because $f''>0$ and the arguments of f on the left-hand-side of equation (9a.39) take larger values than on the right-hand side.

To summarize, the government's optimal contract will induce the high-ability firm to exert the first-best efficient effort level and achieve the first-best cost level, but will yield that firm a positive profit. The contract will induce the low-ability firm to exert something less than the first-best effort level and result in a cost level higher than the first-best, but the firm will obtain zero profit.

***9a.5 The Groves Mechanism** The mayor of a town is considering installing a streetlight costing \$100. Each of the five houses near the light would be taxed exactly \$20, but the mayor will only install it if he decides that the sum of the residents' valuations for it is greater than the cost.

The problem is to discover the valuations. If the mayor simply asks them, householder Smith might say that his valuation is \$5,000, and householder Brown might say that he likes the darkness and would pay \$5,000 to *not* have a streetlight, but all the mayor could conclude would be that Smith's valuation exceeded \$20 and Brown's did not. Talk is cheap, and the dominant strategy is to overreport or underreport.

The flawed mechanism just described can be denoted by

$$M_1 : \quad \left(20, \sum_{i=1}^{5} m_i \geq 100\right), \tag{40}$$

which means that each resident pays 20, and the light is installed if the sum of the messages exceeds 100.

An alternative is to make resident i pay the amount of his message, or pay zero if it is negative. This mechanism is

$$M_2: \quad \left(Max\{m_i, 0\}, \sum_{j=1}^{5} m_j \geq 100\right). \tag{41}$$

Mechanism M_2 has no dominant strategy. Player i would announce $m_i = 0$ if he thought the project would go through without his support, but he would announce up to his valuation if necessary. There is a continuum of Nash equilibria that attain the efficient result. Most of these are asymmetric, and there is a problem of how the equilibrium to be played out becomes common knowledge. This is a simple mechanism, however, and it already teaches a lesson: that people are more likely to report their true political preferences if they must bear part of the costs themselves.

Instead of just ensuring that the correct decision is made in a Nash equilibrium, it may be possible to design a mechanism which makes truthfulness a **dominant-strategy mechanism**. Consider the mechanism

$$M_3: \quad \left(100 - \sum_{j \neq i} m_j, \sum_{j=1}^{5} m_j \geq 100\right). \tag{42}$$

Under mechanism M_3, player i's message does not affect his tax bill except by its effect on whether or not the streetlight is installed. If player i's valuation is v_i, his full payoff is $v_i - 100 + \sum_{j \neq i} m_j$ if $m_i + \sum_{j \neq i} m_j \geq 100$, and zero otherwise. It is not hard to see that he will be truthful in a Nash equilibrium in which the other players are truthful, but we can go further: truthfulness is weakly dominant. Moreover, the players will tell the truth whenever lying would alter the mayor's decision.

Consider a numerical example. Suppose that Smith's valuation is 40 and the sum of the valuations is 110, so the project is indeed efficient. If the other players report their truthful sum of 70, Smith's payoff from truthful reporting is his valuation of 40 minus his tax of 30. Reporting more would not change his payoff, while reporting less than 30 would reduce it to 0.

If we are wondering whether Smith's strategy is dominant, we must also consider his best response when the other players lie. If they underreported, announcing 50 instead of the truthful 70, then Smith could make up the difference by overreporting 60, but his payoff would be -10 $(= 40 + 50 - 100)$ so he would do better to report the truthful 40, killing the project and leaving him with a payoff of 0. If the other players overreported, announcing 80 instead of the truthful 70, then Smith benefits if the project goes through, and he should report at least 20 to obtain his payoff of 40 minus 20. He is willing to report exactly 40, so there is an equilibrium with truth-telling.