Final Examination, G601 ANSWERS.

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Since the answers were so heterogeneous, I did not assign partial-credit point values in grading these. Rather, I looked for perfect answers, and then gave overall grades for each exam based on my impression of all the answers, noting mistakes where I saw them.

1. (10 points) The payoff matrix below describes profits for two firms depending on whether Apex expands or maintains its current capacity and whether Brydox shrinks its capacity or exits altogether. The decision is made simultaneously.

		Brydox	
		Shrink	Exit
Apex	Expand	-2,-1	4,0
	Maintain	0,1	2,0

(a) (6 points) What are the pure strategy equilibria for this game, if any?

ANSWER. The pure strategy equilibria are (Expand, Exit) and (Maintain, Shrink).

(b) (4 points)What are the mixed strategy equilibria for this game, if any?

ANSWER. Let e be the probability that Apex expands and s be the probability that Brydox shrinks. In equilibrium Apex's expected payoffs from its two pure strategies are equal, so

Payoff (Expand) = -2s +4(1-s) = Payoff(Maintain) = 0s + 2(1-s),

so -2s +4 - 4s = 2-2s, so 2 = 4s, and s =0.5.

Brydox's expected payoffs from its two pure strategies are equal, so

Payoff (Shrink) = -1e + 1(1-e) = Payoff(Exit) = 0

so -e +1 -e = 0 and e =0.5.

c) (3 points) Now suppose that Apex makes its decision first. Draw a game tree to represent the new game.

ANSWER. DIAGRAM

(d) (3 points) What is the equilibrium of the new game?

ANSWER. In equilibrium, Apex chooses Expand and Brydox chooses Exit.

2. (5 points) Last year Apex paid its salesmen a fixed salary, and they sold 40 million dollars worth of merchandise. The total salary bill was 30 million dollars. This year, Apex has decided to pay its salesmen a base salary of zero, but a commission of 75 percent of their sales revenue. What can we expect to happen to the salary bill this year? Explain.

ANSWER. The salary bill will rise. With the increased incentives, salesmen will work harder, selling more than 40 million dollars. This will cause the commissions to exceed 30 million.

3. (5 points) Two hot dog vendors are deciding where to locate on a NASCAR racetrack which is circular. One vendor chooses location first, then the other chooses, and then they compete in prices, having fixed their locations. If we think of the racetrack as a clock, which is the most likely locations for them?
(a) Both at three o 'clock.
(b) One at three and one at 6 o'clock.
@ (c) One at three and one at 9 o'clock.
(d) One at three and one at 12 o'clock.
(e) Both at 12 o'clock.

4. (5 points) Why are workers on a team choosing their efforts like the prisoners in the prisoner's dilemma?

ANSWER. Workers on a team are punished or rewarded jointly for the output of the team, but each one chooses his own effort level. Team assignments such as the G100 assignments are an example. The workers' best outcome is if all choose high effort. Each worker, however, would like to have the others choose high effort and he himself choose low effort. Moreover, he will want to choose low effort even if the others also choose low effort. Thus, all the workers will end up choosing low effort, a worse outcome than if all chose high.

In the prisoner's dilemma, the prisoners' best outcome is if both choose to keep quiet. Each prisoner, however, would confesses. Thus, both prisoners end up confessing, a worse outcome than if both keep quiet.

5. (10 points) George and Arthur are thinking of investing in more shares of Apex. Both of them believe that Apex's profits will be higher this year with probability .7. George discovers that Apex's profits are actually lower this year, so rather than buying more shares, he sells them. Arthur still doesn't know what

Apex's profits will be, but he happened to overhear George calling his broker to sell shares-- or perhaps I shouldn't say "happened", because he purposely sat near George, hoping to overhear George telling someone about Apex's profits. So Arthur sold his shares too, rather than buying more shares.

Construct a game tree with appropriate information sets to illustrate this game. Make up payoffs at the end such that each player is behaving rationally to maximize his payoff.

ANSWER. Diagram. (get it from th eG300 final)

6. (13 points) (a) What would you have Margaret Slade do to improve her paper?

ANSWER.
This paper is poorly written, and the poor writing shades into methodological flaws such as not looking hard enough at the institutional context and not looking for simple patterns in the data. It is, however, a publishable paper because it does have the great virtue of collecting some very interesting data, asking a good question, and very carefully using particular techniques to address that question.

Some things that are not flaws:

1. Quoting a business speech. Perhaps not as long a quote was needed, but background information on the industry is very important for an industry-specific paper like this one. More such information was needed, not less, since one speech is better than nothing, but not as reliable as industry statistics and reports by outside observers such as stock analysts.

2. Using first person writing, e.g. ``I did an OLS regression first," instead of passive-voice writing, e.g., ``An OLS regression was done first." Oddly enough, the standard writing manuals (Fowler, Strunk and White, etc.) all say to avoid the passive voice, but high school English teachers seem to give the opposite advice. The reason to avoid the passive is that it usually requires more and longer words and that it conveys less information, often leading to ambiguity about who actually did what is being described. Both of those bad features can be useful in bureaucratic writing, but avoid them if you want to be clear.

ANSWER.
(a) Fixable flaws.

1. The abstract, and indeed, the entire paper, is unclear about what actually is being tested. This is fixable, because something fairly simple is being tested, and that is known to the author, if not to the reader: whether firms respond to price changes by their competitors and whether they respond symmetrically to increases and decreases in price. This should be made clear.
Also, the economic interpretation should be stated separately from the mathemetical interpretation (e.g., the results should be stated in everyday language as well as in terms of properties such as stationarity).

2.

On p. 262 and elsewhere it is said that prior to 1981, the gasoline market in Canada was very stable and that price dispersion was `almost nonexistent`, a very surprising situation. What is the evidence for this?

3. On p. 263, it is said that traditionally Canadian prices were lower than U.S. prices. Why? Evidence? There is the Thompson speech, which says the Canadian government held down prices, but it doesn't say how the government held them down. Were there price controls? Reduced taxes?

4. p. 263. The paper says that before the price war an `unanticipated demand shift' occurred. Was this an increase, or a decrease in demand? (vague terms like `shift' are always bad if you mean increase or decrease in particular.) It seems to be a decline, from the context-- that at the same prices, less gasoline was demanded. I wonder whether there was not an increase in refinery capacity at the same time. And I am still puzzled by how all this fits together, because the Thompson speech says that by 1983 prices were very high in Canada--- *after* the demand slump.

5. Footnote 6. Three of the original 13 stations in the dataset were dropped-- which is a big reduction in sample size. It was good to give short explanations for why they were dropped, but it would have been good to say how these unusual stations behaved during the price war, since they represent about a quarter of the market.

6. Also, even if the model would not explain the 3 dropped stations' behavior because they were different from the rest, those stations' prices would certainly affect the prices of the 10 stations retained. To make things worse, those stations' prices would be correlated with the prices of stations that were retained. Dropping the stations entirely results in a very serious missing variable problem. If, for example, one dropped station went crazy and cut its price 50 percent, and the only other nearby stations cut its price by 10 percent in response, and the station whose behavior the estimated equation is focussing on cut its price by 20 percent, the mis-estimated equation would conclude that our station respond to a price cut with double the size of the first cut-- 20 percnet as a response to 10 percent.

7. Don't use passive writing. On p. 264, we see that stations `are characterized`, that groups `were identifed` and theat behavior `was seen to be`.

8. Why is the data at the start of the price war missing?

9. p. 266. The paper says that net prices are used `because changes in wholesale-purchase price and taxes are immediately passed on to the consumer in this market." That is not at all obvious, and would itself require empirical study. This is particularly so if Canadian gasoline prices did not increase along with world oil prices, as was claimed earlier in the paper.

10. It would have been helpful to have a chart showing the path of taxes, a wholesale price index, and a retail price index.

11. Table 1, and the tables to follow, just give the notation for variables, not the names. This is particularly bad because the paper has many equations with notation scattered all over, making it hard for the reader to find what a particular bit of double-subscripted notation means.

12. There should be a table of summary statistics for the data. This table is shocking by its absence. Five different residual diagnostic tests, but we don't get to see the average price of gasoline!

13. Table III is located far from its mention in the text. Tables should always be on the same page or one page away from where they are mentioned. If that is impossible, then both the table and the text should mention which page the other is on.

14. On p. 269 and elsewhere: Try to avoid referring to coefficients and variable notation in the text without including the English word for them. Otherwise, the reader must constantly flip pages to find out what a sentence means. Don't say ``All four estimated alpha's are positive..."

15. Since Table III apparently is an undependable set of regressions, it might have been better not to have it as a separate table, but instead to present the results in the text, or in a smaller table. Presenting 5 different residual diagnostic tests for a regression that flunk them is overkill.

16. Table IV has the main results. The title should indicate this.

17. The results would be easier to interpret if instead of having one coefficient represent the effect of price changes and another the additional effect of a price increase, the set-up was changed to have one coefficient represent the effect of a price decrease and another the effect of a price increase. The substantive results would be the same, I would think.

(b) Does the paper have any weaknesses so inherent that you could not give advice on how to fix them?

ANSWER. (b) Unfixable flaws.

1. On page 273 Slade says that it is impossible to draw general conclusions "from a single test". That is quite right. And the problem is not quite that only a single test is used, but that it is only a single observation-- one price war in one product in one location. This is a case study-- which is OK, though, since there are likely to be so many differences between markets for different products, times, and locations. Since it is a case study, though, it would have been nice to have more "soft" detail.

2. Figure 1 shows us that in the 90 days of data there are really about 7 periods of change. So really this paper is about how stations responded to 7 instances of price change by other stations, a small sample size.

3. Does the fancy econometric analysis add anything to what we might have gotten by carefully eyeballing the data? Another approach to the same data would have been to look carefully at each of the 7 price change episodes, noting which individual stations led the price changes and which followed quickly or late, with lots of tables showing what happened, but no formal tests. Since we're just looking at one little market anyway, and station-specific effects may be very important, wouldn't this do just as well or better?

7. (12 points) Explain what the following terms mean:

(a) Selling short.

(b) Buying on margin.

(c) Balance sheet

(d) C.P.A.

(e) Consol.

(f) Accumulated depreciation.

ANSWER. “Buying on margin” is the practice of borrowing money from your broker to buy stock, leaving the stock under his control as collateral for the loan.

“Selling short” is the practice of borrowing stock from your broker and immediately selling it, in the belief that the stock price will later fall. Later, you must buy back the stock at the market price to repay your broker.

The “coupon rate on a bond” is the percentage of the principal that the bond pays out each year. This is to be distinguished from the yield on the bond, which is the percentage of the current price that the bond pays out each year.
A phrase such as "the bond's interest rate" is vague and might refer to either one of these.

A Balance sheet is a statement of the assets and liabilities of a company at one point in time.

A C.P.A. is a certified public accountant,

A Consol is a perpetuity, a security that pays a fixed amount each year forever.

Accumulated depreciation is the sum of past values of depreciation. Depreciation is the subraction from an asset’s value to account for its decline in value over time.

8. (10 points) Consider the following game. Nature chooses 90 percent of workers to be "steady," with productivity p=x, and 10 percent to be "wild," with productivity p=x-y.
9.

2. A worker decides whether to marry or not. Marriage adds utility u=m for a steady worker and utility u=-z for a wild worker.

3. The employer, observing whether the worker is married but not whether he is wild, offers him a wage w_m or w_u in competition with other employers, depending on whether he is married or not. The employer has no intrinsic reason to care whether the worker is married or not. Wild workers are less productive, but whether they are married has no effect on their productivity. The only significance of marriage for the employer is its informational value as a signal of steadiness.

What will the equilibrium be if $z>y$?

ANSWER. There is only a single equilibrium. If z is large enough (namely, greater than y), the employer will pay wages of $w_u=x-y$ and $w_m=x$, the steady worker will get married, and the wild worker will stay single.

9. (10 points) The table below shows the payoffs for the following game. Sally has been hired by Rayco to do either Job 1, Job 2, or to be a Manager. Rayco believes that Tasks 1 and 2 have equal probabilities of being the efficient ones for Sally to perform. Sally knows which task is efficient, but what she would like best is a job as Manager that gives her the freedom to choose rather than have the job designed for the task. The CEO of Rayco asks Sally which task is efficient. She can either reply ``Task 1,'' ``Task 2,'' or be silent. Her statement, if she makes one, is an example of ``cheap talk,'' because it has no direct effect on anybody's payoff.

	Sally's Job		
	Job 1	Job 2	Manager
Task 1 is efficient (.5)	2,5	1,-2	3,3
Sally knows			
Task 2 is efficient (.5)	1,-2	2,5	3,3

Payoffs to: (Sally, Rayco).

(a) If Sally did not have the option of speaking, what would happen?

ANSWER. Rayco would make her a Manager. Rayco's payoff is 3 then, but a deviation to either Job 1 or Job 2 would yield a payoff of $.5(5) + .5(-2) = 1.5$. Sally has no choices to make.

(b) There exist perfect Bayesian equilibria in which it does not matter how Sally replies. Find one of these in which Sally speaks at least some of the time, and explain why it is an equilibrium. You may assume that Sally is not morally or otherwise bound to speak the truth.

ANSWER. The key to answering this question and part (c) is to know what a perfect bayesian equilibrium is: a strategy for each player, plus any out-of-equilibrium beliefs that are needed. Someone who remembers that a strategy must specify what Sally does in each of the two states of the world and what Rayco does in response to each of Sally's three possible actions is a long ways towards answering the questions correctly. Here, try the following equilibrium:

Sally: Always say ``Task 1.'' Rayco: Give Sally the job as Manager, regardless of her message. Out-of-equilibrium belief: Rayco thinks the probability that Task 1 is efficient is .5 if Sally says Task 2 or is silent.

Sally's payoff is 3, and she cannot change it by deviating. Rayco's payoff is 3, but a deviation to either Job 1 or Job 2 would yield a payoff of $.5(5) + .5(-2) = 1.5$.

This is an example of a ``babbling equilibrium," so called because the uninformed player treats the informed player's cheap talk as meaningless babbling.

(c) There exists a perverse variety of equilibrium in which Sally always tells the truth and never is silent. Find an example of this equilibrium, and explain why neither player would have incentive to deviate to out-of-equilibrium behavior.

ANSWER. Sally: Say Task 1 if Task 1 is efficient. Say Task 2 if Task 2 is efficient. Rayco: If Sally says Task 1, give her Job 1. If Sally says Task 2, give her Job 2. If Sally is silent, give her Job 1. Out-of-equilibrium belief: If Sally is silent, then Task 1 is efficient.

Sally will tell the truth because if she deviates and the wrong task is assigned, her payoff will be 1 instead of 2. In particular, if she deviates and is silent, she will be given Job 1. Rayco has no incentive to deviate, because given that Sally always tells the truth, Rayco's payoff would fall from 5 to -2 from a deviation. If Sally is silent, which never happens in equilibrium, then Rayco's belief requires that Rayco give her Job 1 in order to maximize Rayco's payoff.

This out-of-equilibrium belief is not particularly plausible, and Farrell and Rabin use this as an example of an implausible equilibrium. It is good for learning how to describe equilibria, though!

10. (10 points) A manager chooses effort level e to maximize his utility, which is $w - e^2/2$. He could have utility of u_0 in an alternative job.

(a) If the owners of the company can perfectly observe the manager's effort, what wage contract would they use?

ANSWER. They would use a contract which pays him, if he exerts the optimal effort level, w high enough that he will achieve a utility level of u ,which is $u_0 - e^{*2}/2$, and otherwise pay him 0.

(b) Suppose the owners observe only their revenue, which is $m = e + v$, where v is a random variable with mean zero. They are risk neutral. They have decided to pay the manager a lump sum of A and a reward of B per unit of revenue. If they do so, what effort level will the manager choose?

ANSWER. The manager will maximize $E(w - e^2/2) = E(A + Be + Bv - e^2/2) = A + Be - e^2/2$, so he will choose e such that

$B - e = 0$, and $e = B$.

c) What levels of A and B should the owners choose?

ANSWER. Expected owner profit is $m - A - mB$, or $e + v - A - eB - vB$. Substituting in the manager's choice of effort, $e = B$, yields profit of
$B + v - A - B^2 - vB$.

The expected value of this, $B - A - B^2$, is maximized subject to

$E(A + mB - e^2/2) >= u_0$
or
$A + B^2 - B^2/2 >= u_0$

This will be binding, so

$A = u_0 - B^2/2$

Substituting this into the profit amounts to the shareholders maximizing $B - (u_0 - B^2/2) - B^2$.

The first order condition is $1 + B - 2B = 0$, so $B = 1$.

Thus, $A = u_0 - 1/2$

11. (10 points) Consider the following auction game:

(0) Nature chooses the value of a vacuum tube being auctioned. The value to Smith of the tube is either 40 or 120 with equal probability, unobserved by either Smith or Jones. Jones's value is either 0 or 100 with independent equal probability, observed by Jones but not Smith. All players are risk neutral.

Start the auction at minute m=1 .
(m.1) Smith decides whether or not to acquire perfect information on his value at cost c=5 .
(m.2) At cost s=1 , Smith may submit a bid ceiling of S to E-Bay.
(m.3) At cost s=1 , Jones may submit a bid ceiling of J to E-Bay.
(m.4) Nonstrategic bidders offer a new bid of b* drawn uniformly from [0, 90]. If this is bigger than the existing biggest nonstrategic bid of N , then b* becomes the new N .
(m.5) E-Bay puts in a bid of b_j =Min (N+5, S+5 , J) for Jones.
(m.6) E-Bay puts in a bid of b_s=Min (N+5, J+5 , S) for Smith.
(m.5) Return to moves (m.2), (m.3), and (m.4) and Smith, Jones, the nonstrategic bidders, and E-Bay repeat their actions. On returning here, go to (m.6).
(m.6) Start the next minute, setting m=m+1 . Return to (m.1) so Smith can decide again whether to acquire information. Continue this process until m= 360, the last minute of the auction.
(7) The highest bidder wins the auction, pays his bid, and gets the vacuum tube.

A winner's payoff is his value for the vacuum tube minus his bid minus T times the number of bid ceilings he submitted minus I if he collected information on his value. A loser's payoff is 0 minus T times the number of bid ceilings he submitted minus I if he collected information on his value.

QUESTION: What is the equilibrium?

ANSWER. This was complex question, so I did not expect you would have time to finish it; I just wanted you to start solving the equilibrium and see how far each of you got. The rest of the test was hard enough that nobody got very far, though.

An equilibrium consists of a strategy for Smith and a strategy for Jones. Consider Jones first. If his value is 0, he will never submit a bid. If his value is 100, he would be willing to win with any price less than or equal to 100. He knows that if Smith's value is 120, and Smith knows that, then Smith will win. He also knows that if Smith's value is 40, he, Jones, will win at a value of either N or N+5.

Now consider Smith. Smith does not know his own value, and must decide whether and when to pay 5 to discover it. He does not care about whether his actions reveal information to Jones, since if Smith's value is 40 Jones will win anyway and if Smith's value is 120 Jones will lose. Smith should put in a bid ceiling of J=40 immediately, since he is willing to pay at least that, and it might be that Jones's value is 0 and no nonstrategic bidder bids over 40.

To decide whether it is worthwhile for Smith to discover his value, we must think about his payoff if he discover it and if he does not. If he does not know his value, what is the most he should bid? If he bids X and wins, his expected payoff is

$-1 -X +.5(40) + .5(120) = -X+ 79.$

Thus, he should not bid more than 79 if he does not know his value. Suppose he knows that Jones's value is 100. Then if he pays 5 to discover his value, his expected payoff is

$-5 + .5(0) + .5(120-100-1) = 4.5,$

because with 50 percent probability he discovers that his value is 120 and so he will be willing to overbid Jones.

Suppose, however, that Smith knew that Jones's value was zero. Then he would want to wait until the nonstrategic bid exceeded 40 and at that point consider whether to acquire information. In fact, he would want to wait until his last opportunity to acquire information. Suppose the nonstrategic bidder's maximum bid is Y > 40. Then Smith's expected payoff if he acquires information is

$-5+ .5(0) + .5(120-Y-5-1) = 52 –Y/2$

This makes X= 104 the biggest value of Y that would make acquiring information profitable compared to just bidding S=40.

The payoffs from bidding above 40 without information and bidding with information are equal if the maximum nonstrategic bid is Z such that $–Z+79 = 52 –Z/2$, so $27 =Z/2$ and Z=54 If the biggest nonstrategic bid is below 54, Smith might as well go ahead and match it without acquiring information. If the biggest nonstrategic bid is above 54, Smith should acquire information and then decide whether to submit S=120 or drop out.

This situation is simplified by the very large number of periods. Since there are 360 draws for the nonstrategic bid, it is almost certain that the biggest nonstrategic bid will be above 80, and in fact it will probably be close to 90. Thus, Smith's should acquire information at once, so that he does not have to bid 5 more than the

highest nonstrategic bidder. The payoff from this strategy will be approximately (not exactly, because the highest nonstrategic bid will almost certainly be less than 90, though very close)

$-5 + .5(0) + .5 [120- .5(90) - .5(100)-1]] = -5 +.5[120- 96] = 7.$

If he were to deviate by not acquiring the information at all, he would not bid more than 79, and his payoff would be close to 0. If he were to deviate by waiting until after the nonstrategic bids, he would simply have to pay 95 instead of 90, and his payoff would also decline. So Smith's equilibrium strategy is to immediately purchase information and then submit S as either 40 or 120.

Since Smith's equilibrium strategy does not depend on his beliefs about Jones, Jones can behave nonstrategically. He should immediately submit either J=0 or J=100, depending on his type.

The game would look quite different if there were only a small number of nonostrategic bids—say, 3 of them. Then we could not deduce that the maximum nonstrategic bid would be above 80, and, in fact, it might not even be above 54. Then, it might happen that Jones would delay bidding until the very end of the game, so as not to provoke Smith into acquiring information.