Should Candidates Flip a Coin if the Difference in Their Votes is Small?

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We are constructing rules for elections between a dishonest candidate and an honest candidate. Denote the dishonest candidate's margin of votes (votes for him minus votes for the honest candidate) by m, his margin of legal votes by x, and the number of illegal votes by N. Both m and x can be negative, indicating a positive margin for the honest candidate, and $m = x + N$.

Let x be a continuous variable distributed by density $f(x)$ with cumulative density $F(x)$.

(A1) The true winning margin density $f(x)$ is strictly increasing in the range $[-2N, 0]$.

The victory rule takes the form $V(m) = p$, where m is the dishonest candidate's margin and p is his probability of victory given that margin.

Society's objective is to maximize the probability of a legitimate victory, $L = 1$, defined as the candidate with the most legal votes being declared the victor.

Symmetry Requirement. If $V(m) = p$, then $V(-m) = 1 - p$.

The Full-Information Rule. $V = 1$ if $m - N \geq 0$; and V= 0 otherwise. (violates symmetry)

The Conventional Rule. $V = 1$ if $m \geq 0$; and V= 0 otherwise.

This is a special case, with $T = 0$, of the following:

The Coin Flip Rule. $V = 1$ if $m \geq T$; $V = 0$ if $m \leq -T$; and $V = .5$ otherwise.

The Reversal Rule. $V = 1$ if $m \in [-N, 0]$ or $m > N$; $V = 0$ otherwise.

The General Rule. $V = z$ if $m \in [-N, 0]$; $V = 1$ if $m > N$;$V = 1 - z$ if $m \in [0, N]$; $V = 0$ if $m < -N$.

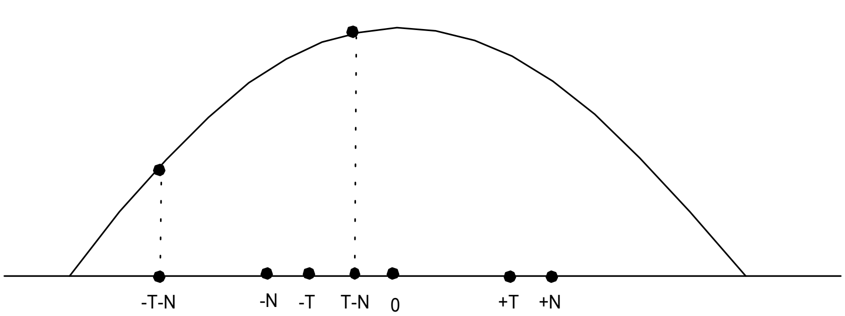

Figure 3

A Model with Endogenous Vote Buying

Let the legitimate vote margin of the dishonest candidate, x, be distributed by a symmetric and unimodal density $f(x)$ with cumulative density $F(x)$. The dishonest candidate, however, begins with an advantage of x_0 legal votes, so the modal outcome in the absence of illegal votes is that he wins by a margin of x_0, or, in the notation, $F(-x_0) = .5$. We allow x_0 to be negative.

Each side can try to buy votes. Candidate 1 (the dishonest candidate) starts out with an advantage of $\underline{N} > 0$ in illegitimate votes. To shift the balance of illegitimate votes, N, the two candidates spend c_1 and c_2. The result is

$$N = \underline{N} + h(c_1) - \alpha h(c_2), \tag{1}$$

where h is increasing and strictly concave, and $\alpha \leq 1$ is a shift parameter representing a possible disadvantage of Candidate 2 in buying votes.

The first order condition for Candidate 1 is

$$\frac{d\pi_1}{dc_1} = -f(-N)h' + f(-N-T)h' + f(T-N)h' - 1 = 0 \tag{2}$$

I would like to answer the following questions:

1. Will a candidate spend more on illegal votes if he has a cost advantage in doing so?
2. Will a candidate spend more on illegal votes if he starts with an advantage in legitimate votes?
3. Does the optimal reversal rule set the threshold T at less than, equal to, or greater than the number of illegal votes chosen in equilibrium?

I might later add that with probability β a candidate does not buy illegal votes. I will have to change all the notation in that draft, since the idea of this is that with probability β a candidate is honest. Adding this uncertainty will make the coinflip rule less attractive.