Excessive Productive Search

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Abstract

Even if (a) search produces real goods by a process with constant returns to scale, and (b) searchers behave atomistically, the equilibrium amount of search can still be excessive from a social point of view. Imposing taxes on search can raise welfare by reducing the number of searchers, despite the transition costs incurred in the move from the untaxed state.

I think I was persuaded by a good referee from Economic Inquiry to abandon this paper.

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1 Introduction.

The equilibria of search models are frequently inefficient, since efficiency usually requires that the actions of an agent be taxed or subsidized if they affect the opportunity sets of other agents, something noted in Mortensen (1982). Depending on the particular model, the amount of search can be either inefficiently low or inefficiently high. It is already well-known that the amount of search is inefficiently high under the conditions of queuing, congestion, and racing. In queuing, the quantity of the good being searched for is fixed, and a longer queue of searchers does not raise the quantity discovered (see, for example, Barzel[1974] on rent control). In congestion, the search process exhibits industrywide decreasing returns to scale, so that adding a new searcher inflicts a real externality on existing searchers (see Dasgupta & Heal [1979] p.55, on fishing). In races— of which patent races are the best known examples— searchers compete in a tournament in which the winner is the only player whose payoff is positive (see Barzel [1968])[1].

This note presents an example of a model in which search is excessive for none of these reasons. Rather, excessive search occurs because the objects of the search are valuable and limited in quantity, leading to competition among searchers that wipes out the value of the rents. A large number of players search for a large but finite number of valuable objects. The stocks of both players and objects are continually replenished, so equilibrium takes the form of a steady state quantity of searchers and objects searched for. It will be shown that the amount of search is excessive in the sense that government policies which shift the market to a new steady state increase economic efficiency, even taking into account the cost of the transition path. Since search is productive, the result is not due to queuing, and since each searcher's instantaneous probability of discovery does not depend on the number of searchers, congestion is not relevant either. Nor is the conclusion due to searchers hurrying to defeat other searchers as in a patent race. In this model, the searchers do not act strategically, and they do not choose the intensity of their search.

The particular context we will use for the search model is a labor market in which "ordinary" jobs are freely available to workers without search, but "good" jobs require search and are limited in number. New unfilled "good" jobs appear regularly over time, and an endogenous number of workers find it individually rational to search for them.

The model is similar to Harris and Todaro (1970), which seeks to explain rural migration to cities. Peasants flock to the city to search for manufacturing jobs, even though not all can succeed and welfare is higher when more peasants stay on the farm. The Harris & Todaro model, however, is an example of queuing: search is unproductive because more migration does not mean more good jobs taken.

A reason for using the labor market as an application is that one argument for unemployment insurance is that an unemployed worker spends too little time searching for a new job unless he is subsidized, a conclusion obtainable from any of several models. One assumption that produces this result is that the worker cannot borrow, so his consumption needs cause him to take a job after too little time spent searching. Such a worker would support a social insurance scheme that pays him while he is searching and taxes him when he is working. Another assumption yielding insufficient search is that each worker's search generates positive externalities, so that agents benefiting from the externalities would be willing to subsidize him. Such externalities arise, for example, when one worker's search improves the wage distribution for the others, or when firms and workers search for each other and the search of one side of the market helps the other side (Diamond-Maskin [1979]).

The present model points towards the opposite conclusion: that job search is excessive, and should be taxed rather than subsidized. I am more concerned with making a point about search than about labor markets, but I hope to have added another element to the discussion of job search. A more complete model of the labor market would describe the employers' side of the market in greater detail than I do, and instead of making the arrival of unfilled good jobs exogenous would explain why good jobs and ordinary jobs coexist. Their coexistence could arise in any of a variety of ways: government regulation, union strength, the agency problem (Shapiro-Stiglitz[1982]), or differences in search costs (Salop-Stiglitz[1977]). Here we will not be concerned with that question, but if the exogeneity of the job distribution is disturbing, the reader may wish to imagine applying the model to search for innovations or minerals, contexts in which exogeneity on the supply side is innocuous.

2 The Model.

Workers are identical, price-taking, and risk-neutral. A worker can either accept an "ordinary" job and receive the wage flow w_o without searching, or join the U_t workers who search for J_t unfilled "good" jobs paying w_g, where $w_g > w_o$. We will take both J and U to be continuous variables, in accordance with the assumption that workers behave atomistically. Time is continuous, the discount rate is r, and the flow cost per unit time spent searching is c.

If there is to be a steady state, new unfilled good jobs must appear in order to replace those removed by successful searchers, and we denote by α the flow rate of appearance of unfilled good jobs. New workers must also appear in order to replace those who take jobs, and we assume that their rate of creation exceeds α so there continue to be enough unemployed workers to fill the new jobs. Provided the rate of worker creation is high enough to meet this assumption, its precise level is unimportant. The values of J and U are endogenous, and depend, as described below, on the flow rate of job creation, the wage differential, and the search cost.

A searching worker receives offers of good jobs according to a Poisson process with arrival rate $F(J_t)$, which is the number of good jobs he would expect to find by searching without stopping for one unit of time if the value of J did not change during that interval. The Poisson search specification is commonly used in search models (see, for example, Loury[1979]). The number of searching workers, U has no direct effect on the arrival rate for an individual worker, $F(J)$. The externality we will discover is not simple congestion, as it would be if F were a decreasing function of U. Assume that $F' > 0$ and $F'' < 0$, i.e. a worker finds a good job more quickly when more are available, but with diminishing returns to the stock of jobs.

The condition for market equilibrium is that the expected benefit from searching for a good job equal the certain low wage foregone by the searcher. The expected benefit is the chance of finding a good job, $F(J)$, times the permanent gain from having a good job, $(w_g - w_o)/r$, minus the search cost *c*. Equilibrium requires that at each instant t,

$$w_o = F(J_t)\left(\frac{w_g - w_o}{r}\right) - c. \tag{1}$$

Equation (1) pins down the stock of unfilled good jobs uniquely. If J_t is too large to solve equation (1) all workers search; if too small, none do.

A steady state is an equilibrium in which the stocks of unemployed workers and unfilled good jobs are constant. Because the number of good jobs found is stochastic, the assumption that J and U are continuous variables is very helpful: the aggregate quantities of workers searching and jobs remaining are certain, and we can dispense with the expectation operators needed otherwise. The rate of creation of new jobs must equal the rate of job-taking for the number of unfilled good jobs to remain constant. Since the number of searching workers is U_t and the probability of a worker finding a good job is $F(J_t)$, the condition for a steady state is

$$\alpha = U_t F(J_t). \tag{2}$$

The market equilibrium is a steady state with the number of unfilled good jobs given by condition (1). The market equilibrium must be a steady state, because otherwise the number of unfilled good jobs would grow or shrink until equation (1) was violated, and although the market equilibrium is only one of many (J,U) steady state combinations, it is the only laissez faire steady state in which workers optimize.

3 Suboptimality of the Market Equilibrium.

A simple comparison of steady states is not interesting from a policy point of view. The optimal stock of unfilled good jobs is infinite, since when there are more unfilled good jobs, workers find good jobs more quickly. The desirability of a large stock of unfilled jobs does not imply, however, that moving away from the market equilibrium is desirable, because increasing the stock of unfilled jobs is costly. The problem is analogous to choosing the capital stock in a growth model; a larger capital stock is better, but sacrifices along the transition path can make it suboptimal to increase savings to enlarge the capital stock. The proper welfare question is whether the market should move from one steady state to another. I will show that the market equilibrium is suboptimal in the sense that a Pareto-superior policy is to move as rapidly as possible to a steady state with more unfilled good jobs and less unemployment.

We will begin with the industry at the market equilibrium, denoted (J_c, U_c), and consider moving to another steady state. Moving to a steady state with more workers searching cannot be efficient, because the low wage foregone by the marginal searcher is greater than his expected gain from search. At the level of individual rationality he should not be searching, and there is no compensating social gain.

Discovering whether it is optimal to move to a steady state with fewer workers searching requires more work.

Proposition 1: *Worker welfare is maximized by moving from the competitive steady state (J_c,U_c) to a steady state (J^*,U^*) in which $J^* > J_c$ and $U^* < U_c$.*

Proof: Choosing the unemployment level over time is a dynamic optimization problem, which can be solved using the maximum principle. The control variables are the $\{U_t\}$, and the state variable is $\{J_t\}$.

The objective functional is the integral over time of the net gain from workers taking good instead of ordinary jobs, minus search costs and lost wages while they search. Each of the U_t workers searching at time t incurs a flow loss of $(w_o + c)$ in foregone wages plus search cost, but finds a good job, a windfall of $\frac{(w_g - w_o)}{r}$, with probability $F(J_t)$.

The initial value of J_t is given by the market equilibrium. New arrivals increase the number of unfilled jobs at rate α and searching workers decrease it at rate $F(J_t)U_t$. A constraint is that U_t must be greater than zero and less than the total number of workers in the industry, which we denote by L_t. The problem is therefore to

$$(3) \quad \underset{\{U_t\}}{Maximize} \int_0^\infty \left[U_t F(J_t)\left(\frac{w_g - w_o}{r}\right) - U_t(w_o + c)\right] e^{-rt} dt$$

$$\begin{array}{lll} such \;\; that & & \\ (3a) & J_0 & = J_C \\ (3b) & \dot{J} & = \alpha - U_t F(J_t) \\ (3c) & 0 \leq U_t \leq L_t. & \end{array}$$

The Hamiltonian is

$$H(U_t, J_t, \lambda_t) = \left[U_t F(J_t)\left(\frac{w_g - w_o}{r}\right) - U_t(w_o + c)\right] e^{-rt} + \lambda_t[\alpha - U_t F(J_t)] \tag{4}$$

Since the Hamiltonian is linear in the control variable, the maximum conditions are both necessary and sufficient, and the problem has a "bang-bang" solution.[2] Until the new steady state is reached, unemployment is either the minimum or the maximum possible under the constraint, so instead of the usual marginal conditions for an interior solution the optimal U_t must satisfy the following:[3]

$$(5a) \quad U_t = L_t \qquad if \quad F(J_t)\left(\frac{w_g - w_o}{r}\right) e^{-rt} > (w_o + c)e^{-rt} + \lambda_t F(J_t)$$

$$(5b) \quad 0 \leq U_t \leq L_t \qquad if \quad F(J_t)\left(\frac{w_g - w_o}{r}\right) e^{-rt} = (w_o + c)e^{-rt} + \lambda_t F(J_t)$$

$$(5c) \quad U_t = 0 \qquad if \quad F(J_t)\left(\frac{w_g - w_o}{r}\right) e^{-rt} < (w_o + c)e^{-rt} + \lambda_t F(J_t)$$

Because the costate variable λ_t is the value of relaxing a constraint it is non-negative. From equation (1), we know that in the market equilibrium

$$F(J_t)\left(\frac{w_g - w_o}{r}\right) = (w_o + c).$$

We therefore know that (5a) is not applicable at the market equilibrium, because it is never true that

$$0 > \lambda_t F(J_t).$$

We can also show that condition (5b) is not applicable at the market equilibrium, by showing that λ_t is strictly positive. Even without analysis we might expect this to be the case: the costate variable tells the value of relaxing the constraint (3b), and the value of increasing the creation flow of new jobs is greater than zero, so λ_t ought to be strictly positive.

The maximum principle generates not only the optimality conditions (5a) through (5c), but also a costate equation which gives the rate of change of the costate variable λ_t. We can use the costate equation (6) to show that λ_t is strictly positive.

$$\tfrac{d\lambda}{dt} = \dot{\lambda}_t = -\tfrac{\partial H(U_t, J_t, \lambda_t)}{\partial J_t} = -U_t F'(J_t)\left(\tfrac{w_g - w_o}{r}\right) e^{-rt} + \lambda_t U_t F'(J_t), \qquad (6)$$

or, after rearranging,

$$\dot{\lambda}_t = -U_t F' \left[\left(\frac{w_g - w_o}{r}\right) e^{-rt} - \lambda_t\right]. \qquad (7)$$

Consider equation (7) as applied to the market equilibrium value U_c, which is positive. If λ_t were zero, then by equation (7), $\dot{\lambda}_t$ would be negative, which would shortly make λ become negative. λ is nonnegative because it is a costate variable, so $\lambda_t = 0$ has led to a contradiction.

Since $\lambda > 0$ under the market equilibrium, condition (5c) applies initially, and the optimal level of search is zero. The stock of unfilled good jobs increases because of new arrivals, and unless J continues to grow forever, which it cannot under an optimal policy, eventually (5b) becomes the applicable condition. Since the optimality conditions for problem (3) are both necessary and sufficient, it is enough to show that there is a policy that satisfies conditions (5b) and (5c), and the costate equation (7). The policy will be to set unemployment to zero until (5b) becomes applicable, and to keep unemployment at a constant U^* thereafter.

To show that a steady state with (J^*, U^*) satisfying (5b) is consistent with the costate equation (7), let us begin by eliminating λ_t from equation (5b), which we can rewrite as

$$\lambda_t = e^{-rt}\left[\frac{w_g - w_o}{r} - \frac{w_o + c}{F}\right]. \tag{8}$$

The derivative of (8) with respect to time is

$$\dot{\lambda}_t = -re^{-rt}\left[\frac{w_g - w_o}{r} - \frac{w_o + c}{F}\right] - \frac{e^{-rt}(w_o + c)F'\dot{J}_t}{F^2}. \tag{9}$$

If the market is at a steady state, then $\dot{J}_t = 0$, and equation (9) becomes

$$\dot{\lambda}_t = -re^{-rt}\left[\frac{w_g - w_o}{r} - \frac{w_o + c}{F}\right]. \tag{10}$$

We can equate (10) with (7) to obtain

$$-e^{-rt}\left[\frac{w_g - w_o}{r} - \frac{w_o + c}{F}\right] = -U_t F'\left[\left(\frac{w_g - w_o}{r}\right)e^{-rt} - \lambda_t\right]. \tag{11}$$

Substituting for λ_t from equation (8), we obtain

$$-e^{-rt}\left[\frac{w_g - w_o}{r} - \frac{w_o + c}{F}\right] = -U_t F'(J_t)\left(\left(\frac{w_g - w_o}{r}\right)e^{-rt} - e^{-rt}\left[\frac{w_g - w_o}{r} - \frac{w_o + c}{F}\right]\right), \tag{12}$$

which simplifies, using the steady state condition (2), to

$$\alpha F'(J)\left(\frac{w_o + c}{F(J)}\right) - r\left[\left(\frac{w_g - w_o}{r}\right)F(J) - (w_o + c)\right] = 0. \tag{13}$$

At the value J_c the left-hand-side of (13) would be positive, because its first term is positive and its second term equals zero by (1). If the zero unemployment policy of (5c) is followed, the value of J increases, decreasing the magnitude of the first term and increasing the magnitude of the second term of (13). Eventually (13) is satisfied. Thus the policy of setting unemployment to zero until (13) is satisfied, and then maintaining it at the level which keeps J at the level defined by (13) is an optimal policy. Since J^* is greater than J_c, equation (1) tell us that U^* is greater than U_c. $\|$

The policy suggested in Proposition 1 can be explained more intuitively. Setting U greater than U_c would be foolish because that both increases the total costs of search and runs down the stock of good jobs. Setting U to zero initially is optimal because the decline in the value of good jobs currently found is outweighed by the increase in the stock of unfilled good jobs to be searched for in the future. Eventually the marginal benefit of this investment in the stock of good jobs becomes less than than the marginal cost of the delay in harvesting them, and the unemployment rate rises to U^*.

The reason the competitive equilibrium is inefficient is that when a worker takes a job, he ignores the harm he has done to other workers. This is a real externality: once that good job is gone, the other workers must search longer if they are to find one. By forcibly restricting search, and keeping the stock of good jobs low, the government keeps the expected cost of finding a good job lower.

The costate variable λ_t has an economic interpretation. λ_t is the marginal benefit of an extra unfilled job at time t, and in the optimal steady state its value is given by equation (8).

$$(8) \quad \lambda_t = e^{-rt}[\tfrac{w_g - w_o}{r} - \tfrac{w_o + c}{F}].$$

When an extra job appears, the optimal policy is to let workers search for and take that job, reducing J_t to J^*. Only the direct benefit, the expected value of good job minus the cost of search, matters to the value of the costate variable. Note also that because of the term e^{-rt}, λ_t falls at the interest rate: an extra unfilled job is worth more now than in the future.

4 Government Remedies.

Instead of directly choosing the unemployment level the government can use market incentives to reach the optimum. Either of two policy instruments will work: a one-time job tax τ_1 on any worker who takes a good job, or a search tax flow τ_2 which any worker must pay as long as he searches. τ_1 and τ_2 are picked to make the market equilibrium condition (1), modified by the taxes, identical to the social optimum condition (5b). Equation (1) becomes

$$(1') \quad F(J_t)[\tfrac{w_g - w_o}{r} - \tau_1] - (w_o + c + \tau_2) = 0.$$

The taxes τ_1 and τ_2 should be chosen to make (1′) equivalent to the social optimum condition (5b), which can be rewritten, letting $\lambda^* = \lambda_0 e^{-rt}$ denote the value of the costate variable in the optimal steady state, as

$$F(J_t)\left(\frac{w_g - w_o}{r}\right)e^{-rt} - (w_o + c)e^{-rt} - \lambda_0 e^{rt} F(J_t) = 0. \qquad (14)$$

If we choose either of the pairs of taxes

$$\{\tau_1 = \lambda_0, \tau_2 = 0\},$$
or
$$\{\tau_1 = 0, \tau_2 = \lambda_0 F(J^*)\},$$

or any of a continuum of intermediate tax packages, then equations (1′) and (14) are equivalent at the optimal steady state. If $J_t > J^*$, then the left-hand-side of (1′) is positive and all workers search, while if $J_t < J^*$, no workers search, just as conditions (5a) and (5c) require.

If τ_1 and τ_2 are picked properly, a market starting at the market equilibrium converges to the optimum. Unemployment equals zero until (5b) is satisfied and continues at the steady state value thereafter. The job tax τ_1 undoes the externality a worker inflicts on other workers when he takes a job. The search tax τ_2 achieves the same effect by equalling not the ex post injury, but the expected injury which the worker's search causes other workers.

NOTES.

1. The term "oversearching" has been used by Kenney & Klein (1983) to describe situations of adverse selection and quality investigation. They use the example of a diamond market in which it is more efficient to give buyers a random draw from packets of diamonds of varying quality, rather than letting them inspect each packet. Such inspection is different from the kind of search discussed in this paper, and their work is best classified under the heading of adverse selection.

2. See Intriligator (1971), p. 366.

3. No transversality condition is specified. Our knowledge that optimal unemployment is never greater than U_c serves as one, telling us that the steady state of (5b) is the terminal state.

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bf QUESTIONS.

1. Should I send this to *Economic Inquiry*?
2. Ought I to call this a note in my cover letter?
3. Is there an interesting point that I am not emphasizing?
4. Is the meaning of Proposition 1 clear?
5. Are there individual sentences that are badly written?
6. Do any of these assumptions need more justification:
   - Exogenous good and ordinary wages.
   - The Poisson process for the arrival of good wages.
   - $F'' < 0$.
   - Specifying J and U as continuous variables.

NOTES.

MAke it very clear what the methodology is, so even the stupidest referee can understand.

Also: parallel efforts to find an invention. (which would disappear with licensing).

Another purpose: this shows how rent-seeking can be a steady state.

(or, of the rent control– what if search did open up a few more apartments?– the effect would be swamped bythe excess search. No– this is congestion.)

I could have a model of a patent race where ther eis one discovery to be made, and it ought to be made in 1990, but it is slightly better to make it in 1980, if one forgets costs. Even atomistic competitors would oversearch.

I should be able to distinguish my model from this. Maybe I could include it as an exmaple, even. In my model there are N goods, and not all of them are discovered.

In this model, the good jobs could be indeed good, high MP jobs, with high wages to attract more labor.